The main goal of the project BOUNDS which we do advertise as a possible academic training opportunity is to develop the theory of stochastic processes whose index sets have no particular structure (for this reason “a random field” is a more suggestive name). The basic questions in the theory concern properties of paths of a given stochastic process. Therefore the study of regularities of paths and estimating distribution parameters such as mean value of the supremum over trajectories are fundamental. The importance of such questions is certified by the new monograph [Ta] of the leading probabilist M. Talagrand, which continues the study on the high dimensional probability started years ago. Among the class of random fields discussed in this book there are a lot of meaningful examples like Gaussian, Bernoulli, canonical, infinitely divisible and empirical processes. Although some progress has been made towards understanding these classes still it is very far from completing the quest.

We do present possible questions that could be the starting point for longer cooperation:

1. **The Ultimate Matching Conjecture**

   The problem explores the question how evenly distributed among the unit hyper-cube are random points drawn from the uniform distribution. For this reason one draws independently a certain number of points in the unite hyper-cube and then with these points fixed once again draws the same number of independent uniformly distributed points. Now the question is to suitably match each point from the first group to a unique point in the second group in order to minimalize distances between the points. There is a well-known Shor's matching theorem which puts some light into the question for the dimension two. There is also an interesting extension of the result known under the name of Ultimate Matching Conjecture which seems to be a challenging question in the theory of fining upper bounds for stochastic process.

   **Goal:** Decide whether or not the ultimate matching conjecture is true.

2. **Study the canonical processes**

   The class of canonical processes contains both Gaussian and Bernoulli processes. Moreover the canonical processes are of meaning for the functional analysis (random Fourier series) as well as for the probability theory itself. The Bernoulli theorem [Be] is an important step towards a complete characterization of sample boundedness for canonical processes. Up to now such characterizations has been known only for special classes of canonical processes. It is believed that the full characterization of sample boundedness
for canonical processes based on independent random variables of log-concave tails is accessible after proving the Bernoulli Conjecture.

**Goal:** To characterize sample boundedness for canonical processes based on variables of log concave tails.

3. **Convex geometry**

There is a hope to establish upper and lower bounds not only for canonical processes based on independent random variables but also for the general dependent case. The theory requires a lot of work even on the basic level, for example what lacks is the basic minoration result that should resemble the Sudakov minoration, which works for Gaussian processes. The attempt to improve this state of knowledge has been made recently which gives a hope to make some progress in this direction.

**Goal:** To establish a bounds for canonical processes based on log-concave one unconditional vectors.

4. **Questions around the Bernoulli theorem**

There are natural questions closely related to the proof of the Bernoulli sample boundedness characterization. They concerns the regularity of the decomposition obtained in the result. The question of K. Oleszkiewicz is whether the weak tail domination implies domination of strong moments for two Bernoulli random series with vector valued coefficients. The same direction explores the question of S. Kwapien who proposed an important extension of the Bernoulli theorem which would explain the almost sure convergence of Bernoulli series in Banach spaces. It states that if a Bernoulli series with coefficients in a Banach space is almost surely convergent then there must exist a certain decomposition of the series into Gaussian and unconditionally convergent series. The result should have immediate consequences for the functional analysis. We would be also happy to understand limit cases when the so called union bound can be applied. There is a nice conjecture that the union bound, which is an alternative approach to the chaining, should fail in the case of independent symmetric Weibull random variables with parameter smaller than one.

**Goal:** Answer some of the questions.

**Bibliography**


I do invite anyone who would like to cooperate with my group!