# 1 - do oddania 4 marca 2020

Name:____________________

**1.1.** Let $G$ be a finite nilpotent group. Does there exist a normal series with quotients being cyclic groups of prime order?

**1.2.** Let $G$ be a nilpotent group. Show that every proper subgroup of $G$ is a proper subgroup of its normalizer in $G$.

**1.3.** Let $p$ and $q$ be prime numbers. Show that the group of order $p^m q$ is solvable. We start with the assertion concerning $p$ – groups. Proof will be by contradiction. Assume that $G$ is the "minimal criminal" i.e. group of smallest order that is not solvable.

I) Remark that $p \neq q$

II) Show that $G$ must be simple.

III) Let $I = P_1 \cap P_2$ be the biggest (in terms of rank) intersection of $p$ - Sylow subgroups. Show that $I \neq \{1\}$.

IV) Let $J =< N_{P_1}(I), N_{P_2}(I) >$. Show that $J$ can not be a $p$ – group (using maximality of $I$ and 1) above). Hence $q \mid |J|$.

V) Let $Q$ be a $q$ – Sylow subgroup contained in $J$. Deduce that every element of $G$ is of the form $xy$, $x \in Q$, $y \in P_1$. Show that fore every $g \in G$, $gIg^{-1} \leqslant P_1$. Show that $G$ is not simple, hence contradiction.

**1.4.** Show that a group of order $p^2q^2$ with $p$, $q$ prime numbers is solvable.