MAGIT Exercises, Series 10

Exercise 1.

Prove that the geometric realization of $\Delta(p)$ is isomorphic to the p-dimensional simplex.

Exercise 2.

Prove that for any simplicial set X and any topological space Y we have

$$\mathrm{Hom}_{\mathrm{Top}}(|X|, Y) \simeq \mathrm{Hom}_{\mathrm{SSet}}(X, S_\bullet Y).$$

Exercise 3.

Let $\varphi : \mathcal{F} \to \mathcal{G}$ be a morphism of sheaves. Show that φ is an isomorphism of sheaves if and only if for all $x \in X$ the induced morphism $\varphi_x : \mathcal{F}_x \to \mathcal{G}_x$ is an isomorphism.

Exercise 4.

Let $\mathbb{Z}_X$ be the sheafification of the constant presheaf $U \to \mathbb{Z}$. Show that $\mathbb{Z}_X$ is isomorphic to the sheaf of locally constant functions in values in $\mathbb{Z}$.

Exercise 5

Let X be a topological space and $i : Y \hookrightarrow X$ a closed subset. Let us set $j : U = X - Y \hookrightarrow X$. Let $\mathcal{G}$ be a sheaf on U and let $j_!(\mathcal{G})$ be the sheafification of the presheaf

$$\mathcal{G}'(V) = \begin{cases} \mathcal{G}(V) & \text{for } V \subset U, \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, we define $i^*\mathcal{F}$ and $j^*\mathcal{F}$ as the sheafification of the presheaves $i^{-1}\mathcal{F}$ and $j^{-1}\mathcal{F}$ defined in Series 5. Show that if $\mathcal{F}$ is a sheaf on X then

$$0 \to j_!(j^*\mathcal{F}) \to \mathcal{F} \to i_*(i^*\mathcal{F}) \to 0$$

is a short exact sequence of sheaves on X.