

SUBJECTS FOR MASTER THESIS

- (1) *Lifting maps from homotopy colimits.* Let \mathcal{C} be a small category, $F : \mathcal{C} \rightarrow \mathbf{Top}$ a diagram of spaces, and $p : E \rightarrow B$ a fibration. Consider a commutative diagram:

$$\begin{array}{ccc}
 \coprod_{c \in \mathcal{C}} F(c) & \xrightarrow{\coprod g_c} & E \\
 \downarrow \subseteq & \nearrow \bar{g} & \downarrow p \\
 \text{hocolim}_{\mathcal{C}} F & \xrightarrow{f} & B
 \end{array}$$

Develop an obstruction theory for existence and uniqueness of a lifting extension \bar{g} .

- (2) *Directed paths on spheres.* We say that a path α in n -cube I^n is *directed* if all its coordinates are non-decreasing. A path in $S^n = I^n / \partial I^n$ is directed if it is a concatenation of images of directed paths in I^n . Let $\vec{P}(S^n)$ be the space of directed paths. Check if the inclusion $\vec{P}(S^n) \subseteq P(S^n)$ is a homotopy equivalence. Hint: use James construction.
- (3) *Components of directed simplicial complexes* Calculate the category of components of a given directed simplicial set. References:
 K.Ziemiański, *A cubical model for path spaces in d -simplicial complexes.* Topology Appl. 159, No. 8, 2127-2145 (2012). MSC2000: *55U10 68Q99
 L. Fajstrup, E. Goubault, E. Haucourt, M. Raussen, *Components of the fundamental category*, Appl. Categ. Structures 12 (2004) 81108.