MAGIT Exercises, Series 12

Exercise 1

Let $\mathcal{U}$ be an open covering of Δ^n by $U_i = \{(x_0, ..., x_n) \in \Delta^n : x_i > 0\}$. Show that

$$\check{\mathrm{H}}^m(\mathcal{U}, \mathbb{Z}) = 0$$

for $m > 0$.

Exercise 2.

Let X be an arbitrary topological space. Prove that for all $n > 0$ and for any open covering $\mathcal{U} = \{U_i\}$ such that $U_{i_0} = X$ for some i_0 and for any sheaf of $\mathbb{Z}_X$-modules M we have $\check{\mathrm{H}}^n(\mathcal{U}, M) = 0$.

Exercise 3.

Let X be a topological space and let $\mathcal{U} = \{U_i\}_{i \in I}$ be an open covering of X. Prove that if $\mathcal{F}|_{U_i}$ is flabby for all i then $\mathcal{F}$ is flabby.

Exercise 4.

Let X be a topological space and let $\mathcal{F}$ be a flabby sheaf of abelian groups on X. Prove that for any open covering $\mathcal{U}$ and all $m > 0$ we have $\check{\mathrm{H}}^m(\mathcal{U}, \mathcal{F}) = 0$.

Exercise 5.

Let F be a sheaf of abelian groups on $I = [0, 1]$. Assume that for every open (connected) interval $U \subset I$ the map $F(I) \to F(U)$ is surjective.

1. Prove that if
$$0 \to F \to G \to H \to 0$$
is an exact sequence of sheaves of abelian groups then for every open interval $U \subset I$ the map $G(U) \to H(U)$ is surjective.

2. Prove that $H^i(I, F) = 0$ for $i > 0$.

3. Is F necessarily flabby?