Subjects for Master Thesis (AW)

1 Real Bott periodicity. Modify the proof of Bott periodicity given in B. Harris, *Bott periodicity via simplicial sets* to obtain the homotopy equivalence $B(\bigsqcup_{n\in\mathbb{N}} BO(n)) = U/O$.

2 Loop space $\Omega SU(2)$: study geometric properties of the cell decomposition given in general in A. Pressley, *Decomposition of the Space of Loops on a Lie Group*. Possible variants: 1) Apply simplicial methods, 2) Study singularities of cells.
See `http://ssdnm.mimuw.edu.pl/pliki/prace-studentow/st/pliki/km.pdf`

3 We consider the space of configurations of affine hyperplanes in $\mathbb{C}^n$ or $\mathbb{R}^n$ („hyperplane arrangements"). This is a topological space with an action of the group affine transformations. Describe topological properties of orbits. Study the Milnor fibration.

4 Let $X_2(n)$ be the set of $n \times n$ upper-triangular matrices (complex or real) satisfying $A^2 = 0$. The group of all upper-triangular matrices acts on $X_2(n)$ with finitely many orbits. See A. Melnikov,
`http://arxiv.org/pdf/math/0312290v2.pdf`
Study the cohomological properties of that decomposition. Compute so called Thom polynomial of orbits.

5 (Characteristic classes) We study complex manifolds with an action of upper-triangular matrices. We assume that there is a finite number of orbits. Relying on the paper J. Huh,
`http://arxiv.org/pdf/1302.5852v2.pdf` check which characteristic classes of the logarithmic tangent bundle (vector fields tangent to orbits) are represented by effective geometric cycles. There are various possible directions of the work: 1) generalize huh result for equivariant cohomology, 2) give a proof in the language of differential forms.

6 Old list `http://www.mimuw.edu.pl/%7Eaweber/semag/mag10.pdf`

7 Let $E \to B$ be a locally trivial fibre bundle, with the fiber F which satisfies the Hard Lefschetz theorem with respect to a cohomology class $[\omega] \in H^2(F)$. If $[\omega]$ is a restriction of a class $[\widetilde{\omega}] \in H^2(E)$ then $H^2(E) \simeq H^2(E) \otimes H^2(B)$. Deligne has shown that this isomorphism can be made canonical when $[\widetilde{\omega}]$ is fixed. An a elementary proof for $B = \mathbb{CP}^n$ is given in
`http://www.mimuw.edu.pl/%7Eaweber/ps/szamotulski.ps`
Give a prof for an arbitrary base.