# 52nd Mathematical Olympiad in Poland

## Problems of the first round, September – December 2000

**1.** Solve in integers the equation

$$x^{2000} + 2000^{1999} = x^{1999} + 2000^{2000}\ .$$

**2.** The points $D$ and $E$ lie on the sides $BC$ and $AC$ of the triangle $ABC$, respectively. The lines $AD$ and $BC$ meet in the point $P$. The points $K$ and $L$ lie on the sides $BC$ and $AC$, respectively and are chosen so that $CLPK$ is a parallelogram. Prove that

$$\frac{AE}{EL} = \frac{BD}{DK}\ .$$

**3.** Find all positive integers $n \geq 2$, such that the inequality

$$x_1x_2 + x_2x_3 + \ldots + x_{n-1}x_n \leq \frac{n-1}{n}\left(x_1^2 + x_2^2 + \ldots + x_n^2\right)$$

is satisfied for all positive real numbers $x_1, x_2, \ldots, x_n$.

**4.** Prove or disprove: into a cube box with edge 4, one can put 65 balls of diameter 1.

**5.** Prove that for all integers $n \geq 2$ and for all prime numbers $p$ the number

$$n^{p^p} + p^p$$

is composite.

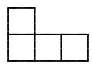
**6.** The integers $a$, $b$, $x$, $y$ satisfy the equality

$$a + b\sqrt{2001} = \left(x + y\sqrt{2001}\right)^{2000}\ .$$

Prove that $a \geq 44b$.

**7.** $ABC$ is an isosceles triangle with the angle $\sphericalangle A = 90°$. The points $D$ and $E$ lie on the side $BC$ and $\sphericalangle DAE = 45°$. The circumcircle of the triangle $ADE$ meets the sides $AB$ and $AC$ in the points $P$ and $Q$, respectively. Prove that $BP + CQ = PQ$.

**8.** For which positive integers $m$, $n$, can the rectangle of dimensions $m \times n$ be cut into the pieces congruent to the one at the figure. Each of the little squares at the figure has side 1.



**9.** Prove that among any 12 consecutive integers there exists an integer which is not equal to the fourth power of an integer.

**10.** Prove that each triangle $ABC$ contains an interior point $P$ possessing the following property: each line passing through $P$ divides the perimeter and the area of the triangle $ABC$ in the same ratio.

**11.** The sequence $(c_1, c_2, \ldots, c_n)$ of positive integers is called admissible if each integer $k \in \{1, 2, \ldots, 2(c_1 + c_2 + \ldots + c_n)\}$ can be represented in the form

$$k = \sum_{i=1}^{n} a_i c_i \quad \text{with} \quad a_i \in \{-2, -1, 0, 1, 2\}\ .$$

For each $n$ find

$$\max\left\{\sum_{i=1}^{n} c_i \ :\ (c_1, c_2, \ldots, c_n) \text{ is admissible}\right\}\ .$$

**12.** Consider the sequences $x_0, x_1, \ldots, x_{2000}$ of integers satisfying

$$x_0 = 0 \quad \text{and} \quad |x_n| = |x_{n-1} + 1| \qquad \text{for} \ \ n = 1, 2, \ldots, 2000\ .$$

Find the minimum value of the expression $|x_1 + x_2 + \ldots + x_{2000}|$ .