2. Consider two subspaces $E_1$ and $E_2$ of $E$. Establish the relation

$$E_i^{\perp} := \mathrm{Anh}(E_i)$$

$$(E_1 + E_2)^{\perp} = E_1^{\perp} \cap E_2^{\perp}.$$

3. Given a vector space $E$ consider the mapping $\Phi: E \to (E^*)^*$ defined by

$$\Phi_a(f) = f(a) \quad a \in E, f \in E^*$$

Prove that $\Phi$ is injective.

4. Suppose $\pi: E \to E$ and $\pi^*: E^* \leftarrow E^*$ are dual mappings. Assume that $\pi$ is a projection operator in $E$. Prove that $\pi^*$ is a projection operator in $E^*$ and that

$$\mathrm{Im}\,\pi^* = (\ker \pi)^{\perp}, \qquad \mathrm{Im}\,\pi = (\ker \pi^*)^{\perp}.$$

Conclude that the subspaces $\mathrm{Im}\,\pi$, $\mathrm{Im}\,\pi^*$ and $\ker \pi$, $\ker \pi^*$ are dual pairs.