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Numeracy 2030

I. *What is Numeracy 2030?*

One of the United Nations targets for sustainable development (target 4.6) is defined as:

> *By 2030, ensure that all youth and a substantial proportion of adults, both men and women, achieve literacy and numeracy.*

The term "numeracy" used here is not directly explained. However, the webpage https://sustainabledevelopment.un.org/sdg4 reporting the progress of the above goal uses several times just the term "proficiency in mathematics". It is then a very capacious notion, which can be understood in a many ways. The minimal interpretation of the term numeracy could be just "the ability to deal with numbers"; at the other end would then be "the ability to deal with the deductive structure of mathematics". Neither of the above seems to be the perfect choice for an average citizen living in XXI century.

Education policy makers and, in particular, school curriculum designers in every country do their best to find a reasonable solution. They usually just derive it from the educational tradition of the country.

The 16 years of the experience of PISA leads in this context to a quite optimistic conclusion: despite numerous particular differences in national math curricula, there is a vast international common ground of understanding what numeracy or proficiency in mathematics means. We owe the identification of this common space of understanding to the PISA Mathematics Framework designers who developed the concept of "mathematical literacy" as, roughly speaking "the ability to apply mathematics to deal with problems encountered in real life". This idea, made precise in the form of the modelling cycle, proved to be a successful tools for international comparisons; over the six cycles of the assessment, the math expert group did not have any serious curriculum mismatch problem reported from a country. Apparently, we have a sound global understanding how 15-year-old students should be able to apply mathematics.

However, from the perspective of a country math curriculum, “mathematical literacy” is usually only part of the picture. In this context, we can hear about “more traditional mathematics”, but also about “digital or ICT numeracy”.

Some education systems are very concept-oriented; their main focus is on a careful selection of adequate math tools and then mastering with the students their most typical applications. While doing so, the available technology supporting the above design of the education process is of utmost importance.

With full respect for this approach, I think that this approach will not be sufficient in 2030. In the next section I will present some context factors which will make it necessary to look for deeper ideas than just selecting the adequate concepts or including machines into teaching mathematics. Concepts and IT will remain relevant, but not the most important.

II. *Context. Drivers of Change*

The world is changing faster and faster. Impressive technological progress, globalization of economy, and rapidly broadening communication tools influence our private and social lives on an unprecedented scale. We are faced with challenges which are of growing complexity, but still which we have to resolve. It becomes urgent to reflect on ways to prepare young generations to best cope with those challenges.

There is a growing feeling that traditional paradigm of education is not a satisfactory answer to the problems we face.

A fast overview of the education systems in different countries shows that there seems to be a general agreement that learning mathematics at school is somehow useful. However, we also witness in many countries a public discussion about the role and purpose of teaching mathematics to everyone. Governments argue for the need of STEM education, which includes mathematics. This is motivated with the growing need of the labor market for people developing and servicing new technologies. This need is a fact, but it is also true that not everybody in the future will become a technician or an engineer. It is also true that using the technologies of today is much more intuitive than 50 years ago and no special knowledge for applying them is necessary.

The most common argument in defense of common mathematical education is its usefulness in various practical situations. However, this argument alone gets weaker with time as a lot of simple activities get automated. In my youth cashiers in stores would multiply and add on paper to calculate the price I would pay; today they just press buttons. Not so long ago we were using printed train timetables to plan travel – it required a good understanding of the time axis and inequalities. Today we just make a direct internet inquiry.

Nevertheless, I claim that the present times and the coming future make mathematical skills more and more useful for everyone.

Many misleading images about the role of mathematics stem from the way it is understood. Many see mathematics just as a useful toolbox. A clear trace of this approach can be found in the school curricula in many countries. They are quite often concentrated on a list of math concepts; they are carefully selected and students are to practice the use of them in predictable situations.

This perspective on mathematics is far too narrow; it overlooks its features which will be of growing importance in the coming future.

Mathematics, as I see it, is a science about objects and notions which are completely defined and independent of their origin or nature. Once we isolate them in some context, they become entities which can be analyzed and transformed in ways called 'mathematical reasoning' to obtain 100% sure and timeless conclusions. What is also important, those conclusions are impartial, without any need for validation by some authority.

This is very close to the perspective on mathematics of the ancient Greeks, the inventors of mathematics. In those days mathematics was encompassing all 'sure' knowledge; physics departed only later. The only difference is that we put aside the divine dimension – Plato's ideas living in a cave are substituted with manmade objects and notions.

In this context, the first reason for the growing usefulness of mathematics in the future is its role as an important part of the cultural heritage of humanity. With the observed incredible speed of diversification of societies, there will be a growing yearning for values common to all. The ways of mathematics are ideal for this purpose – culturally unbiased, a secure ground for seeking common understanding.

There are beautiful charts of this heritage that are today obscured or just forgotten. We teach students in elementary schools about angles created by a pair of parallel lines intersected by a third line. Not everybody realizes that this tool was powerful enough to enable Eratostenes to measure the radius of the Earth. We also learned at school about similar triangles. How many of us (and of our teachers) know that this tool together with very simple astronomical observations allowed Arystarch around the year 300 to estimate the distances from Earth to Moon and Sun and the sizes of both bodies? According to Archimedes, Arystarch, on the base of his calculations, suggested that it is unlikely that Sun goes around the Earth. There are many other examples of great achievements from the past based on the simplest mathematical principles which, when brought to light, can be admired like Bach's music or Leonardo da Vinci paintings, as proofs of the greatness of humanity.

There is also another dimension in which we will observe the growing role of mathematics education in the future. In a diversified and yet tightly interlocked by e-communicators society there is a growing role of honest negotiations and discourse. (I stress 'honest' to put aside Schoppehauer's 'discoveries'.) It will become more and more important to teach students at school how to make reasonable arguments and be sure that they are right. The arguments they make should be strong enough to withstand criticism, and yet, whenever possible, they should avoid referring to authorities (e.g. 'Google says so'). This is part of the fundamental competence to make independent judgements and taking responsibility for them[1]; in the social context it is not enough to be right; we must be able and ready to present our arguments and to defend them. Learning mathematics, with its 100% clarity of contexts, is a perfect opportunity to practice and develop the ability of this kind of argumentation.

The sad side of our social life which manifested itself just recently is the 'post-factual' (some even say: 'post-truth') era. I am afraid that this phenomenon will continue and spread. Therefore it is of utmost importance to equip our students with tools which they can use to defend themselves from lies. Quite often some fluency in logical reasoning is sufficient; a lie usually includes some hidden contradiction. The alertness of young minds towards possible contradictions can be developed most easily in good classes of mathematics.

One more feature of the present and future world is the exponential growth of the number of contexts with which humans have to interact. In consequence, we must be flexible and ready to deal with surprises. Algorithmisation of activities is not any longer a satisfactory strategy. That means that preparing students for a fluent reproduction of knowledge and trained practices is not sufficient from the perspective of future needs. They should be taught to expect and to be ready to actively react to completely new problems and situations. We have in this field a lot to do. Too often our students consider the answer: 'I have never seen in class anything like this before' as a satisfactory justification of taking no action. A winning strategy for the future requires a radical change of this attitude. This should start in math classes, where at least the contexts are crystal clearly defined.

At last, we should acknowledge and assimilate in our thinking the expansion of new information technologies. There are many dimensions of this phenomenon.

The simplest is automation of many operations which in the past were to be performed manually. We should reflect on the amount of time and scope of procedures taught in math classes which an effectively performed by machines. This is a natural process; slide rules disappeared from classrooms half an age ago; algorithm for calculating a square root of a number disappeared even earlier. The algorithm for long division is still present in school curricula, but it will probably pretty soon share the fate of the above. However, we must be careful to delegate to the machine the act of calculation only, not the understanding of its sense. A student should become a careful judge of the machine's work. Only by critical thinking they will be able to eliminate a possible error (usually of human nature).

Another dimension of IT is the reach world of possible experiments and simulations. Today it is usually used in math classes as a support to learn today's curriculum content. To develop the abilities described in previous paragraphs we will have to use the technology in more creative ways. Computers, for example, can support proving, that something is true. This will require special efforts to illustrate the need of an actual proof, even though something may seem true on a computer screen. Computer modelling can be quite misleading, e.g., due to numerical complications of the calculation model implemented in the device. Thus, critical

[1] E2030 Conceptual Framework: key competences for 2030, (DeSeCo 2.0), page 5

thinking is again a key issue. Despite this dangers, computers offer a broad field for students' creativity, including building their own models and testing them.

Another great opportunity brought by computers into math education is the possibility to deal with big data. Today's world is more and more complex[2] and its multiple dimensions are represented by terabytes of data. Making sense of this data is one of the biggest challenges of humanity that we will face in the future. Our students should be familiarized with such data and the exponential growth of some parameters of our world, illustrated by this data.

The power of mathematics, from its very beginnings, lies in the ability of reducing complex contexts to set of simple basic principles. Euclid's 'Elements' constituted the first spectacular success in this field; he was able to reduce all known ancient geometry to conclusions from 5 simple assertions. Today's math theories are not less successful (including the studies on chaos). Good mathematics education should build the attitude for hunting for those 'prime principles' in well designed, yet quite complicated contexts.

I would consider it a total failure of math education, if it resulted in uncritical trust in our computers' judgments.

III. *Numeracy in 2030 – the main objectives*

From the section on challenges it clearly follows that the main shift in our understanding of numeracy should concern more complex competences and a significant shift in attitudes. In particular, the primary role will be played by the processes and students' involvement in practicing them; the content issues (the choice of 'key concepts') will play an important, yet secondary role.

There are three main skills which should, in my opinion, guide the teaching of mathematics for the future. These are:

- Mathematical reasoning and argumentation,
- Strategic thinking,
- Mathematical modelling.

The abilities to ***reason rationally*** and to present arguments in honest and convincing way are the skills which are getting more and more important in today's world. This remark reaches far beyond mathematics, but it can be most effectively learned and practiced within mathematics, just because it has the advantage of a well-defined context, which creates a comfortable training environment.

Some argue that just a course of formal logic could bring a better result. In my opinion, reasoning is best learned by practicing it and then the clarity of context is fundamentally

[2] The World Bank data on world's education systems use 16 000 variables.

essential. Manipulating simple, well-understood objects (like numbers or geometric figures) is a starting point for reasoning. Initial formalization of the arguments diverts the attention from the basic relations with the given context and often cripples students' activities.

The main obstacle here is students' attitudes. Taking on reasoning needs an active attitude; making an argument requires courage, as it involves quite often challenging someone's opinion.

The first issue is crucial. The main barrier is lack of confidence of students – they quite often do not believe they can do anything about a problem they have never seen before. This lack of confidence has partially source in bad pedagogy. Some teachers expect a student to present a complete, 'smooth' solution and do not value unsuccessful attempts, which discourages their students even to try an attack on the problem. Some others claim that mathematical reasoning is reserved for the gifted students only, attending special classes. Students, hearing that, feel justified to restrict their efforts to the basic routines.

The same teachers are pleasantly surprised once they try to involve students into mathematical reasoning. To many students it is a refreshing exercise which challenges their common sense. On such occasions one can often observe a spontaneous and effective involvement of students which are not the best in class in routines.

This kind of change of paradigm of teaching is a challenge for many teachers. It requires, of course, perfect understanding of the problem discussed and readiness to (fast) react to completely unpredictable students' ideas. In my opinion, to meet the challenges of the near future, it is absolutely necessary that all math teachers are equipped with those skills.

There is one more condition: teachers should be patient partners. Our students have a great potential; it takes a lot of patience to make it surface. Studies show that quite often teachers who inspire their students to reason, give them precise hints – step after step – to get to the successful end. This increases effectiveness of the teaching process, but deprives the students of the joy of search. It is a pity, but we can still find students' paper work, dismissed by a teacher as worth nothing due to its untidy form, which hides a testimony of three consecutive attempts to solve the problem, only the last, written across the sheet of paper, being the right one.

The mathematical reasoning is not easy. In PISA, there are not too many items that require this kind of skill (but still there are a few). This was caused partly by the intervention of psychometricians who rightly claimed that the measurement power of items that only a very small fraction of students can solve makes little sense. However, in the context of all above remarks, I would suggest to increase significantly the proportion of reasoning items, with a shift in its marking: even an attempt to solve a problem never met before should be properly acknowledged by a partial credit. In the long run, students who make the (even finally unsuccessful) attempt to fight with a problem should be considered to have better *Numeracy 2030* skills than those just fluent in routines.

The courage issue is another thing. Many students, coming from very authoritarian environments, have a satisfactory ability to reason and yet they avoid reporting it. The big advantage of mathematics, when compared with other areas, is that it is authority-free; with a right argument even an elementary school student can challenge a university math professor, and the latter will be rather delighted than offended by this fact.

Mathematical reasoning can be practiced in numerous contexts; it need not be necessarily just proving theorems. The teaching practice provides many results of great ideas how to get students involved in creating networks of implications and counterexamples. W. Guzicki, a teacher from Poland, describes in his book[3] how 13-year old students are able produce complex chains of sound implications explaining their decisions while solving a Sudoku puzzle. A list of useful techniques, which can be applied to develop students' reasoning skills can also be found in the book "Art of Insight" by S. Mahajan[4]. This includes proportional reasoning, dimension analysis, lumping etc. These techniques, when applied in the context of calculations, deepen the understanding of the sense of performed operations.

Another important skill of *Numeracy 2030* is ***strategic thinking***. The essence of this aspect of numeracy is the ability to make responsible choices in the course of action while dealing with a problem. In today's teaching of mathematics too often we see the rule: one problem – one solution. Some textbooks look almost like cookbooks: first a (typical) problem is presented with a complete solution, next a very similar problem is provided with gaps in the solution, to be filled by a student, and a list of clones of the problem is offered, to be solved by the student, almost blindly following the solution outlined before. This is undoubtedly an effective way of memorizing certain algorithms, but – in my opinion – it is hardly sufficient for the needs expected in the future. After this kind of training students are fully dependent on the set of procedures they got at school. In the fast changing world it is hard to expect that they will encounter in practice exactly one of those problems. Usually modifications of the solution method will be needed. It is not a coincidence, that even the 'self-training' techniques of Artificial Intelligence are more sophisticated than that.

The attitude of readiness to modify the approach is one of the most important skills in mathematics. Without this ability hardly any progress in mathematics would be possible.

The last of the three most important skills of *Numeracy 2030* is ***modelling***. This interface aspect of numeracy is crucial for various applications of mathematics. The art of isolating the most important features of investigated objects and their interactions to form a model, i.e., a completely defined mathematical object, is not easy. However, any application of mathematics to a real life problem must start with this step. Of course, the step of modelling refers to knowledge outside mathematics, specific for the domain to which the given practical problem belongs.

The aspect of modelling is adequately presented in the PISA Mathematics Framework, in the form of the Modelling Cycle. It essentially consists of three steps: building a mathematical model, resolving a math problem and then the validation of the obtained results, possibly leading to a model modification and the next full cycle. In my opinion, this description does not require any substantial modifications.

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[3] W. Guzicki, 'Rozszerzony program matematyki dla gimnazjum'

[4] https://mitpress.mit.edu/sites/default/files/titles/free_download/9780262526548_Art_of_Insight.pdf

The roots of the above ideas are present in the concept of ‘mathematical literacy’, as defined in the PISA Mathematics Framework.

In my opinion, a mathematics curriculum for *Numeracy 2030* should concentrate on the above skills. The content knowledge should be a tool for developing those skills, not an aim of its own.

IV. *Numeracy 2030 – content and skills suggestions*

The three key skills of *Numeracy 2030* should, of course, be developed and practiced on the base of some content knowledge. Notice that the general character of those skills makes them, in a certain sense, content independent: they can be effectively developed with respect to any mathematical concept.

There is a remarkable agreement on global scale with respect to the content of mathematical education. This observation has been confirmed by the experience of the Math Expert Group for PISA. During over 10 years of my work in this group, we encountered no single objection from a participating country to proposed test items, referring to the lack of fit of the used concepts.

However, one can list certain content areas, which seem to be relatively new for the traditional mathematics curricula, which would enable effective modelling of relatively new and yet important phenomena. These were discussed at the Center for Curriculum Redesign, and presented at one of the CERI Steering Committee meetings. Many current developments in technology and social life suggest that dealing with complex systems and big data and using algorithmic thinking to enhance models were mentioned as concepts of increasing importance.

PISA Mathematics Framework has identified four content areas (for 15-year-olds): Quantity, Shape and Space, Change and Relationships, Uncertainty and Data. These areas cover the content of mathematics classes in most countries; the names of the areas were created to move away from traditional names lie algebra, geometry, etc., to avoid strong, and yet internationally diverse connotations. At the CERI meeting, there were also a number of detailed suggestions for shifts of interest in certain PISA content areas, like:

Quantity: shift from explicit calculations to smart estimations,

Shape and Space: use more nonstandard shapes, better modelling objects in the surrounding world,

Change and Relationships: stress more exponential growth and proportional thinking,

Uncertainty and Data: use conditional probability, handle combinatorial situations, and include elements of game theory.

The above list should be understood not as a call for global change. No rapid changes of curricula are possible. Curricula are very much dependent on the actual teachers' skills and knowledge. Therefore countries are changing them very carefully, taking into account many inside factors.

However, most curricula designers analyze needs of the future. Therefore the above content remarks can be understood as a learned guess of the potential directions of change in the future.

When discussing changes in curricula, we always have a discussion about the proportion of the factual knowledge a student has to acquire versus knowledge that can be, if necessary, obtained on demand from outside sources, like the internet. In my opinion, practice resolves this dilemma pretty well. Amount of memorized knowledge in mathematics has significantly decreased in many countries over the last decades. On the other hand, relying only on the memory of 'the cloud' is risky; as someone said: 'if that was a good strategy, then rats living in a library would be much smarter than the other. Experiments do not confirm that.'

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Teaching and learning of mathematics encounters certain difficulties, which are domain specific. The nature of mathematical truth is binary; there is no shadow area, where students (and teachers) feeling insecure can hide. When you fail to solve the problem, there is no way to deny it. On the other hand, solving a problem can be an individual victory over own weakness and be a driving force for deeper engagement in mathematics and possibly elsewhere. This duality can be depressing and lead to 'math anxiety'; therefore it requires special emotional handling.

The problem of math anxiety, not observed for other school subjects, is quite serious. It is very important to make attempts to identify its true reasons in individual cases. Quite often, it is a result of some "math defeat" in the past. Concentration on the development of the complex skills, described above requires a high level of interaction, which creates opportunities to identify and overcome the source of the trouble. On the other hand, just practicing routines creates an environment with possibilities to hide true reasons of problems. To see that it is enough to look into the PISA reports – in some countries a surprisingly high fraction of students admit that they memorize mathematics!

Another danger is continuously increasing participation of students in networking which can possibly spread math anxiety as a plague. Already in the pre-computer era it was spreading – everybody has heard someone saying 'I am a humanist, hence I am not able to do math'. Of course, the same technology, when properly used, can be a very effective tool to fight it. Learning mathematics without accompanying positive emotions can be a pure torture. On the other hand, positive emotions can make learning mathematics a fascinating intellectual game. The main objectives of *Numeracy 2030* can be achieved only when we convince students to play an active role in the education process: they should select the model parameters, invent and present mathematical arguments, discuss and possibly change the path of action in search for a problem solution. This kind of teaching is absent in many classrooms where students just sit, listen to teacher's explanations and then do practice exercises, following the algorithm, earlier presented by the teacher.

The global paradigm of teaching mathematics depends on solving problems. Achieving goals suggested for the *Numeracy 2030* needs special types of problems. They should enable discussions between students, collaborative problem solving and active use of tools, including software. These contexts require social skills, but they also create great opportunities to develop them.

V. *Important characteristics of Numeracy 2030*

Transferability

The main objectives of *Numeracy 2030* clearly have a scope exceeding the borders of mathematics. Making logical inferences from agreed assumptions and making arguments about them is a common feature of science and culture of social interaction. If this skill is sufficiently well practiced within mathematics, it will, to some extent, serve well in other contexts. Values like courage to express an opinion and defending it, critical analysis of arguments presented by others are of growing importance in today's world, in which misinformation, manipulation of emotions and demagoguery are facts.

Also persistence in seeking the right way of solving a problem, developed together with strategic thinking, can be easily transferred to other school subjects. Their content will have to reflect more and more complicated relations of the surrounding world, both physical and social. This will work better in those subjects, where the tasks offered to students require their active action .

Finally, modelling can potentially give splendid opportunities to interact with other school subjects, as mathematics is obviously present, to some extent, in all of them.

In my opinion, if we focus mathematics on the main skills, which are reasoning, strategic thinking, and modelling, it can rescue itself from the many concerns of being isolated as a subject with technical concepts or equations.

Measurability

Mathematical skills have been measured in domains of mathematical reasoning and argumentation, strategic thinking or modelling since long. The same kinds of math problems can be used to measure the knowledge needed to find solutions. However, as mentioned earlier, measurement of the complex skills should be equipped with a system of partial credits, reflecting student's engagement in the solution process. Especially, when it comes to evaluating whole education systems, it is of utmost importance to catch the proportion of students who did not reach the final answer, but struggled to find it in inventive ways. From this perspective, the concept of 'double coding' from PISA 2003 could be again put into use. In particular, it would allow identifying main solving strategies applied by students.

Measuring attitudes is more difficult. Of course we can indirectly infer about them from the cognitive measurement. Connections between the shown abilities and attitudes can prove fascinating. With all respect for psychology, I trust more those indirect inferences than tests designed especially for the purpose, without any relation to students' cognitive efforts. For example, a student's paper presenting many attempts to find a solution, written one on top of the other, is – in my opinion – a much more credible proof of his/her persistence than a result of a self-evaluation: how persistent are you? A patient investigation of a scrambled student's word is an incredibly rich source of information about the attitudes.

Sequencing

Experiences of good teachers I met in Poland suggest that one can start working on the three main objectives from the first day of school. Of course, the content should fit to the students' age. This kind of pedagogy requires an increased level of communication between teachers and students, among students and between students and various resources, including databases and software.

It is clear that such a change will be more demanding for teachers. Teachers who are not able to build students' positive emotions towards mathematics will have hard time to motivate them to engage into activities that are assumed in the *Numeracy 2030* objectives. Those teachers will probably not change their pedagogy, unless properly supported.

In many countries such positive change is stimulated by the external examination system. For instance, when the leaders of the system start including problems whose solving requires mathematical reasoning or strategic thinking, all accompanying industries (textbook publishers, teacher support services) will follow and the process of implementation of the ideas of *Numeracy 2030* can be initiated.

Once students get interested in active play with mathematics, they will also make efforts to look up other potential sources of information and ideas, such as, e.g., Khan Academy, the YouTube channels etc. If not, the abundance of additional sources of knowledge will be of very restricted use.

VI. *Beyond 2030*

It is very hard to predict the future, but some present developments indicate the direction in which we are moving. The fast developing AI (artificial intelligence) can in not so distant future dramatically change our world. This can be similar to the First Industrial Revolution: simple jobs will be overtaken by androids and just disappear from the job market. Reproduction of knowledge or of skills in humans will become obsolete – machines will outperform us here in the most obvious way.

The AI will, with time, probably also outperform us in the more complex skills – the SI (superintelligence) is also around the corner. We were shocked not so long ago about computers winning chess games with grandmasters; we were hoping that the game of go or poker are too complicated for machines. Already today it is not true.

I hope that AI of the future will be intelligent enough not to intimidate the humans with its advantages. However, I am afraid that people without complex skills will have a big problem to find a satisfactory way of life.