

Large Project 2010

This year's Large Project will be a joint project for all students participating in the course. In this project we shall write a "professional" package for evaluating contingent claims on the FX market. The term "professional" means that, contrary to the standard courseworks, this software has to be written for an end-user who uses financial instruments in its everyday operations and is not an expert in numerical analysis and computational finance. This means that we have to feed the system with data which are readily available from Bloomberg or Reuters services. Finally, we have to offer instruments and methods which are in demand, i.e. industry standards and not "white elephants".

The project will be the continuation of the project from the last year. Hence, a number of functions is already available. We shall modify existing functions and write new specific for the FX market.

We shall write the project as a library of functions performing different tasks. We have also to extend the main module (main programme) which manages all functions, supplies data to them, provides the communication between the functions and solves the problem of communication with a future end-user interface. The version of this module written in web-octave is ready and it needs only suitable adjustment to new functionality.

To make the project accessible we will limit our interest to most popular simple instruments: currency options, currency swaps and currency futures. In evaluating derivative instruments we shall explore the Black-Scholes analytic formula but also numerical PDE methods for solving the Black-Scholes equation.

In the course of the project the following tasks are essential:

1. Extension of the existing calendar. For the description of the existing calendar and problems which have to be solved in calendar construction see the description of the calendar functions on the web page and the file `conventions.pdf` which describes conventions used on financial market. For FX transactions the existing calendar has to use simultaneously calendar data for 2 (or more) currencies (financial centers).
2. Processing input data – term structure of interest rates. Now we shall need term structures for several currencies at the same time. Recovering the term structure of interest rates from the prices of traded instruments (LIBOR rates, FRA rates and swap rates) is complicated because it requires full use of the calendar. The existing functions are based on a term structure of discount factors, where discount factors are quoted together with dates to which they refer. We shall use the same structure but now, having calendar functions, we shall produce a function which will calculate discount factors from real market quotations. We need only proper interpolation of rates. One such interpolation is already implemented. We shall enrich it by other methods.
3. Processing input data – term structure of implied volatilities. Available volatilities are implied volatilities for a very limited number of compound options. We have to implement a procedure by which a volatility smile is recovered from these data. This volatility smile is available only for a limited tenor of calendar dates. In addition we have to implement interpolation procedure between available dates.
4. Construction of an European PDE engine – a module for solving the Black-Scholes equation for European instruments with arbitrary payoff function and (almost) arbitrary boundary data together with sensitivity parameters. This module will be used for pricing instruments in more complicated models (CEV, local volatility). The main goal is to construct a code for pricing and hedging variety of barrier options.

5. Construction of an American PDE engine – a module for solving Black-Scholes equation for American (Bermudan) instruments with arbitrary payoff function and (almost) arbitrary boundary data together with sensitivity parameters. This module will be used for pricing and hedging standard American instruments. This can be a difficult task as standard SOR algorithm is not very effective and we have to find a better algorithm for American instruments.
6. Construction of functions calculating prices of different instruments from the Black-Scholes formula or simple parity considerations. This task can be divided on pricing options, futures, currency deposits and swaps.
7. Calibration of local volatility surface from the implied volatility data. This seems to be a difficult task as already two groups in previous years attempted to solve this problem without success.
8. Documentation. It is absolutely essential to describe carefully the theoretical background of every function. But also the documentation of the computer code has to be provided. A particular attention should be put on the structure of the programme (block scheme), the format of input and output data, signaling choices, etc.

The project will consist of the following tasks:

1. Calendar functions.
2. Processing input data — preparing discount factors table.
3. Construction of implied volatility surface.
4. Construction of local volatility surface.
5. Construction of an European PDE engine and its application to pricing of different options (mostly barrier).
6. Construction of an American PDE engine and its application to pricing of different options.
7. Pricing instruments from the Black-Scholes formula, parity identity and time value of money considerations.
8. Pricing complicated instruments using PDE engine.
9. Implementing all functions in web-octave.

Literature comments

- Calendar functions — main code is available, main source of information are web pages, but you can also consult the course coordinators.
- Preparing discount factors table — knowledge for financial engineering, but you can also consult the course coordinators.
- Construction of implied volatility surface — general idea is in the book by Gatheral and papers by Castagna & Mercurio.

- Construction of local volatility surface — a number of different papers provided by the course coordinators.
- Construction of an European PDE engine — knowledge from the course and consultations with the course coordinators. Additional information about pricing of different options can be get from the book by Wystup.
- Construction of an American PDE engine — knowledge from the course and papers provided by the course coordinators.
- Pricing instruments from the Black-Scholes formula — knowledge for financial engineering and from the book by Wystup. List of instruments will be provided by the course coordinators.
- Pricing instruments using PDE engine — list of instruments and additional papers will be provided by the course coordinators.

References

The are two fundamental references about FX market:

1. Alexander Lipton – *Mathematical Methods for Foreign Exchange*, World Scientific 2007.
2. Uwe Wystup – *FX Options and Structured Products*, J. Wiley 2007.

The book of Lipton is more mathematical and the book of Wystup contains many practical informations. Both are in some form available at the course coordinators.

In addition there are plenty of books and papers devoted to specific problems.

1. Yves Achdou, Olivier Pironneau – *Computational Methods for Option Pricing*, SIAM 2005.
2. Jim Gatheral – *The Volatility Surface*, J. Wiley 2006.
3. Justin London – *Modeling Derivatives in C++*, Wiley 2005. (Library)
4. Antonio Castagna, Fabio Mercurio – Consistent Pricing of FX Options, Preprint. (pdf)
5. Antonio Castagna, Fabio Mercurio – The vanna-volga method for implied volatilities, *Risk*, January 2007, 106–111. (pdf)
6. Cheng Wang – Stable Local Volatility Calibration Using Kernel Splines, MSc Thesis, Waterloo 2008. (pdf)
7. Gabriel Turinici – Calibration of local volatility using the local and implied instantaneous variance, *J. Comp. Finance*, **13** No 2 (2009), 1–18. (pdf)
8. Wlodzimierz Walus – On approximation of discount factors and yields. (pdf)