

GA Series 1

k below is algebraically closed.

Ex. 1.

Let $\varphi : \mathbb{A}^1(k) \rightarrow \mathbb{A}^3(k)$ be a morphism given by $t \rightarrow (t^3, t^4, t^5)$ and let $C = \varphi(\mathbb{A}^1(k))$. Show that C is an affine algebraic set and find $I(C)$.

Ex. 2.

Show that $I(C)$ from the previous exercise is not generated by 2 elements.

Ex. 3.

Show that there exist $f, g \in k[x, y, z]$ such that $C = V(f, g)$, where C is as above.

Ex. 4.

Show that $\det[x_{ij}] - 1 \in \mathbb{C}[\{x_{ij}\}_{i,j \in \{1, \dots, n\}}]$ is irreducible. Conclude that $\mathrm{SL}(n, \mathbb{C})$ is an affine algebraic set. Is $\mathrm{GL}(n, \mathbb{C})$ an affine algebraic set?

Ex. 5.

Let $U = \mathbb{A}^n(k) - \{0\}$ for some $n \geq 2$. Show that (U, \mathcal{O}_U) is not isomorphic to an affine algebraic set in the category of locally ringed spaces.