Automata based verification over linearly ordered data domains

to be presented at STACS `11

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INRIA and ENS Cachan

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University of Warsaw

Motivation

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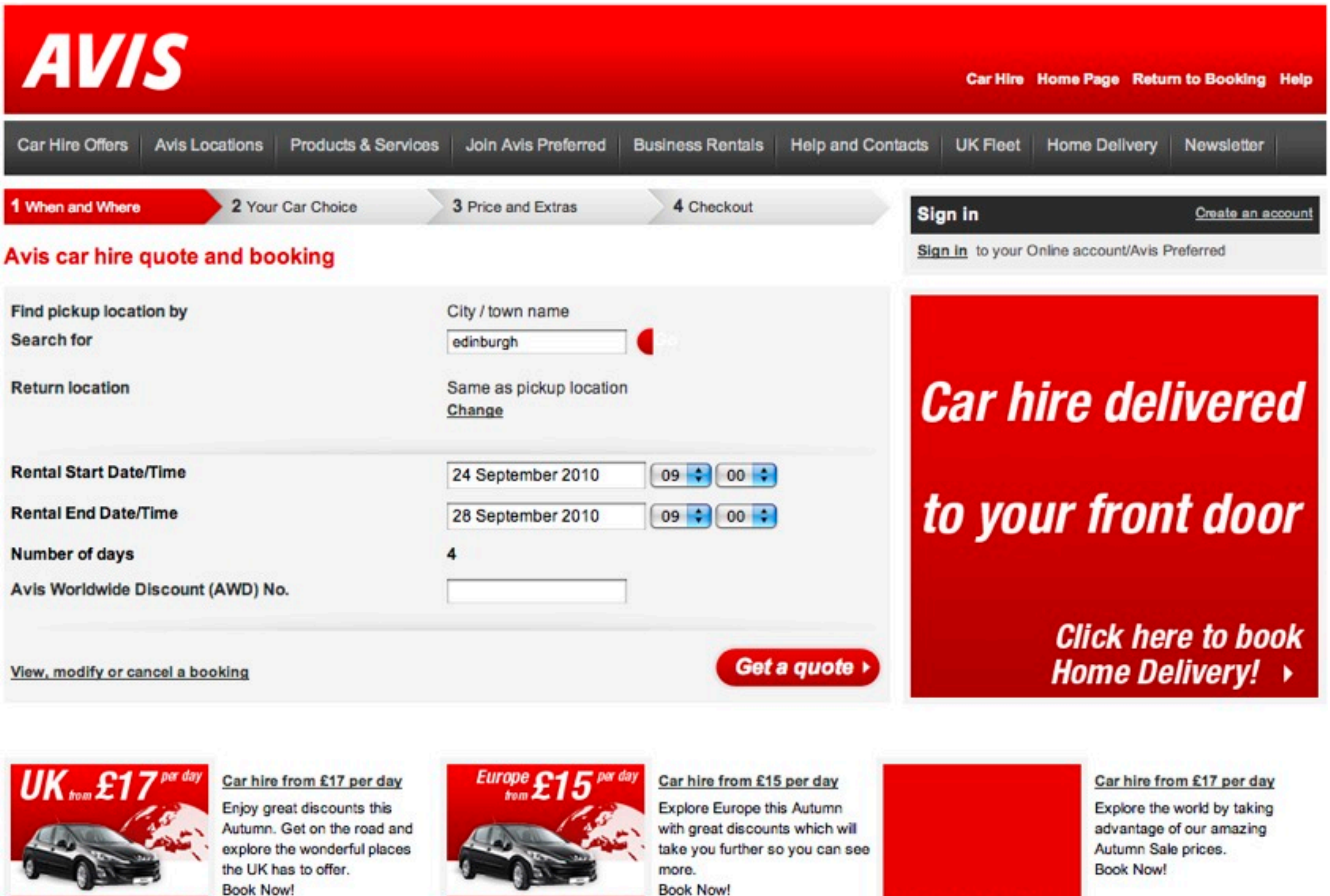
Given a system which refers to a database and data values, verify properties of the system



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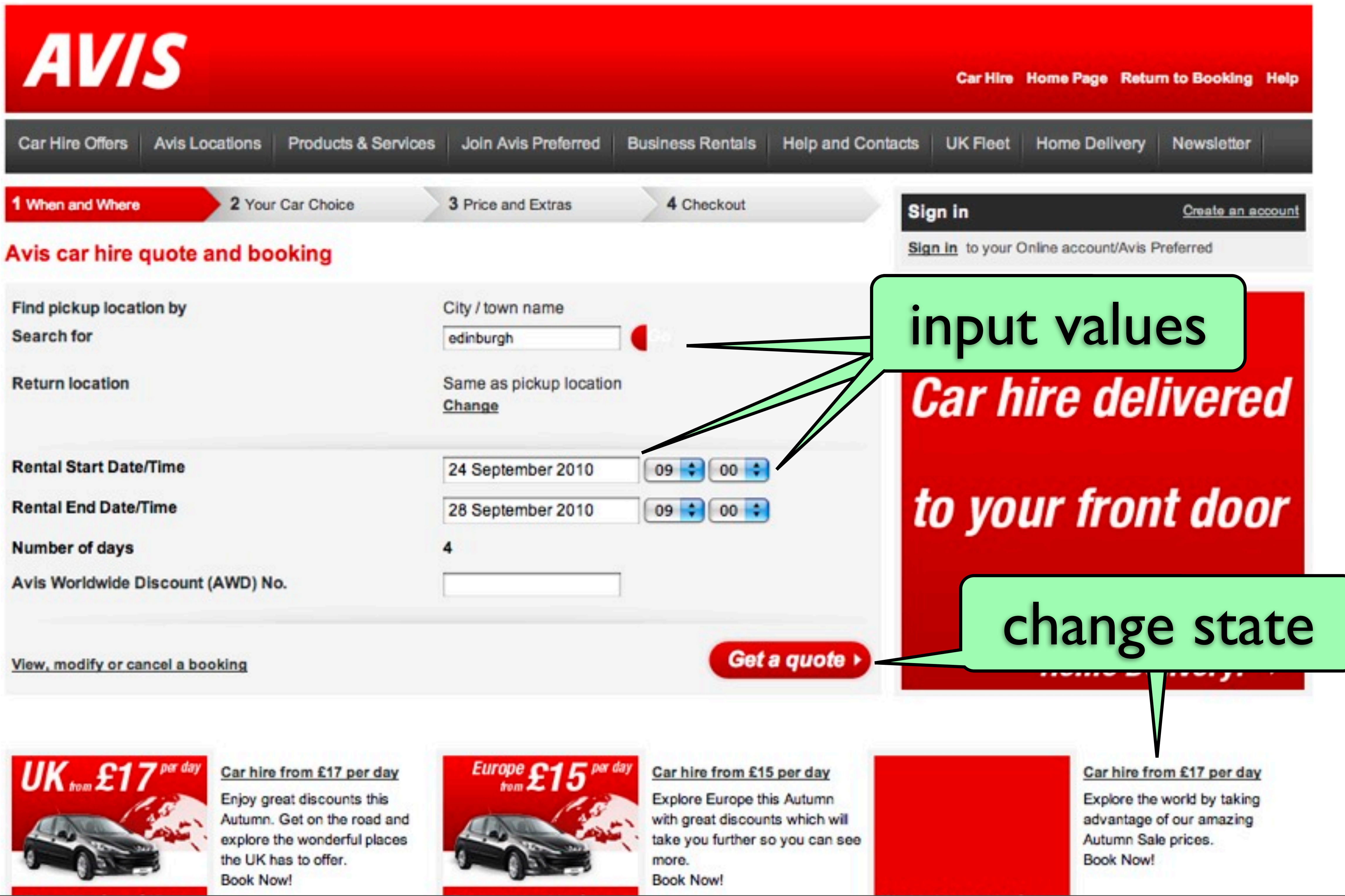

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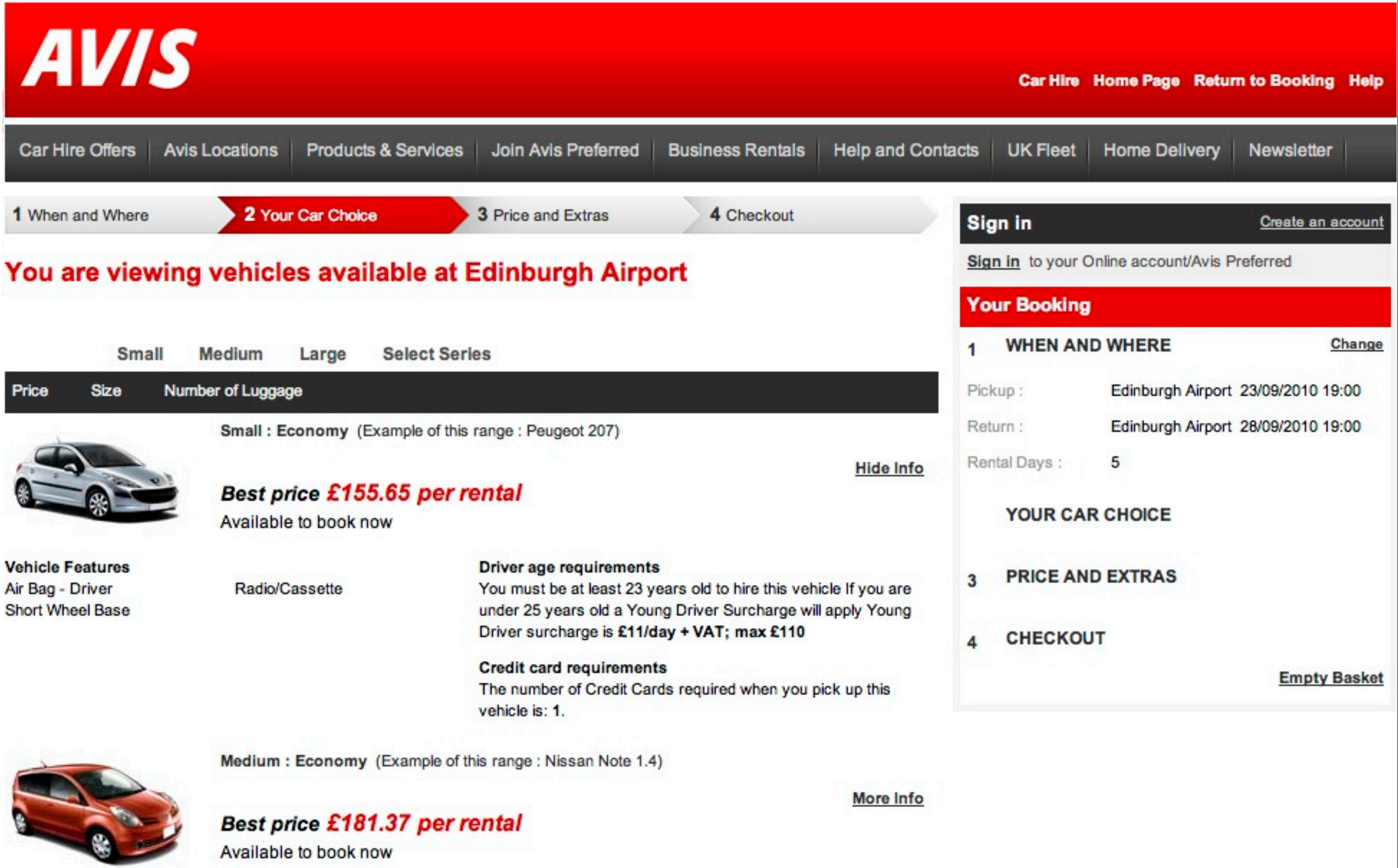

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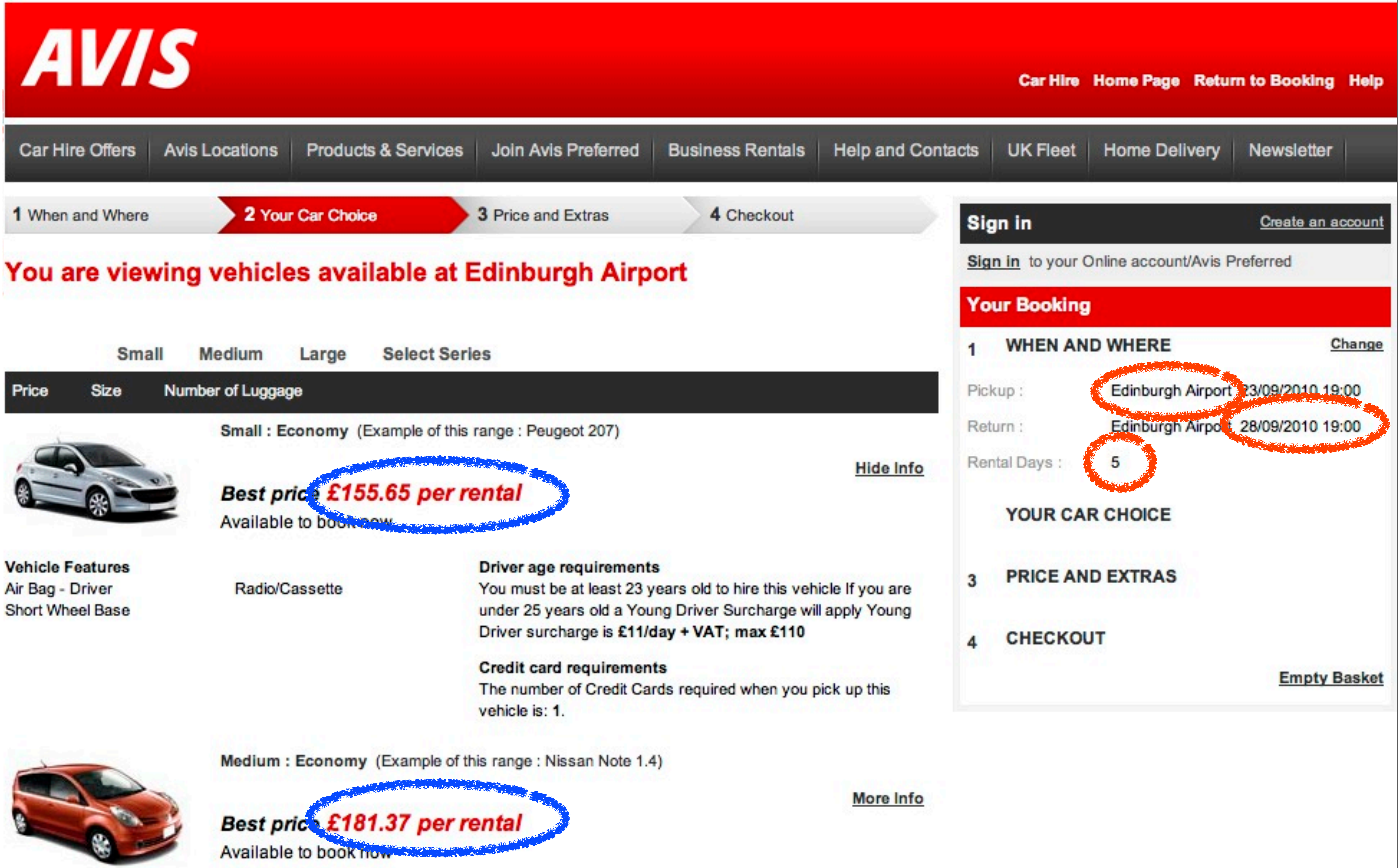

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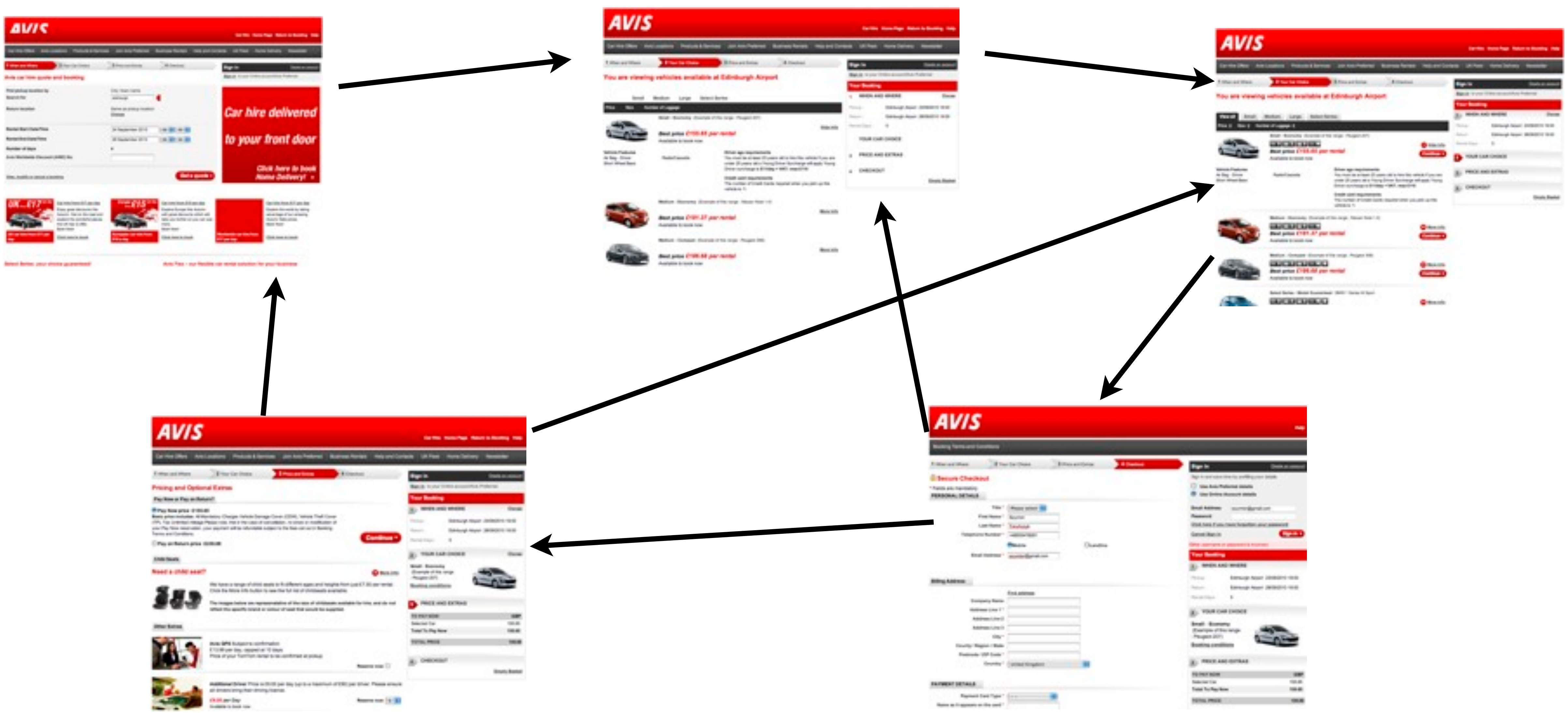

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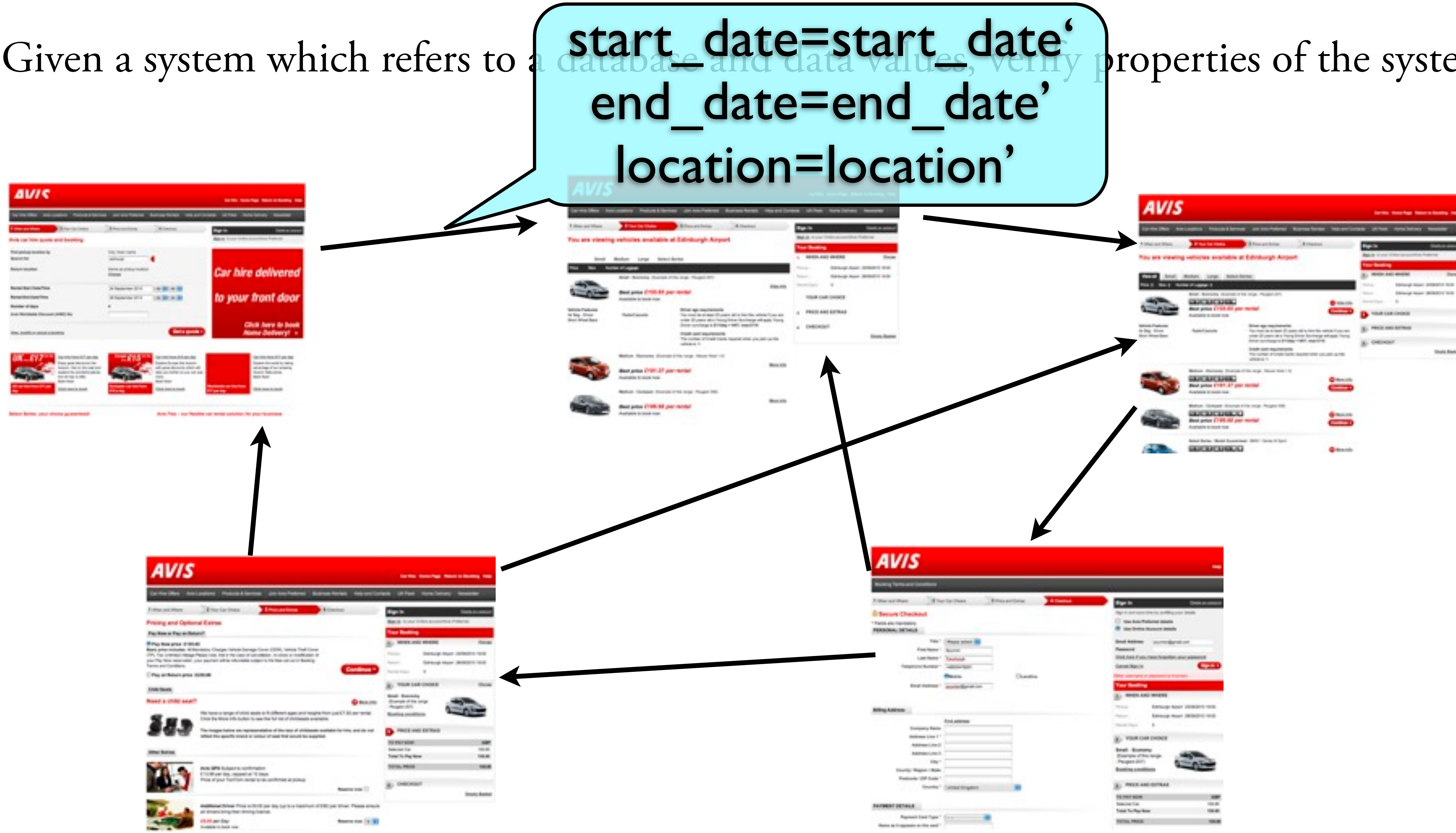

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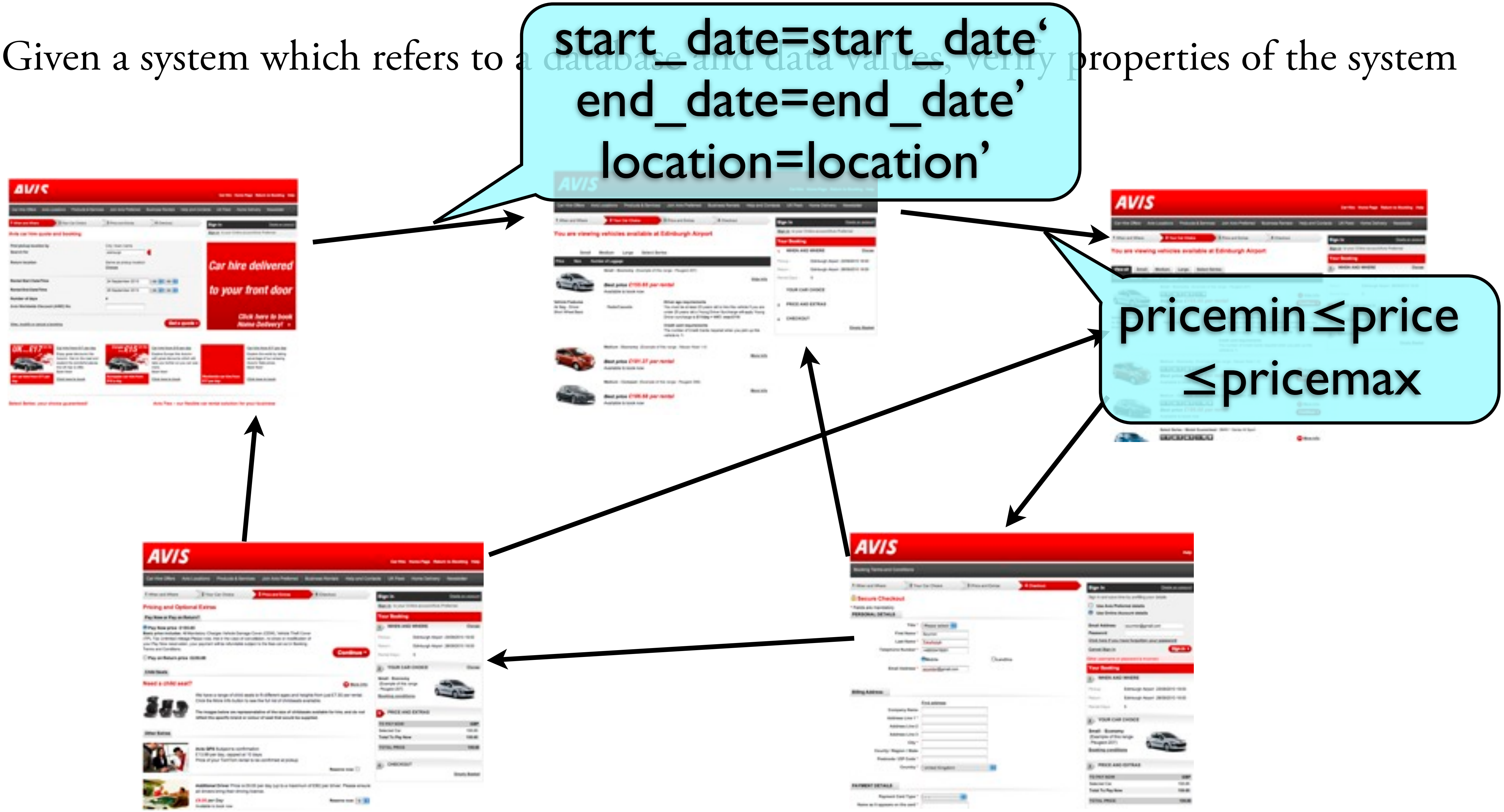

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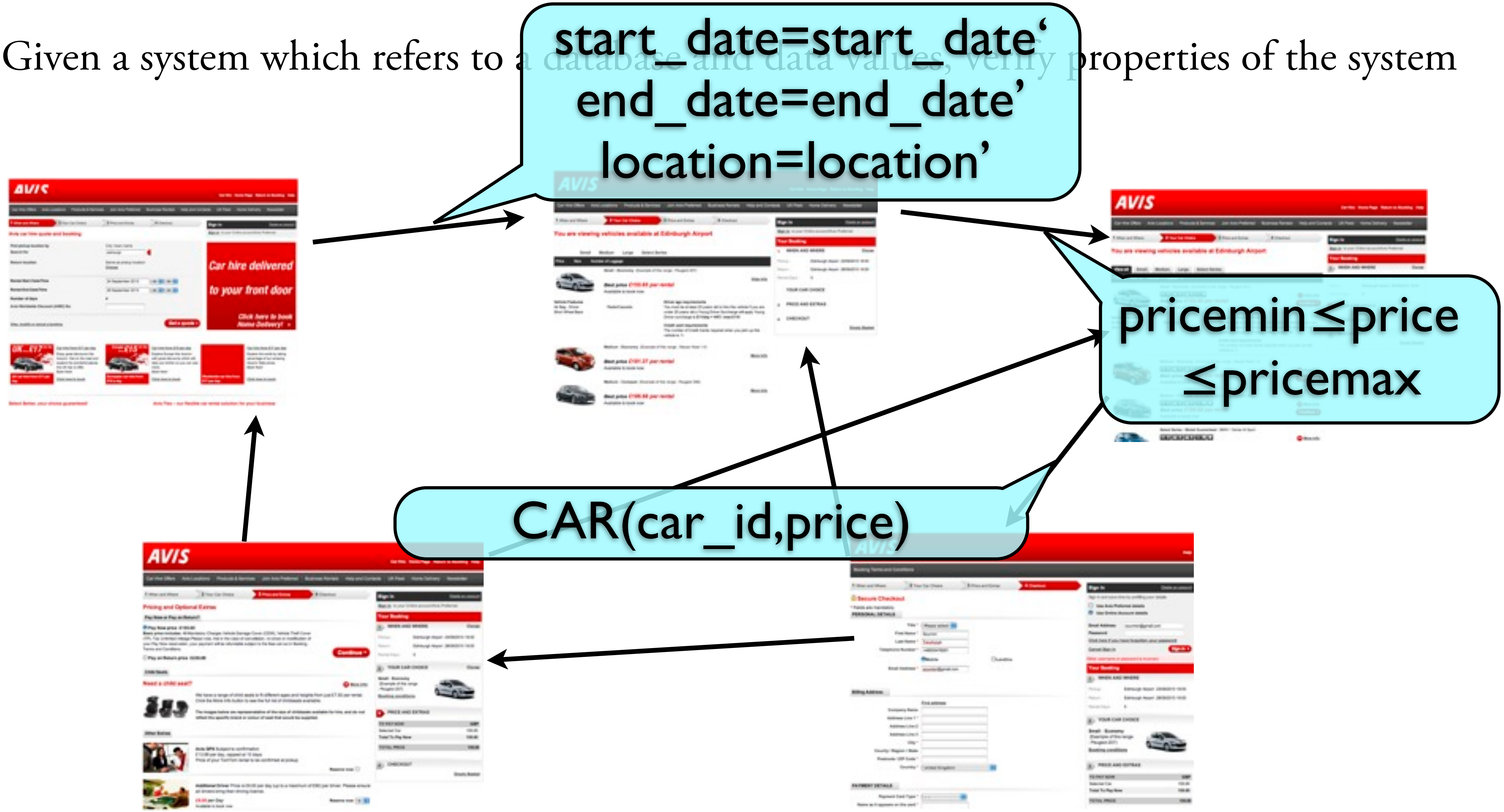

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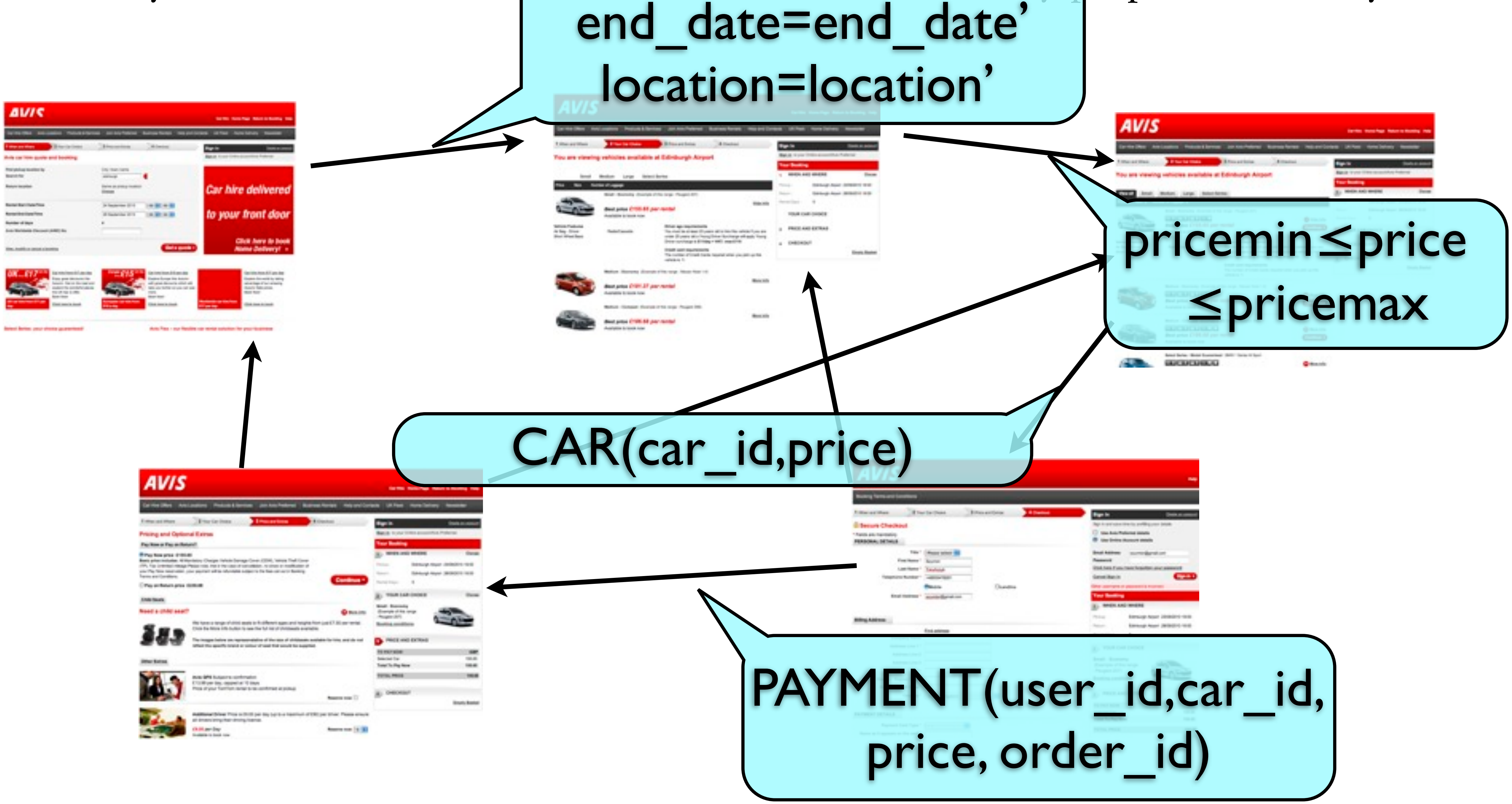

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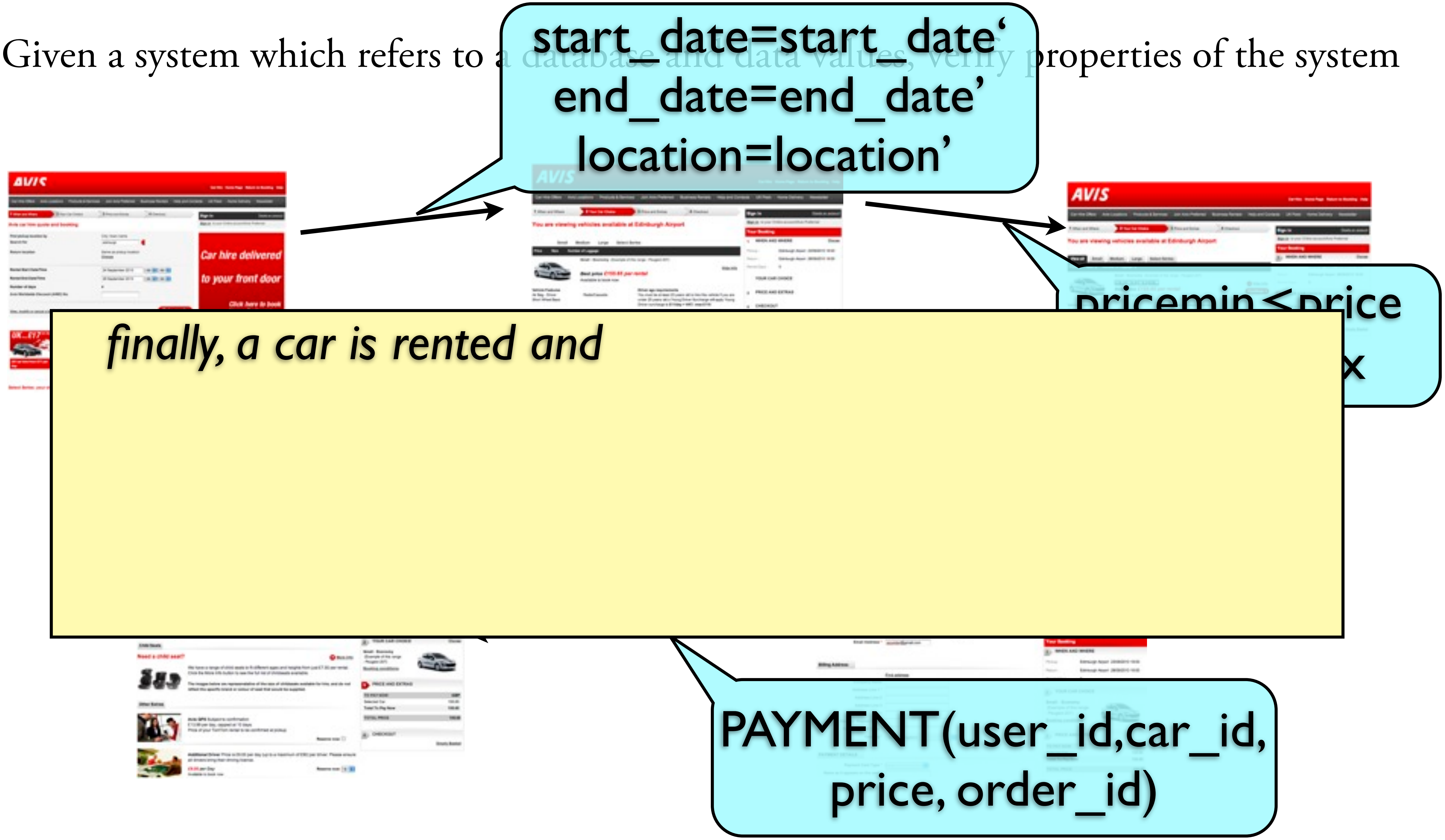

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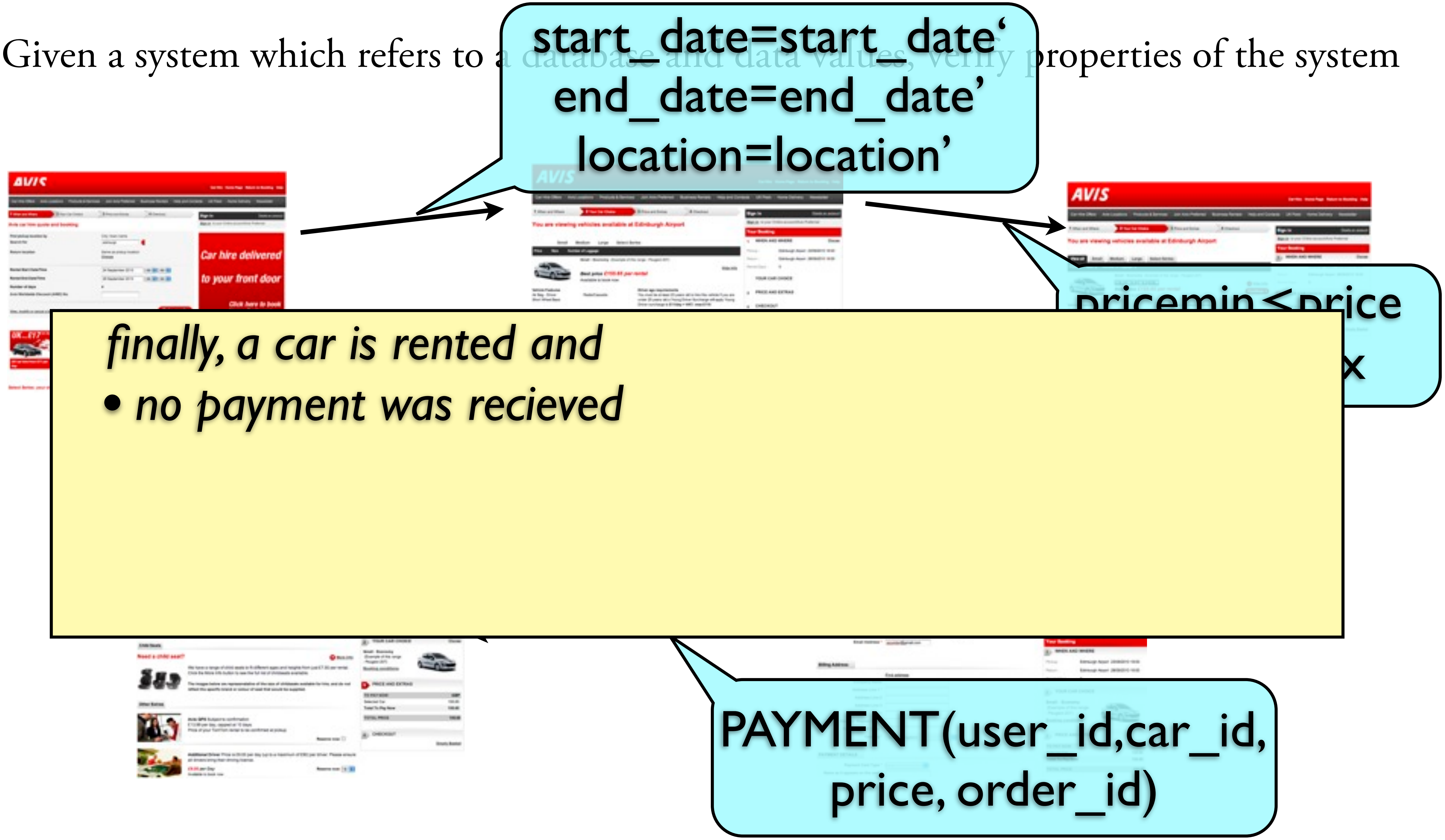

finally, a car is rented and

- *no payment was recieved*

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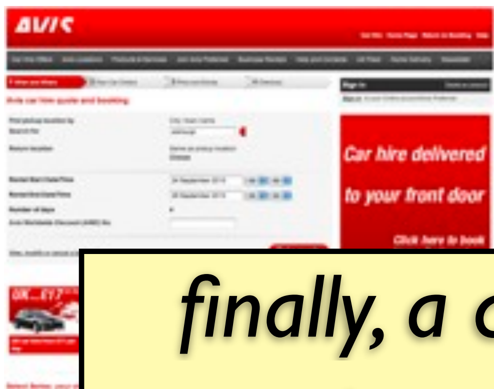
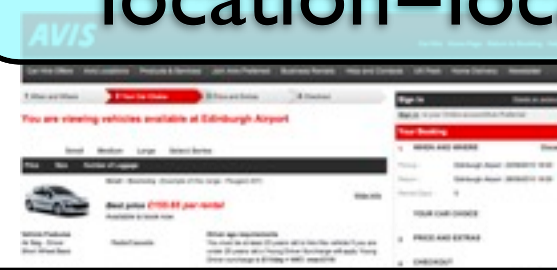


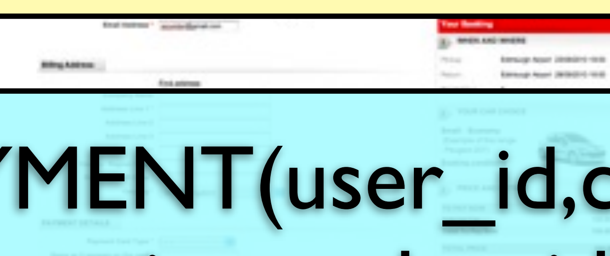
start_date=start_date'
end_date=end_date'
location=location'



pricemin≤price
x

finally, a car is rented and

- *no payment was recieved*
- *a payment was recieved but* payed=0



PAYMENT(user_id,car_id,
price, order_id)

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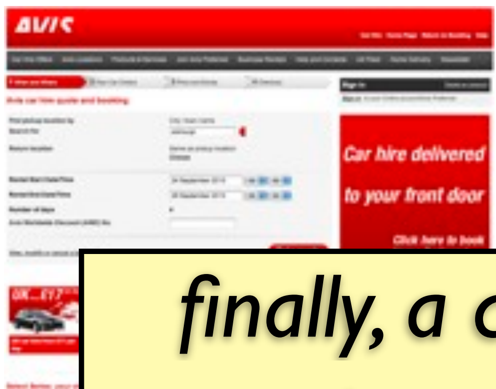
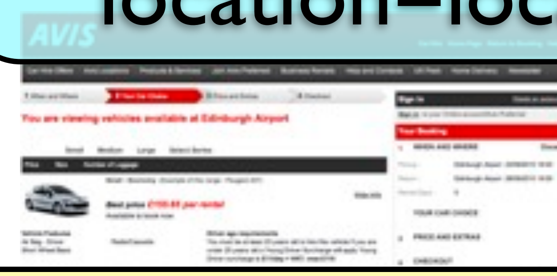
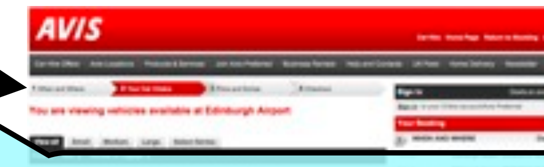
location=location'







pricemin≤price
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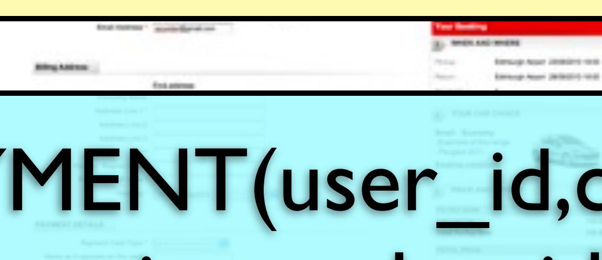
finally, a car is rented and

- *no payment was recieved*
- *a payment was recieved but* payed=0
- *there was a payment, but* payed<100 & end_date>2050



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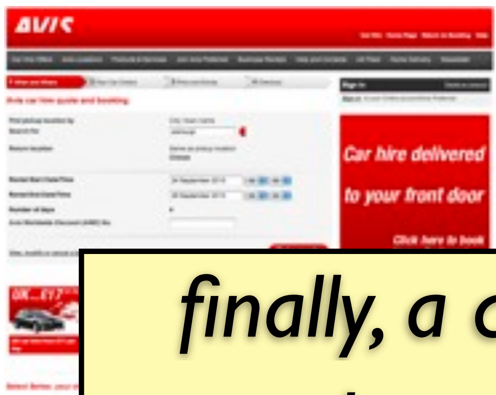

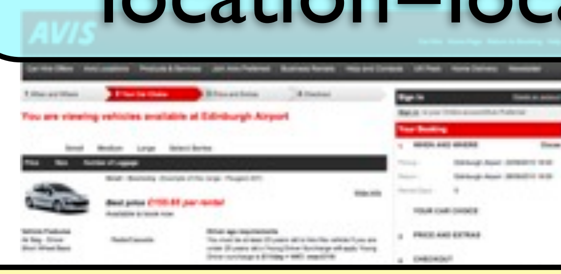

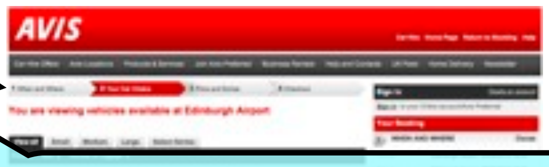

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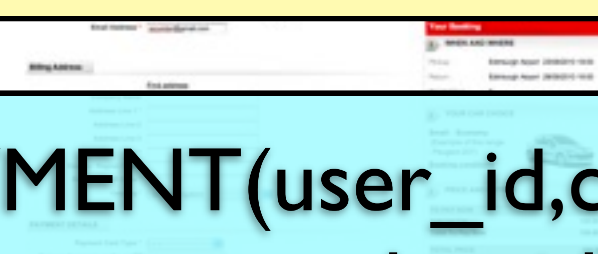

PAYMENT(user_id,car_id, price, order_id)

Outline

1. Define automata model
2. Analyze automata over dense orders
3. Analyze automata over discrete orders
4. Add database and infinite runs

Extended automaton

a definition that captures timed automata, or vector addition systems

D – fixed domain

Q – finite set of states

X – finite set of variables

$D^X = \{ v: \; X \to D \}$ – space of variable valuations

$Q \times D^X$ – space of configurations

restricted form

$\delta \subseteq (Q \times D^X) \times (Q \times D^X)$ – set of allowed transitions

$I \subseteq (Q \times D^X)$ – set of initial configurations

$F \subseteq (Q \times D^X)$ – set of final configurations

Examples

- *Vector Addition System*: $D = \mathbb{N}$

$$(q, v) \xrightarrow[v' = v + w]{} (q', v')$$

- *Timed Automata*: $D = \mathbb{R}$

$$(q, v) \xrightarrow[c_2' = 0]{c_1 < 2} (q', v')$$

- (*Lossy*) *channel system*: $D = \{a+b\}^*$

$$(q, v) \xrightarrow[c_1' = c_1 \cdot b]{first_a\,(c_1)} (q', v')$$

Our setting
(without the database)

The domain:

$$\mathscr{D} = \langle\ D, < ,\ P_1,\ P_2,\ P_3 ,\ldots, P_l\ \rangle$$

linearly ordered set

unary predicates
(subsets of D)

Examples

$\langle\ \mathbb{N}, <, 0, 100, P_{even}, P_{prime}\ \rangle$

$\langle\ \mathbb{Q}, <, 0, 100, P_{integer}, P_{<\pi}\ \rangle$

$\langle\ \{a+b\}^*, <_{lex}, P_{(ab)^*}\ \rangle$

Transitions:

$$I, F \subseteq (Q \times D^X)\ , \qquad \delta \subseteq (Q \times D^X) \times (Q \times D^X)$$

are specified by quantifier free formulas over $\mathscr{D}$

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states

$\mathcal{A}$

τ_I: $(x>0) \wedge (y=0)$

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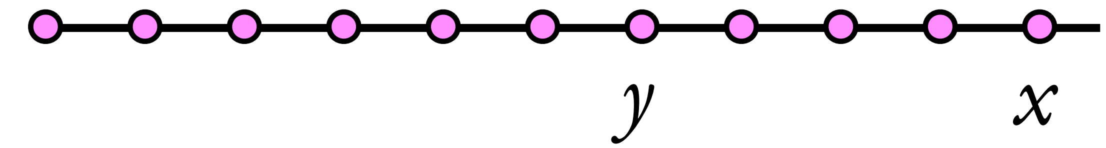

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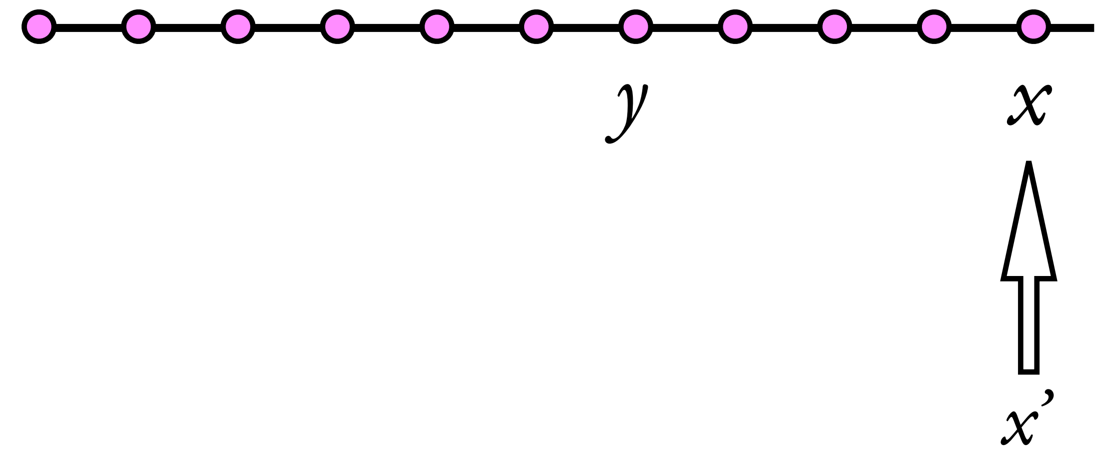

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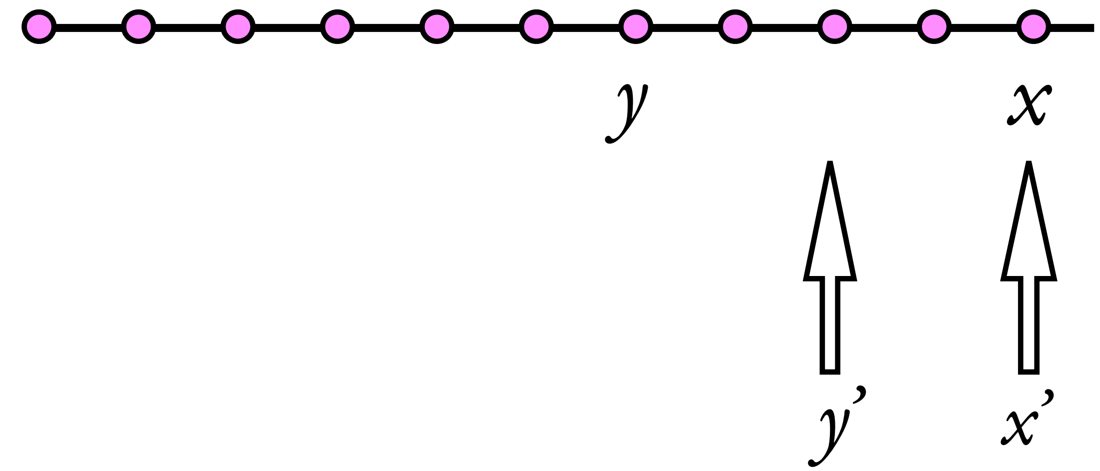

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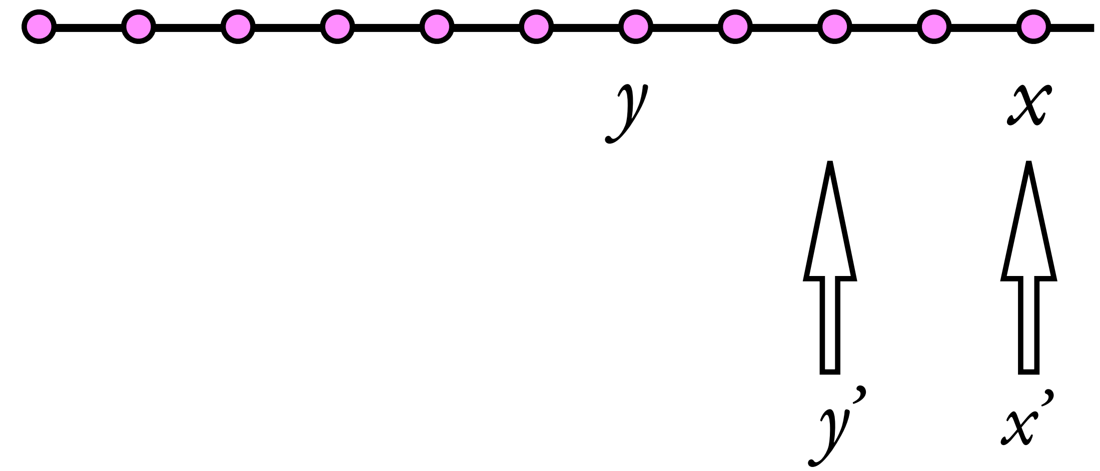

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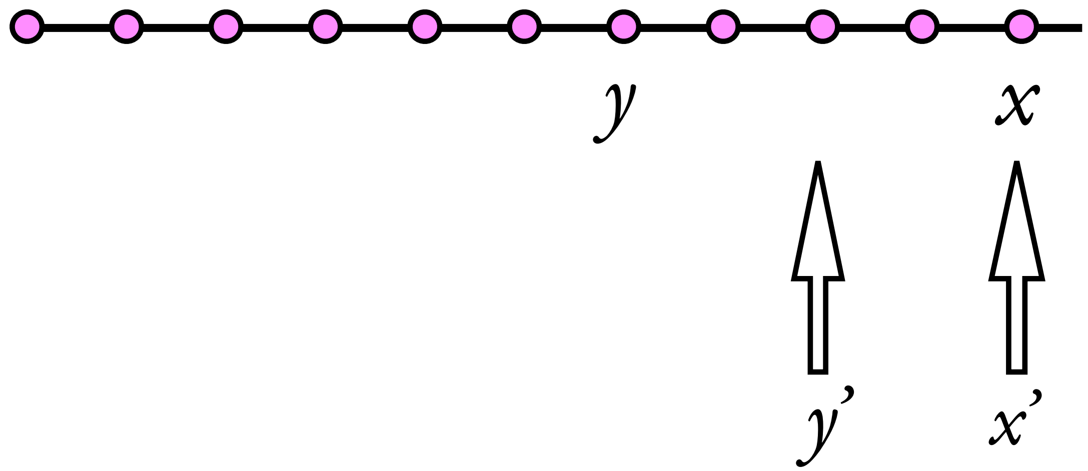

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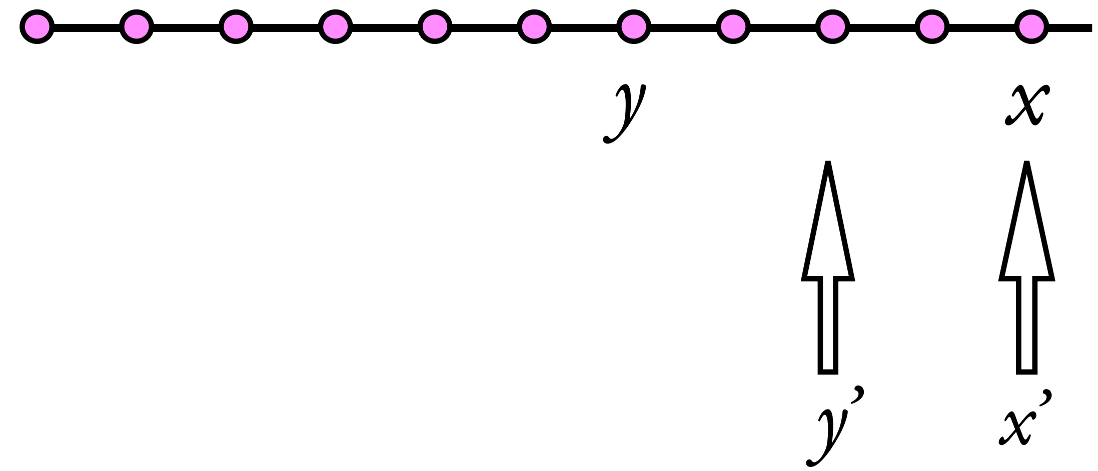

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x

y

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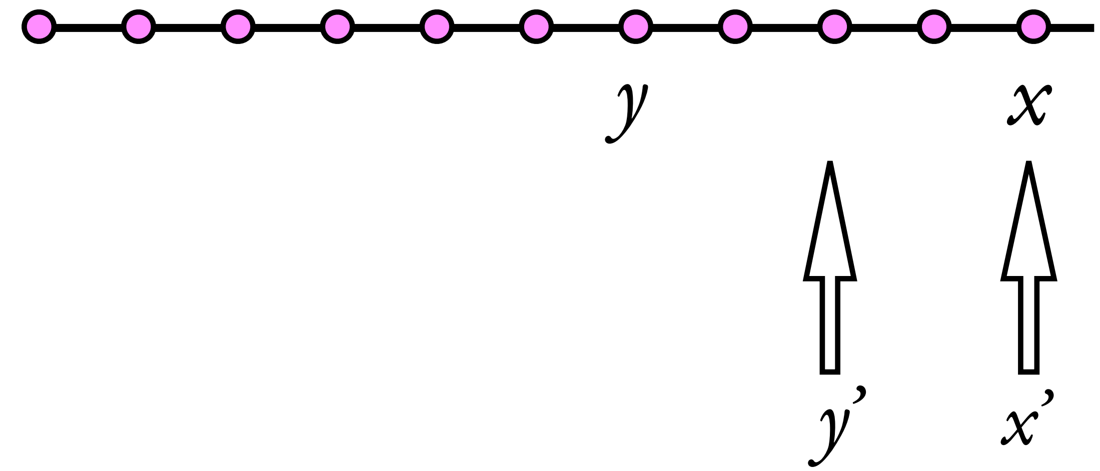

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x	5
y	0

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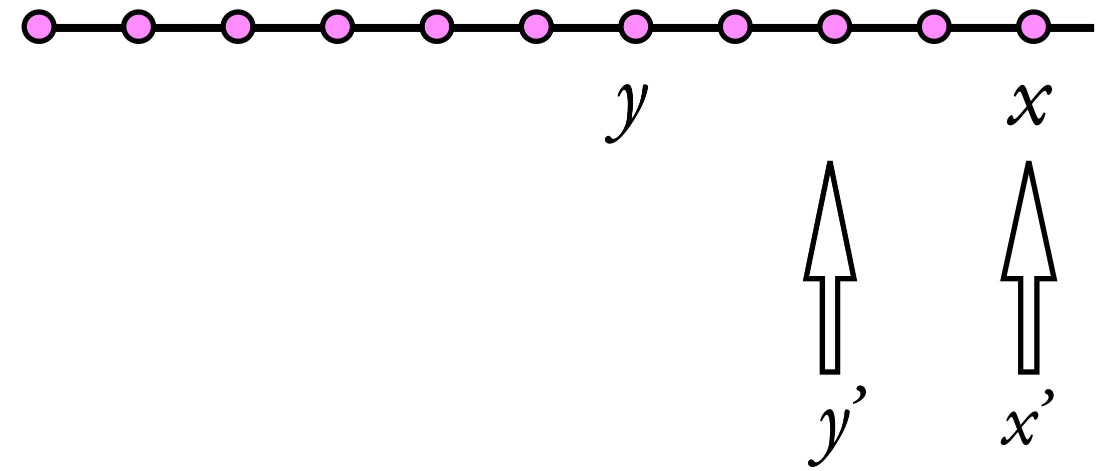

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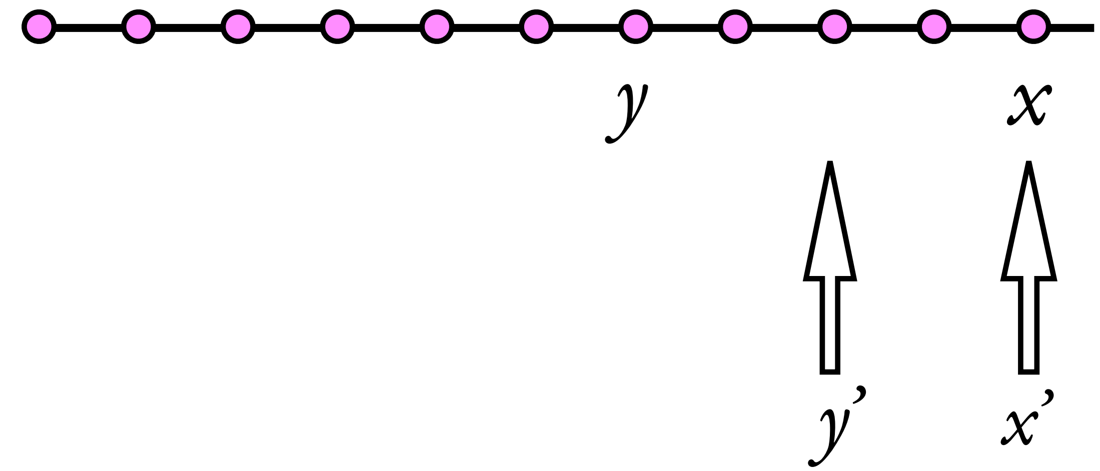

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x	5	5	5
y	0	1	2

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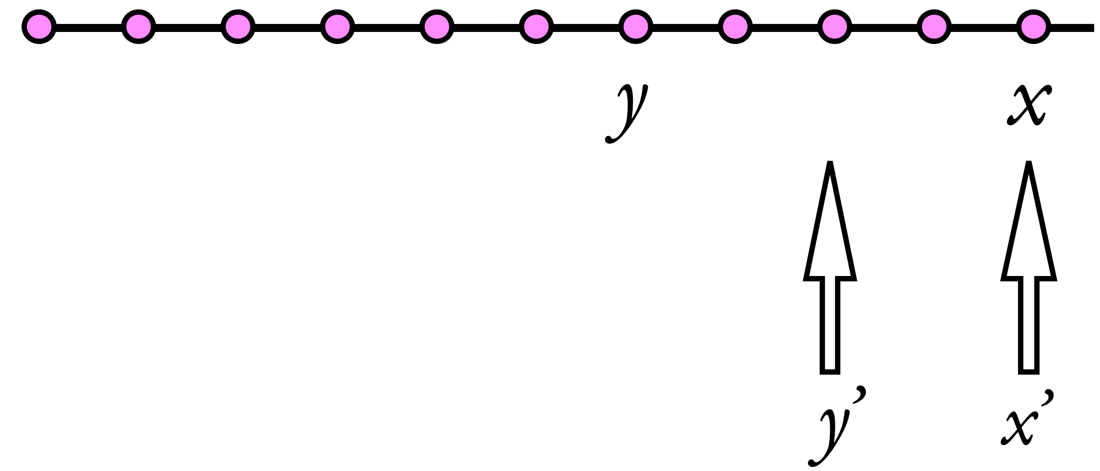

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a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x	5	5	5	5
y	0	1	2	4

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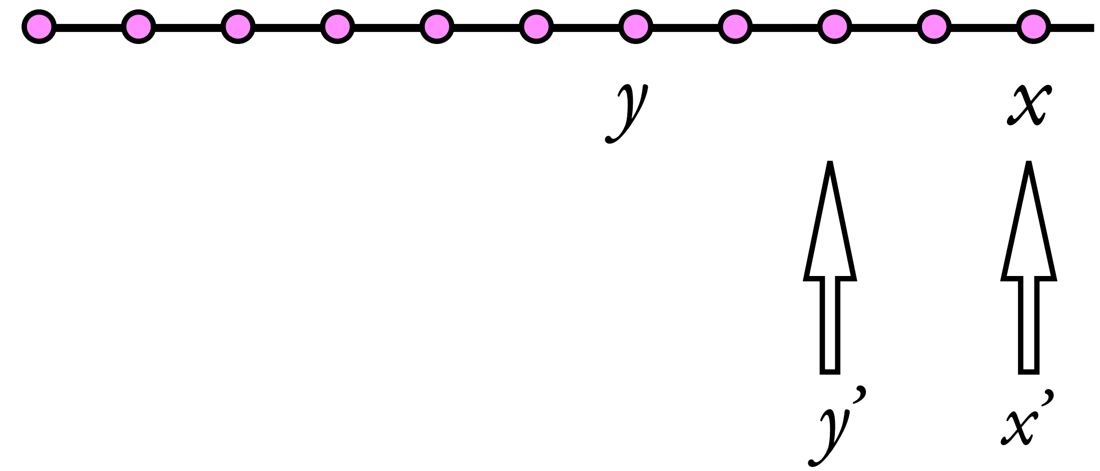

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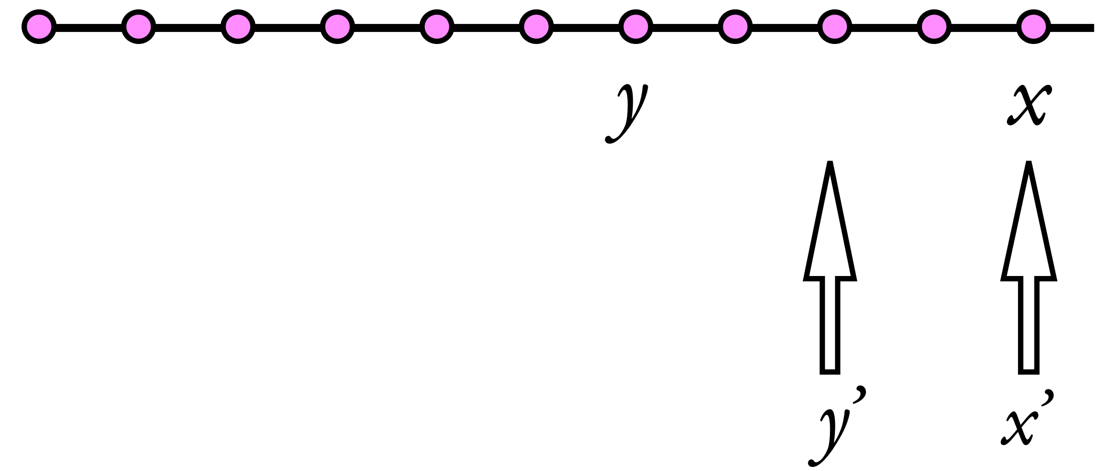

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x	5	5	5	5	5	5
y	0	1	2	4	0	3

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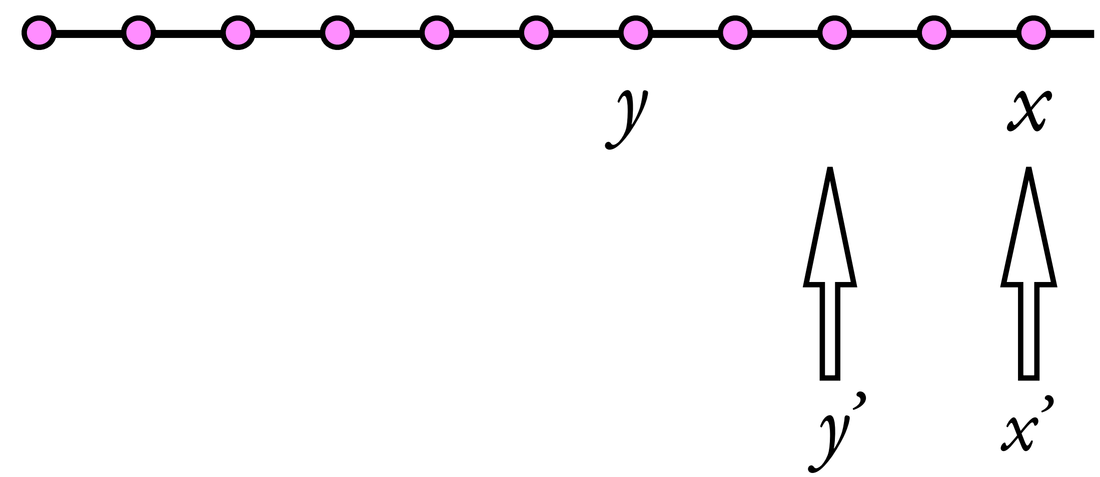

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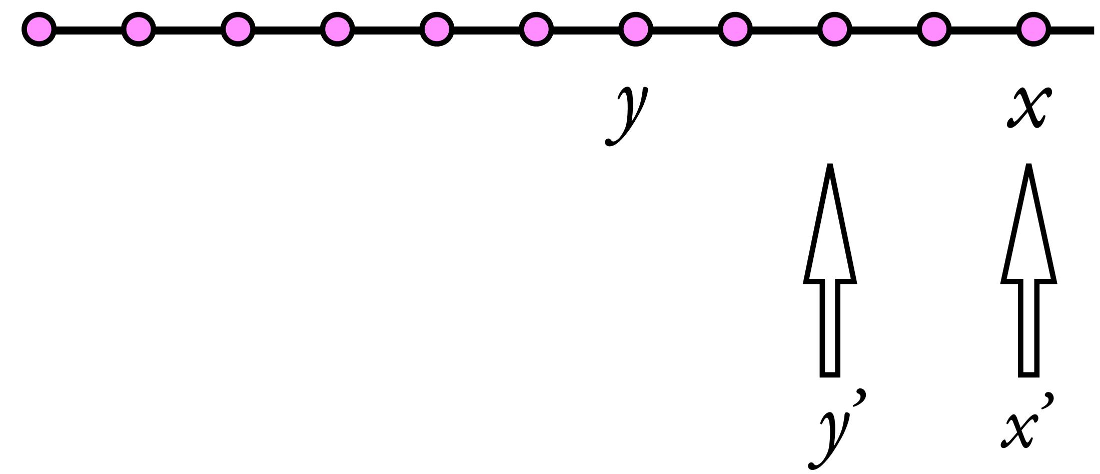

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x	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$

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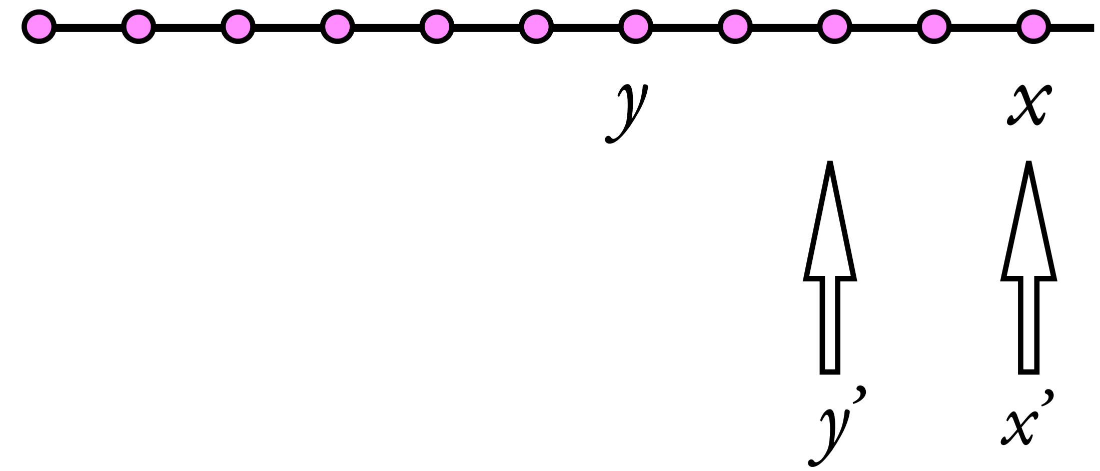

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x	5	5	5	5	5	5	5	5	5
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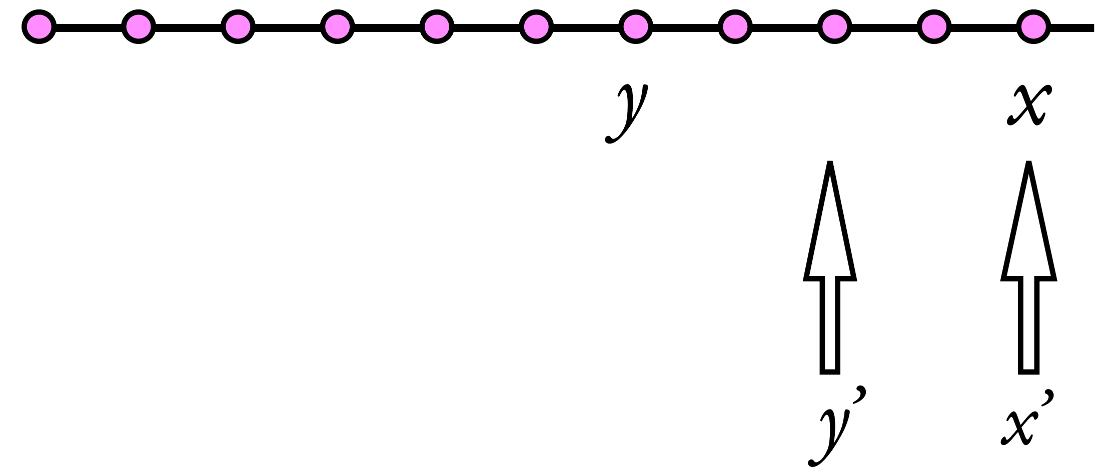

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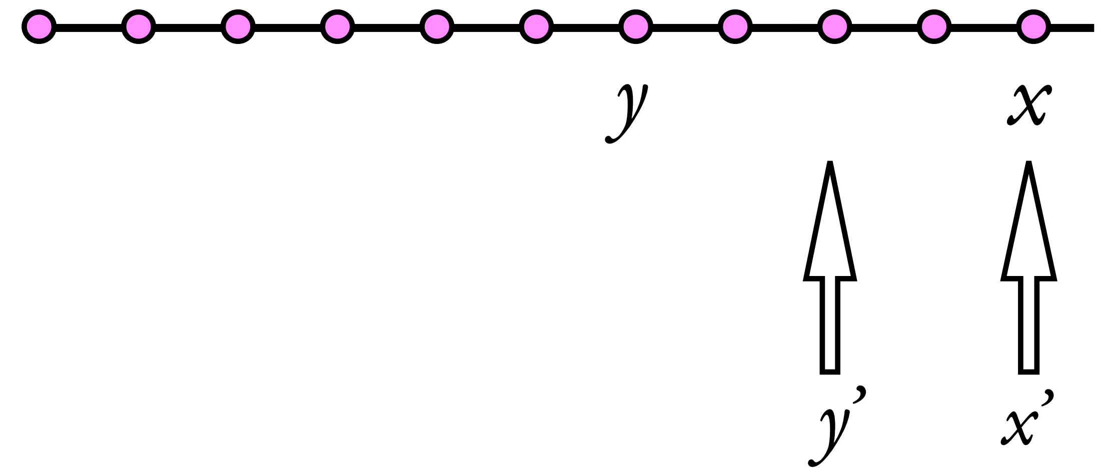

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x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$

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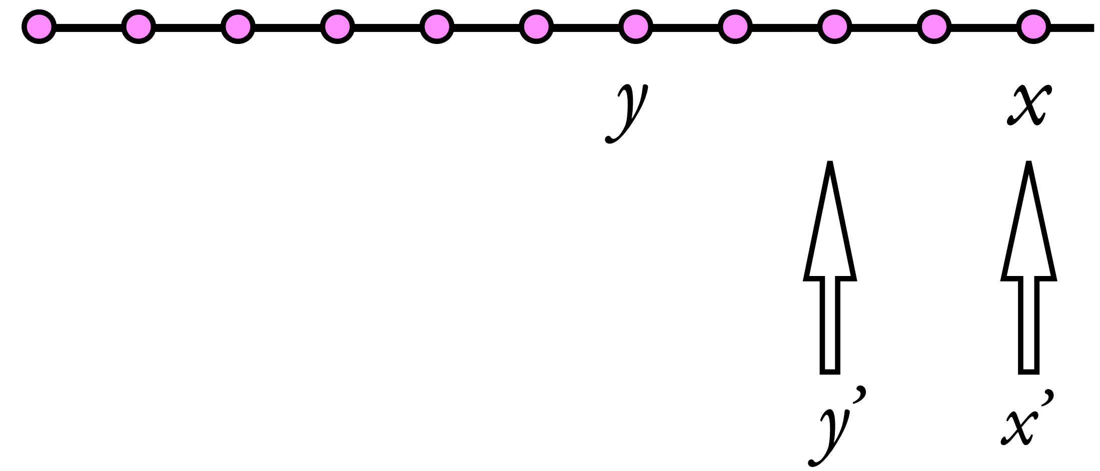

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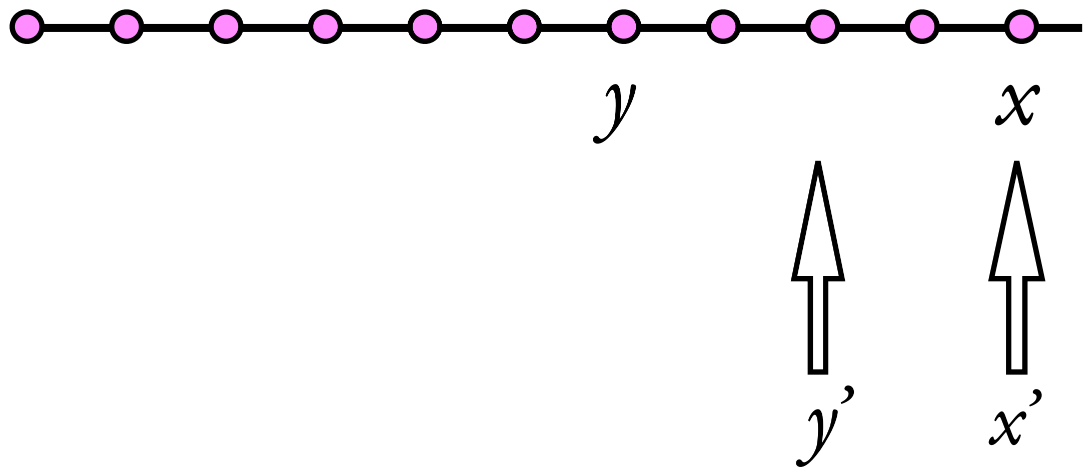
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

Example D-automaton $\mathcal{A}$

x, y – variables of $\mathcal{A}$

no states



$\mathcal{A}$:

- τ_I: $(x>0) \wedge (y=0)$
- δ_a: $(x'=x) \wedge (y<y'<x)$
- δ_b: $(x'=x) \wedge (y'=0)$
- τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

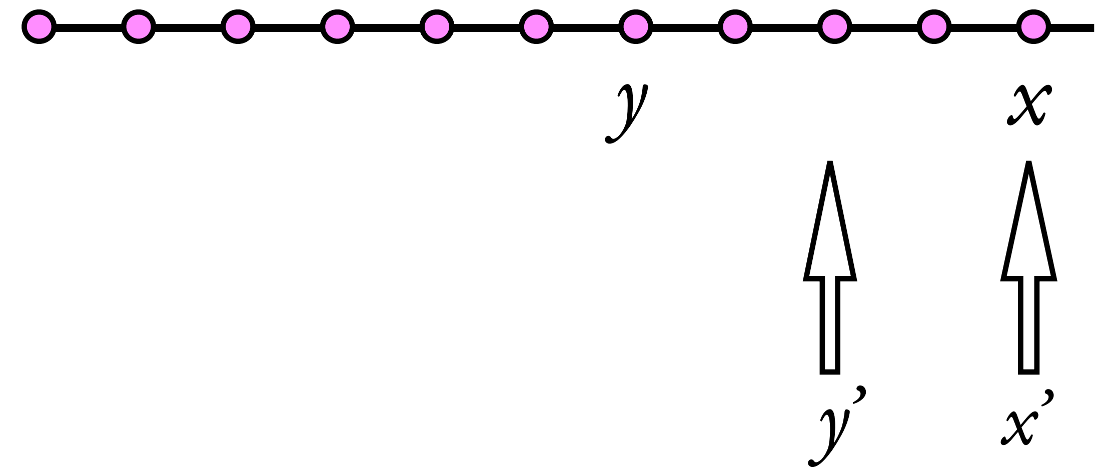
a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$\mathcal{A}$:

- τ_I: $(x>0) \wedge (y=0)$
- δ_a: $(x'=x) \wedge (y<y'<x)$
- δ_b: $(x'=x) \wedge (y'=0)$
- τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

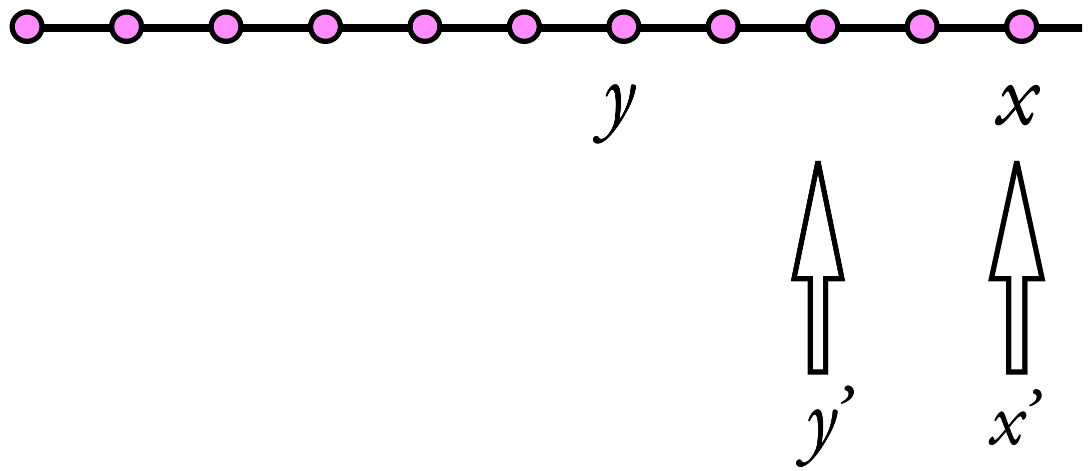
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$\mathcal{A}$:

- τ_I: $(x>0) \wedge (y=0)$
- δ_a: $(x'=x) \wedge (y<y'<x)$
- δ_b: $(x'=x) \wedge (y'=0)$
- τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

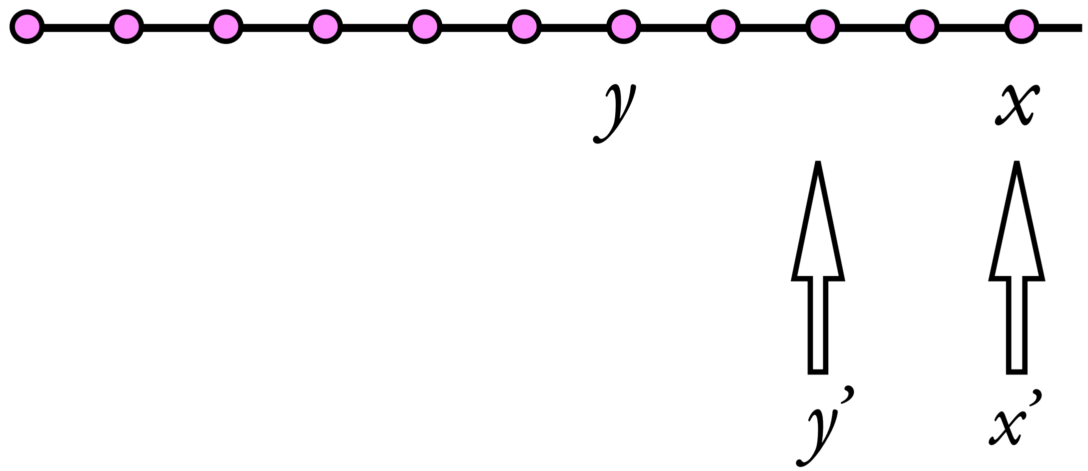
accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

Example D-automaton $\mathcal{A}$

x, y – variables of $\mathcal{A}$

no states



$\mathcal{A}$

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

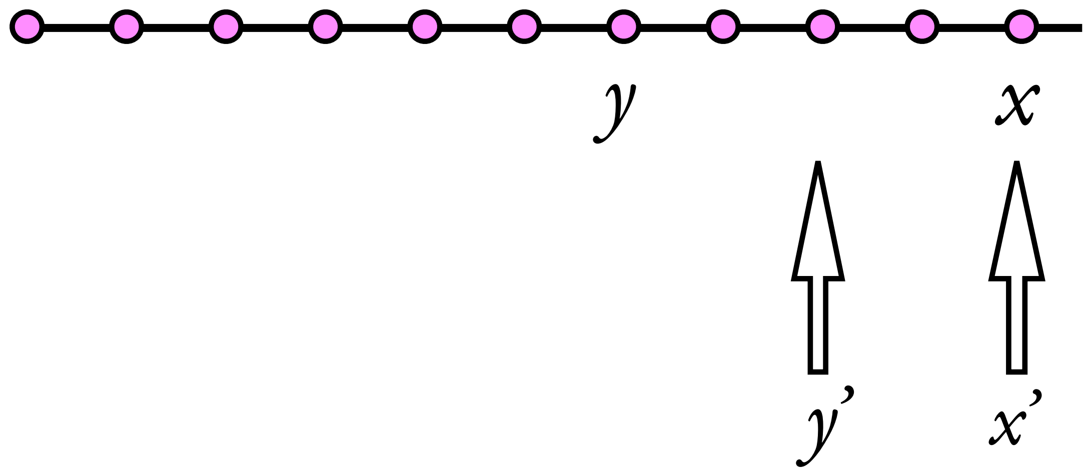
a finite run over $\mathbb{N}$:

a a a b a b a a a a

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$$\mathcal{A} \begin{cases} \tau_I: & (x>0) \wedge (y=0) \\ \delta_a: & (x'=x) \wedge (y<y'<x) \\ \delta_b: & (x'=x) \wedge (y'=0) \\ \tau_F: & (y=1) \end{cases}$$

a finite run over $\mathbb{Q}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a$

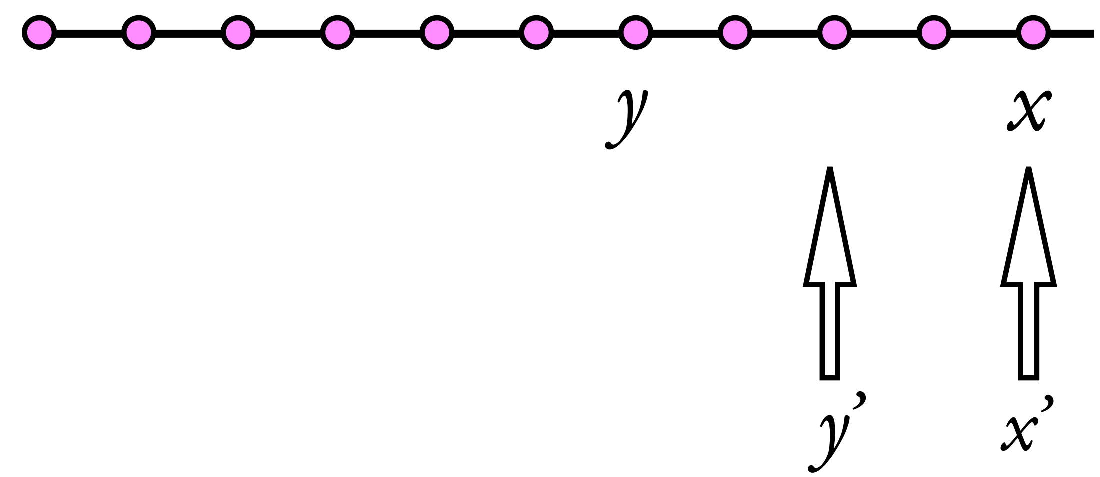
x

y

Example D-automaton $\mathcal{A}$

x, y – variables of $\mathcal{A}$

no states



$\mathcal{A}$:

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

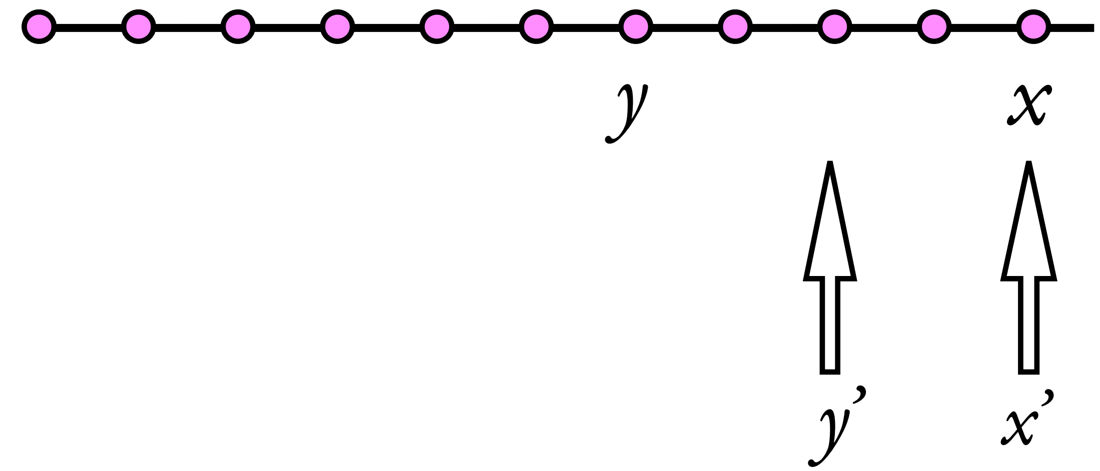
a a a b a b a a a a

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$\mathcal{A}$:

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δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a$

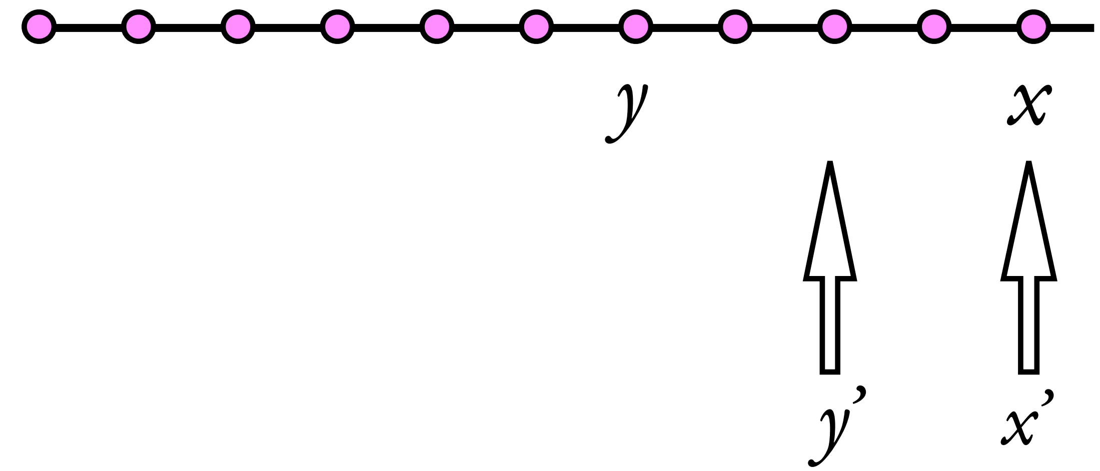
x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

accepted language: $(a^*b)^*a$

Example D-automaton $\mathcal{A}$

x, y – variables of $\mathcal{A}$

no states



$\mathcal{A}$

- τ_I: $(x>0) \wedge (y=0)$
- δ_a: $(x'=x) \wedge (y<y'<x)$
- δ_b: $(x'=x) \wedge (y'=0)$
- τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

		a	a	a	b	a	b	a	a	a	a	a
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

		a	a	a	b	a	b	a	a	a	a
x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

accepted language: $(a^*b)^*\,a$

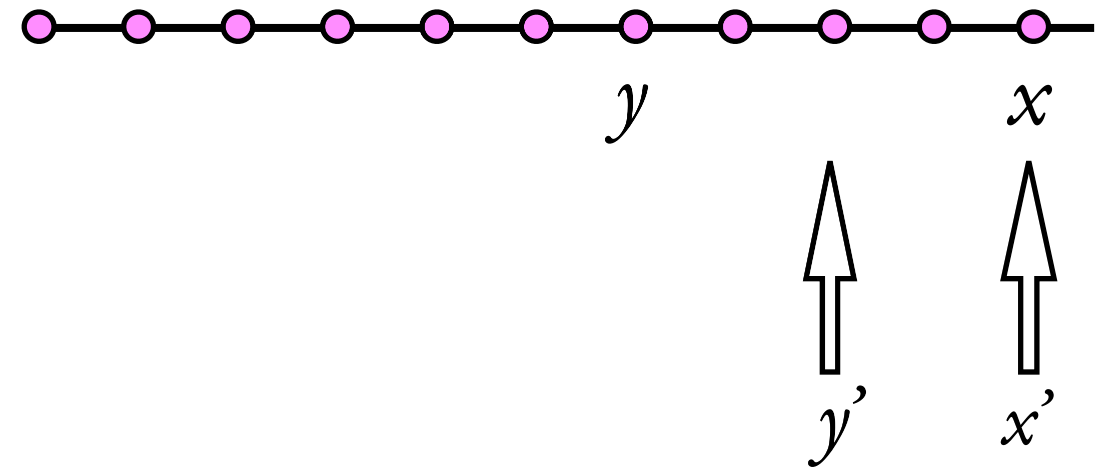
an infinite run over $\mathbb{N}$:

		a	a	a	b	a	b	a	a	a	a
x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$$\mathcal{A}\ \begin{cases} \tau_I: & (x>0)\ \wedge\ (y=0) \\ \delta_a: & (x'=x)\ \wedge\ (y<y'<x) \\ \delta_b: & (x'=x)\ \wedge\ (y'=0) \\ \tau_F: & (y=1) \end{cases}$$

a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

a a a b a b a a a a

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

a a a b a b a a a a

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

accepted language: $(a^{B}b)^{\omega}$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x' > x) \wedge R(x) \wedge \neg S(x, y')$

δ_b: $(x' = 0)$

τ_F: $(x=1)$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
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δ_a: $(x' > x) \wedge R(x) \wedge \neg S(x, y')$

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EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M?*

a finite run over $\mathbb{N}$:

a a a b a b a

Adding database constraints

$x,y,z,\dots$ – a finite set of variables
$R,S,\dots$ – a finite set of relational symbols

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δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

a a a b a b a

$\}\exists$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

a a a b a b a

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$ $\Big\}\exists$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

a a a b a b a

x 0 2 3 4 0 3 0 1

R={(0),(1),(2),(3),(4)} S={} } $\exists$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

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δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

		a	*a*	*a*	*b*	*a*	*b*	*a*	
x	0	2	3	4	0	3	0	1

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$ $\Big\}\exists$

accepted language: $(a^*b)^*a$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_I: $(x{=}0)$

δ_a: $(x'{>}x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'{=}0)$

τ_F: $(x{=}1)$

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a a a b a b a

x	0	2	3	4	0	3	0	1

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$ $\}\exists$

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

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$R, S, \ldots$ – a finite set of relational symbols

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δ_a: $(x' > x) \wedge R(x) \wedge \neg S(x, y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

a a a b a b a

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={} } $\exists$

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

a a a b a b a a b a a…

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
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τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

a a a b a b a

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={} $\Big\}\exists$

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

a a a b a b a a b a a…

$R=\{(0),(1),(2),(3),(4)\}$ S={}

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x' > x) \wedge R(x) \wedge \neg S(x, y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={}

$\}\exists$

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ b\ a\ a \ldots$

x 0 1 3 4 0 3 0 1 4 0 1 3

$R=\{(0),(1),(2),(3),(4)\}$ S={}

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x' > x) \wedge R(x) \wedge \neg S(x, y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

	a	*a*	*a*	*b*	*a*	*b*	*a*	
x	0	2	3	4	0	3	0	1

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$ $\}\exists$

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

	a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*b*	*a*	*a*…	
x	0	1	3	4	0	3	0	1	4	0	1	3

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$

accepted language: $(a^B b)^\omega$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

Decide emptiness of *D*-automata over ω-words, with databases

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

a a a b a b a

x	0	2	3	4	0	3	0	1

R={(0),(1),(2),(3),(4)} S={} } ∃

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

a a a b a b a a b a a…

x	0	1	3	4	0	3	0	1	4	0	1	3

R={(0),(1),(2),(3),(4)} S={}

accepted language: $(a^{B}b)^{\omega}$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

1. infinite, ordered data

Decide emptiness of D-automata over ω-words, with databases

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

x: 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$ $\}\exists$

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ b\ a\ a \ldots$

x: 0 1 3 4 0 3 0 1 4 0 1 3

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$

accepted language: $(a^B b)^\omega$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

1. infinite, ordered data

Decide emptiness of D-automata over ω-words, with databases

2. unbounded database

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over ℕ:

$a\ a\ a\ b\ a\ b\ a$

x	0	2	3	4	0	3	0	1

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$ } ∃

accepted language: $(a^*b)^*a$

an infinite run over ℕ:

$a\ a\ a\ b\ a\ b\ a\ a\ b\ a\ a \ldots$

x	0	1	3	4	0	3	0	1	4	0	1	3

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$

accepted language: $(a^B b)^\omega$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

Decide emptiness of D-automata over ω-words, with databases

1. infinite, ordered data

2. unbounded database

3. infinite runs

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over ℕ:

$a\ a\ a\ b\ a\ b\ a$

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={} $\}\exists$

accepted language: $(a^*b)^*a$

an infinite run over ℕ:

$a\ a\ a\ b\ a\ b\ a\ a\ b\ a\ a \ldots$

x 0 1 3 4 0 3 0 1 4 0 1 3

$R=\{(0),(1),(2),(3),(4)\}$ S={}

accepted language: $(a^B b)^\omega$

Known results
deciding emptiness (for ω-runs)

	no database	with database
$\mathbb{N}$,=	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q}$,<	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
$\mathbb{N}$,<	Čerans (94) NONPRIMITIVE	?

Known results

deciding emptiness (for ω-runs)

	no database	with database
$\mathbb{N}$,=	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q}$,<	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
$\mathbb{N}$,<	Čerans (94) NONPRIMITIVE	?
$\mathscr{D}$,<	PSPACE	PSPACE

Known results

deciding emptiness (for ω-runs)

	no database	with database
$\mathbb{N}$,=	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q}$,<	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
$\mathbb{N}$,<	Čerans (94) NONPRIMITIVE	PSPACE
$\mathscr{D}$,<	PSPACE	PSPACE

Known results

deciding emptiness (for ω-runs)

	no database	with database
$\mathbb{N}$,=	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q}$,<	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
$\mathbb{N}$,<	PSPACE	PSPACE
$\mathscr{D}$,<	PSPACE	PSPACE

Outline

1. Define automata model
2. Analyze automata over dense orders
3. Analyze automata over discrete orders
4. Add database and infinite runs

Dense case

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- finite words
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The region construction

$0<y<1<x$, $0<x<1=y$, $0=1<y<x$, $0<1=y<x$, $0=y<1<x$, $0=y<1=x$, ...

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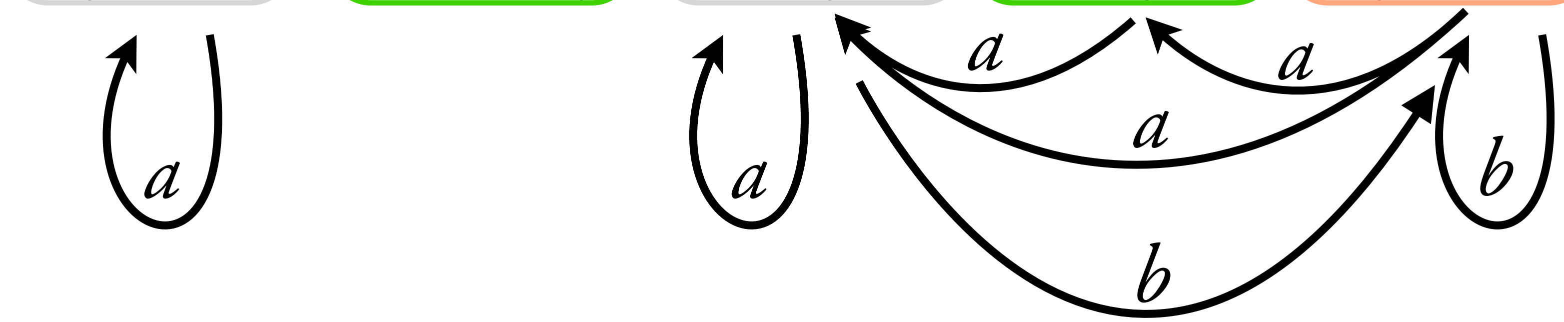

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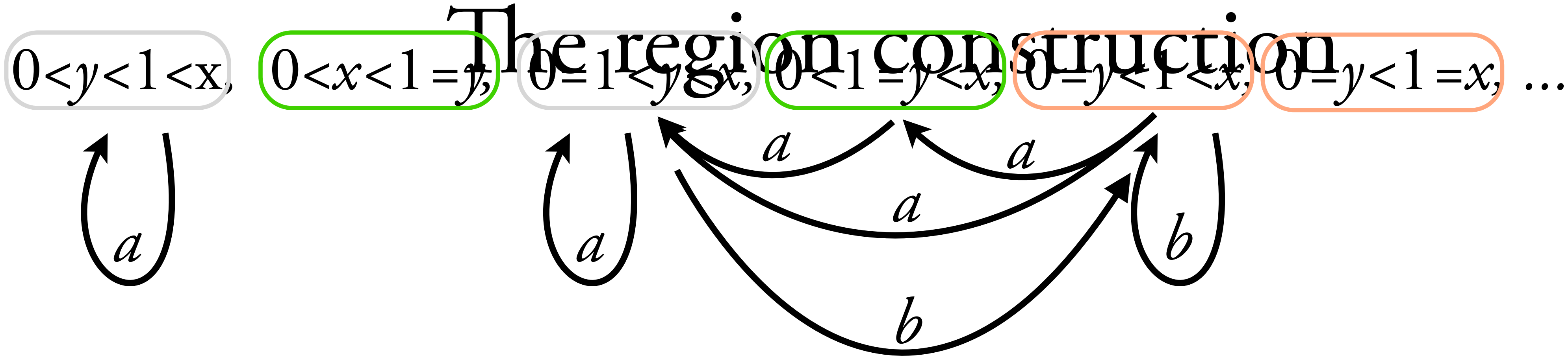

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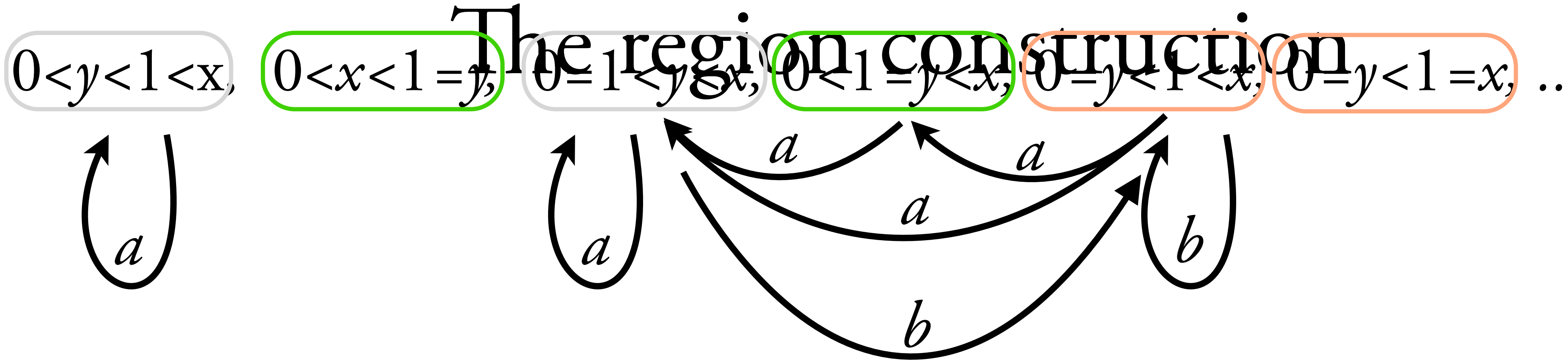

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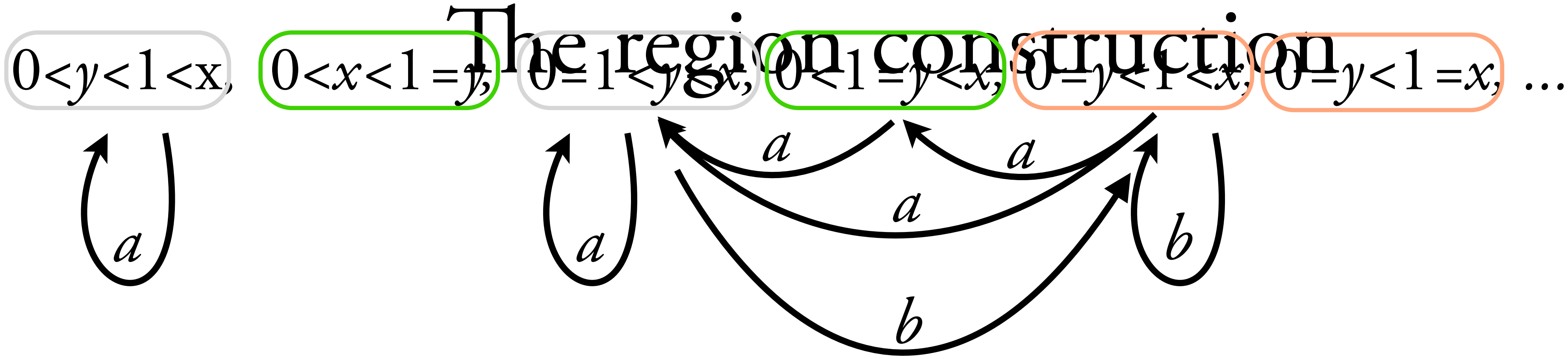

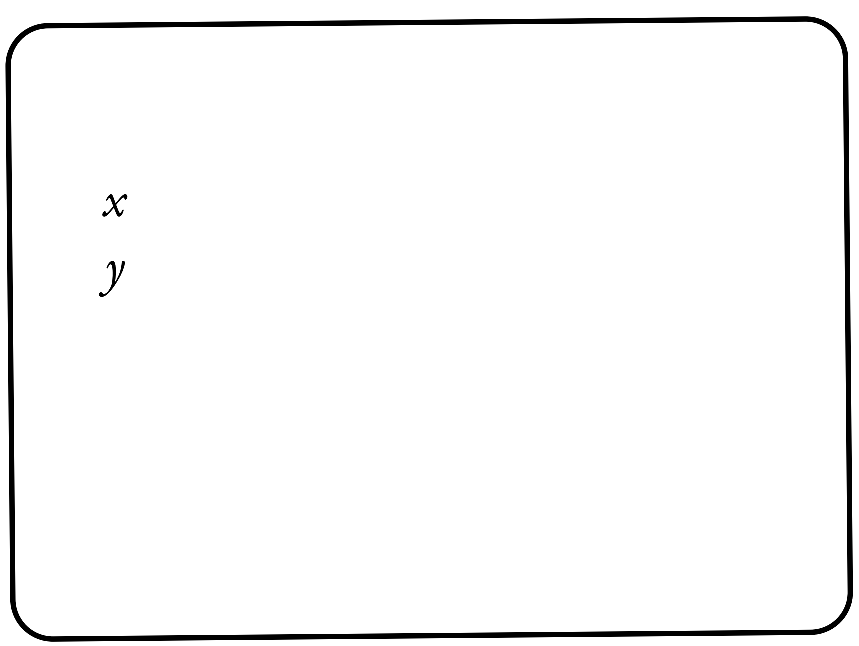

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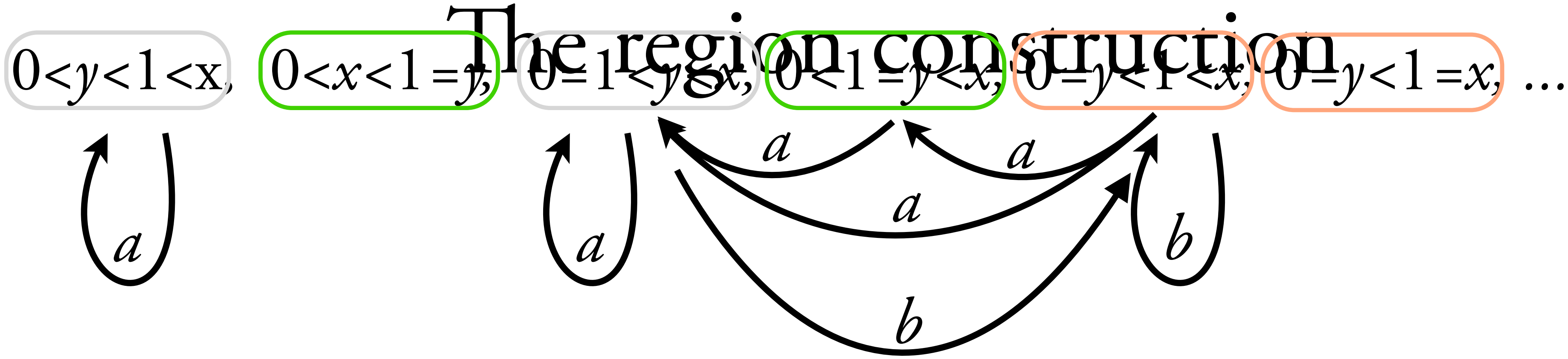

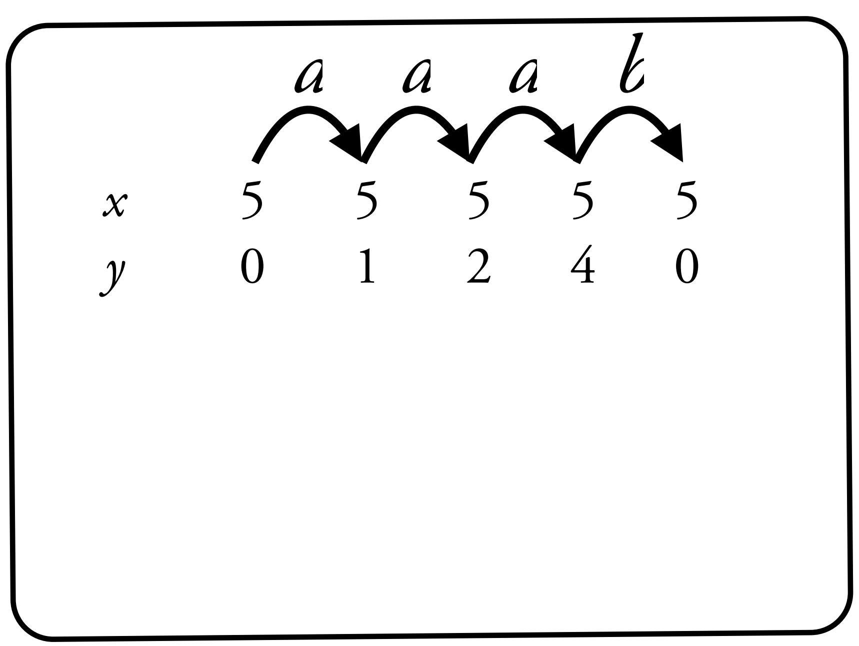

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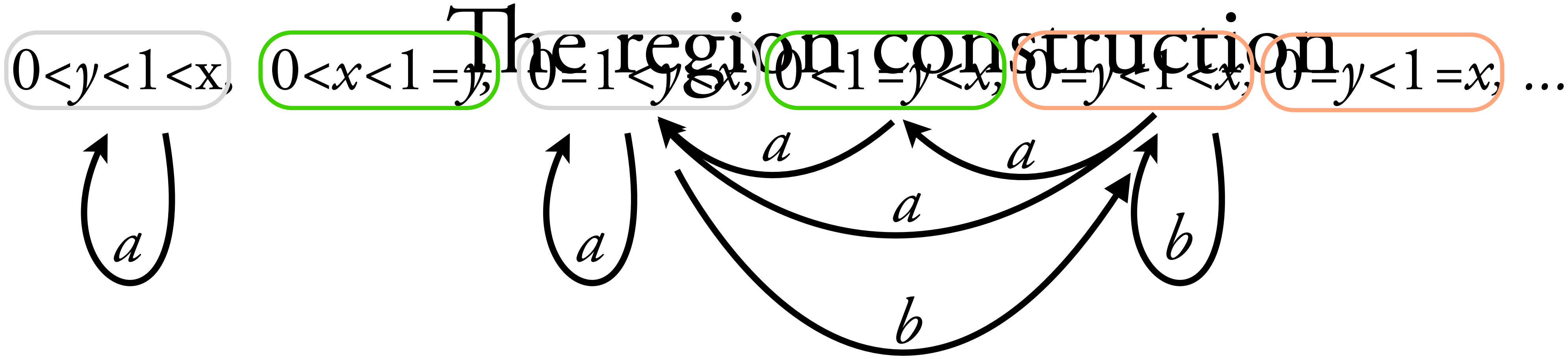

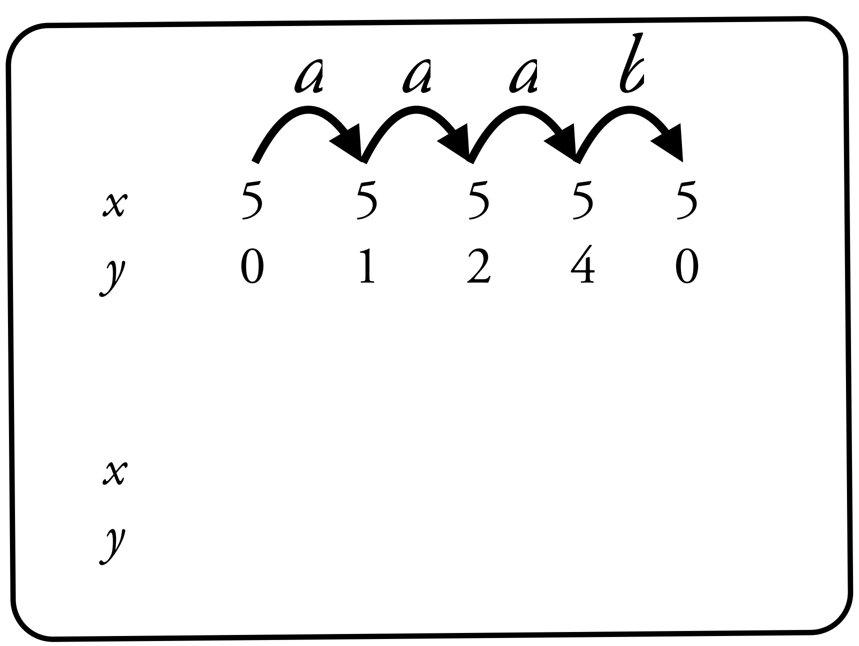

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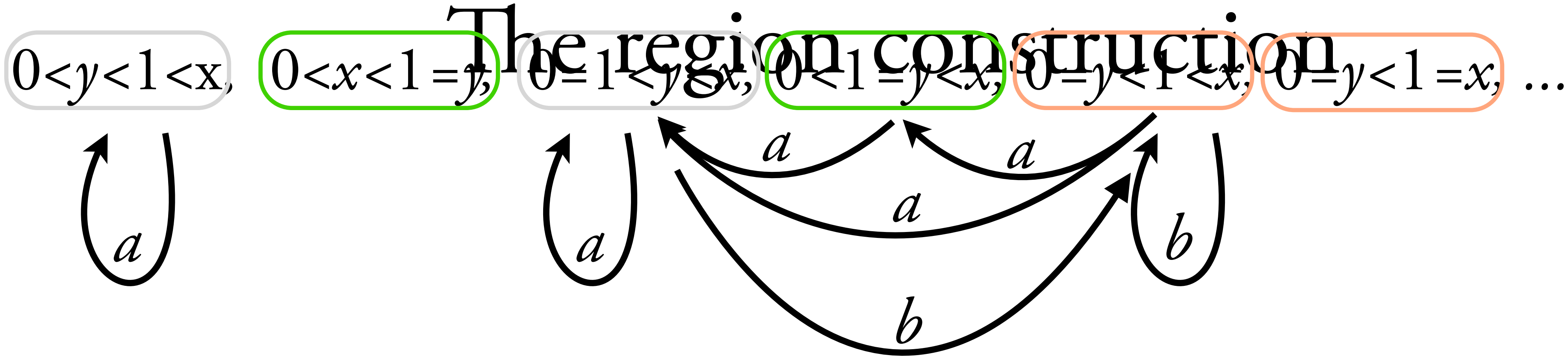

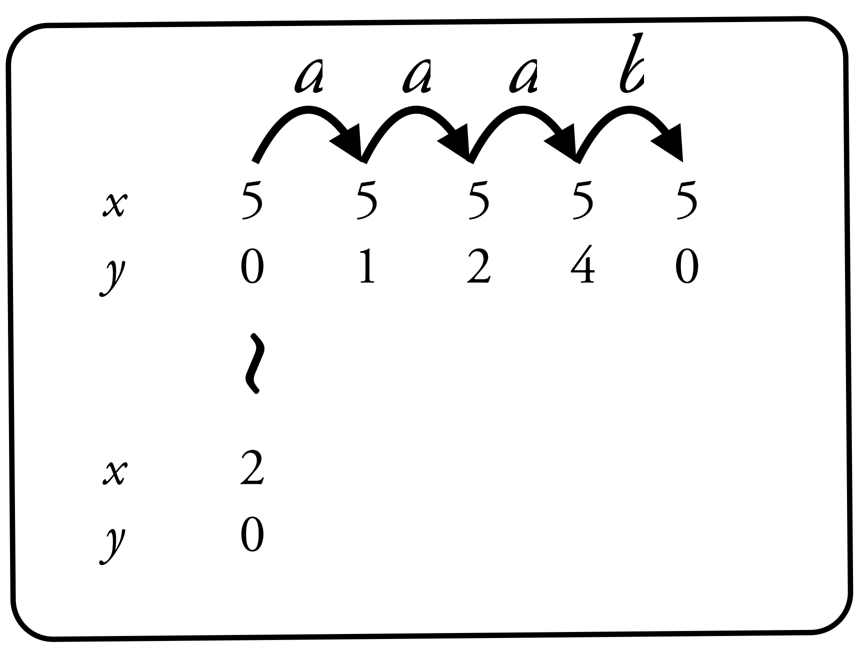

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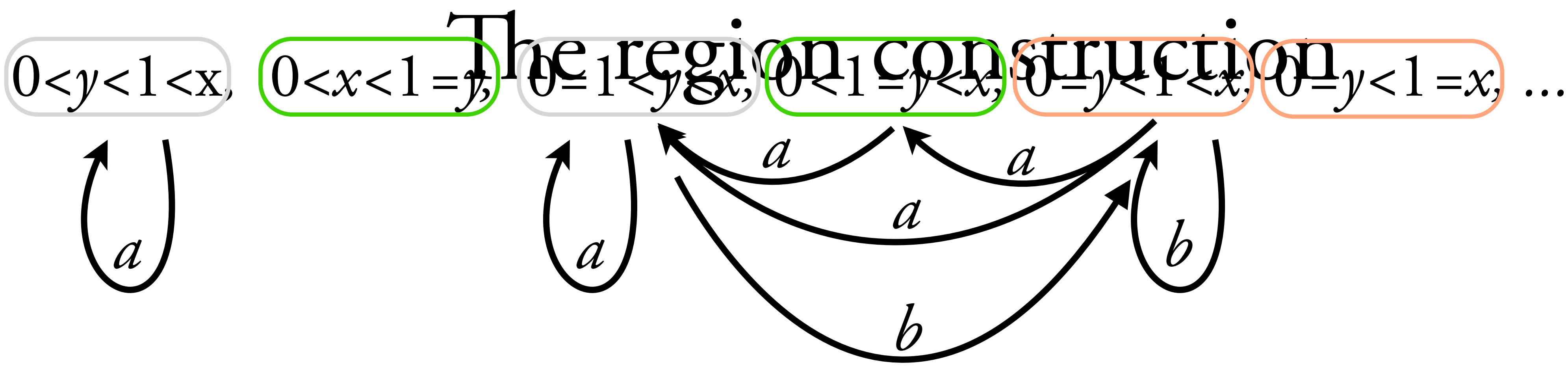

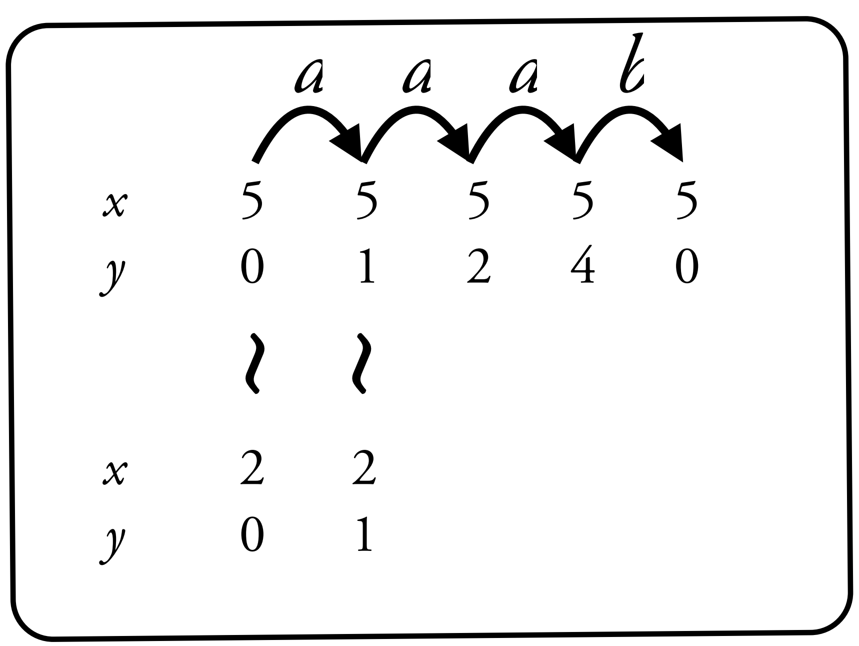

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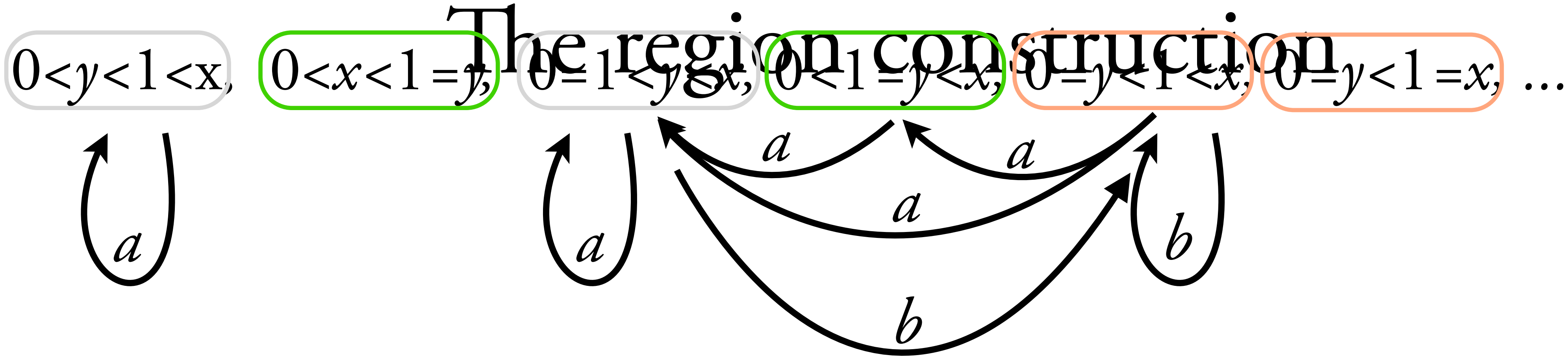

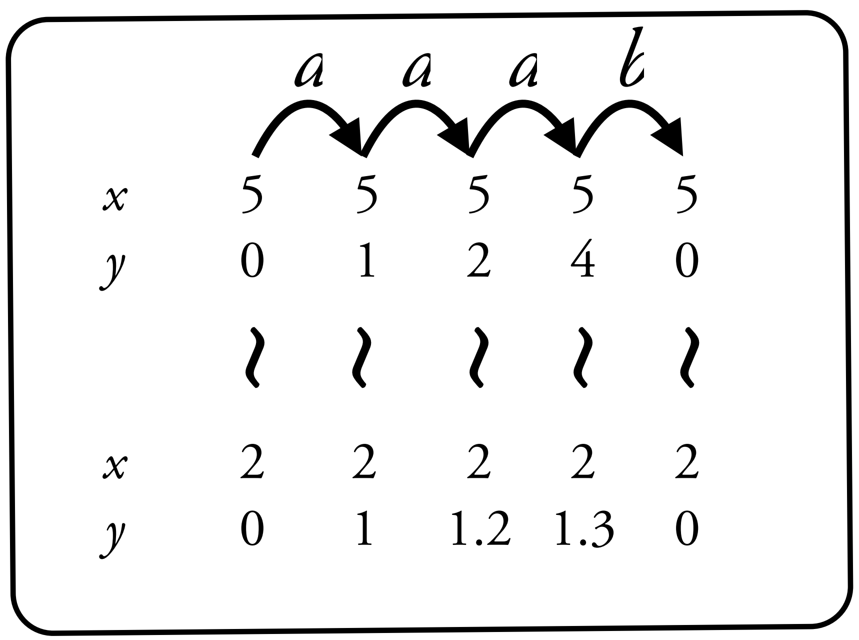

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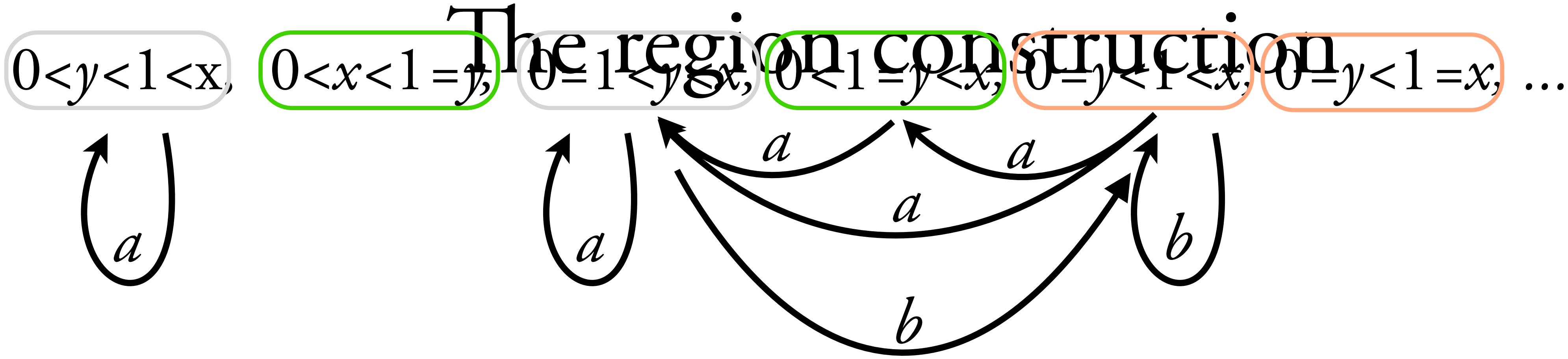

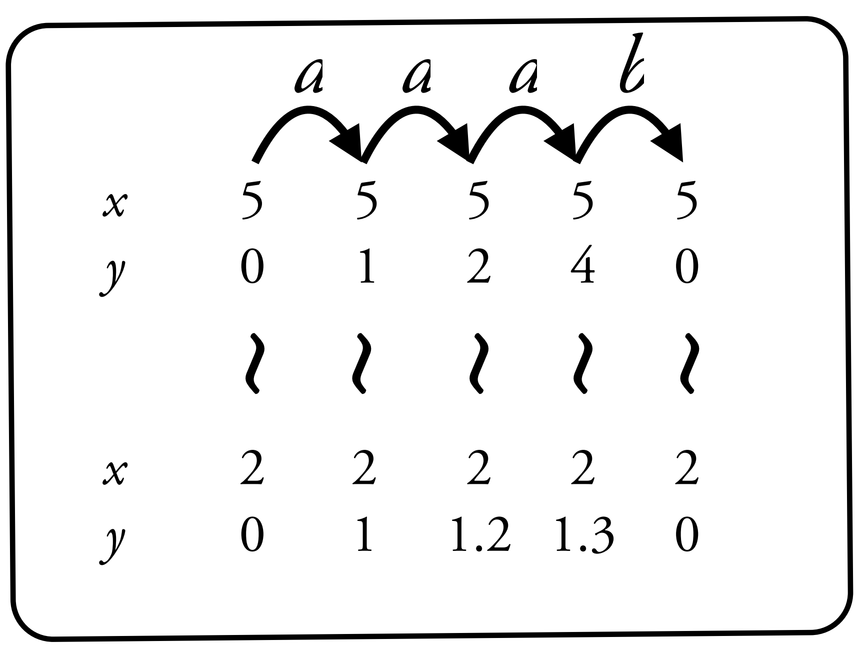

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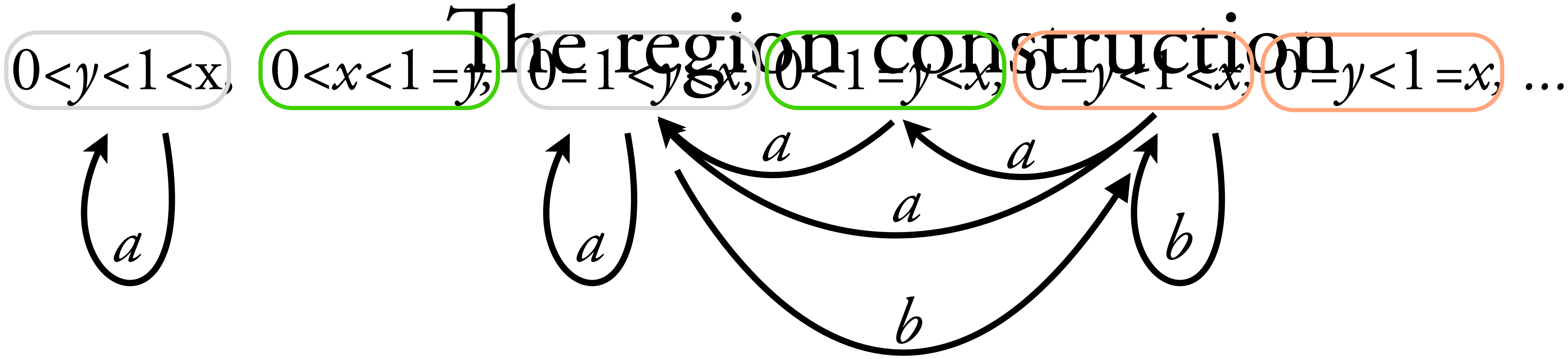

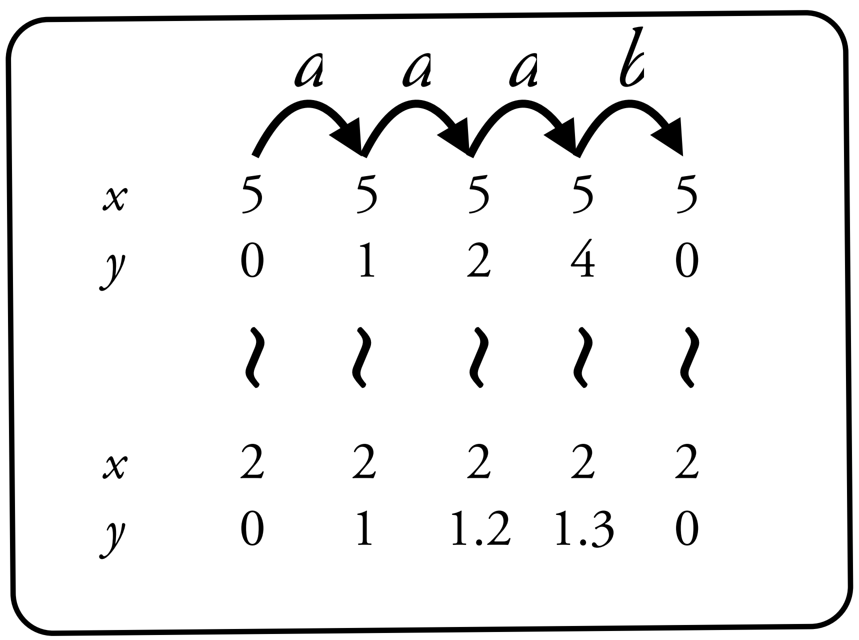

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accepted language: $(a+b)^*a$

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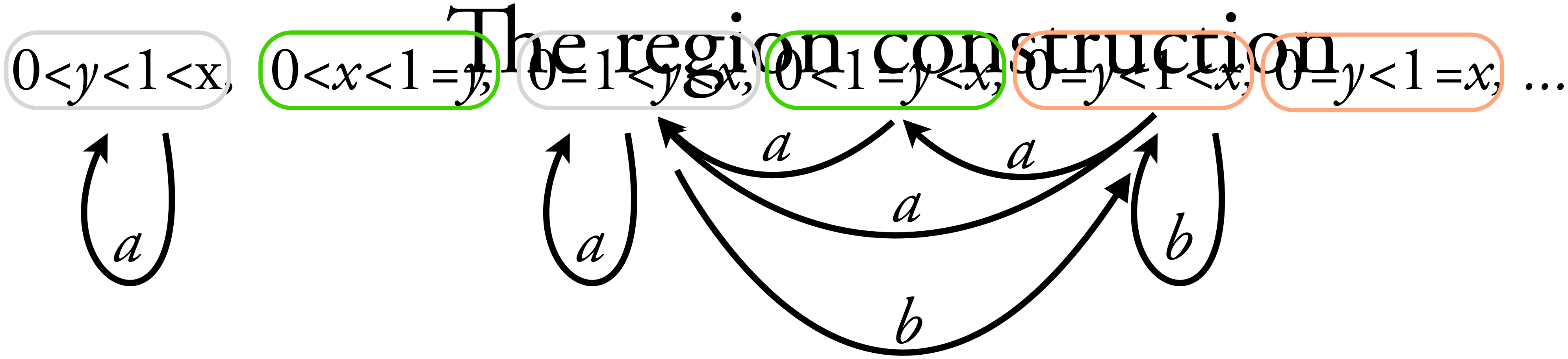

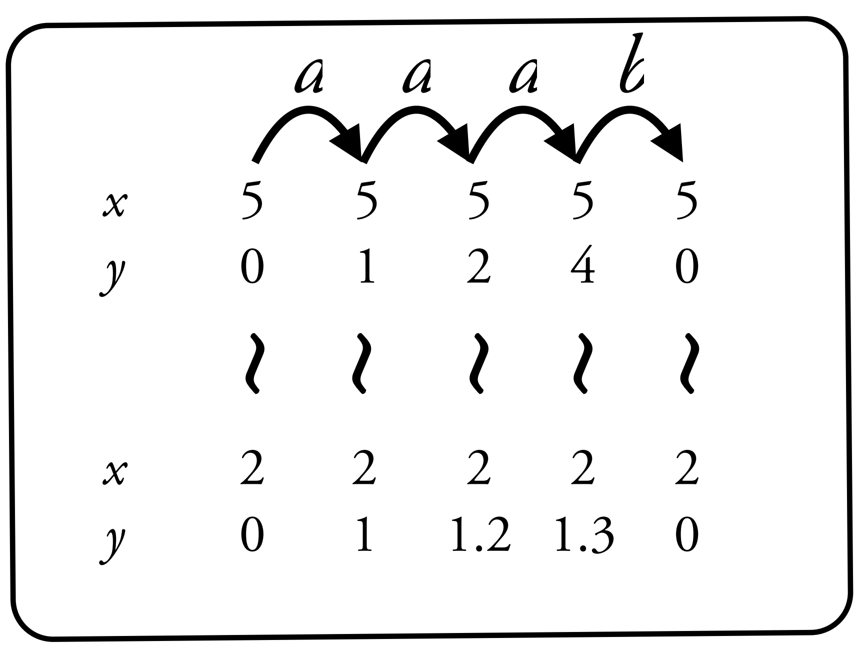

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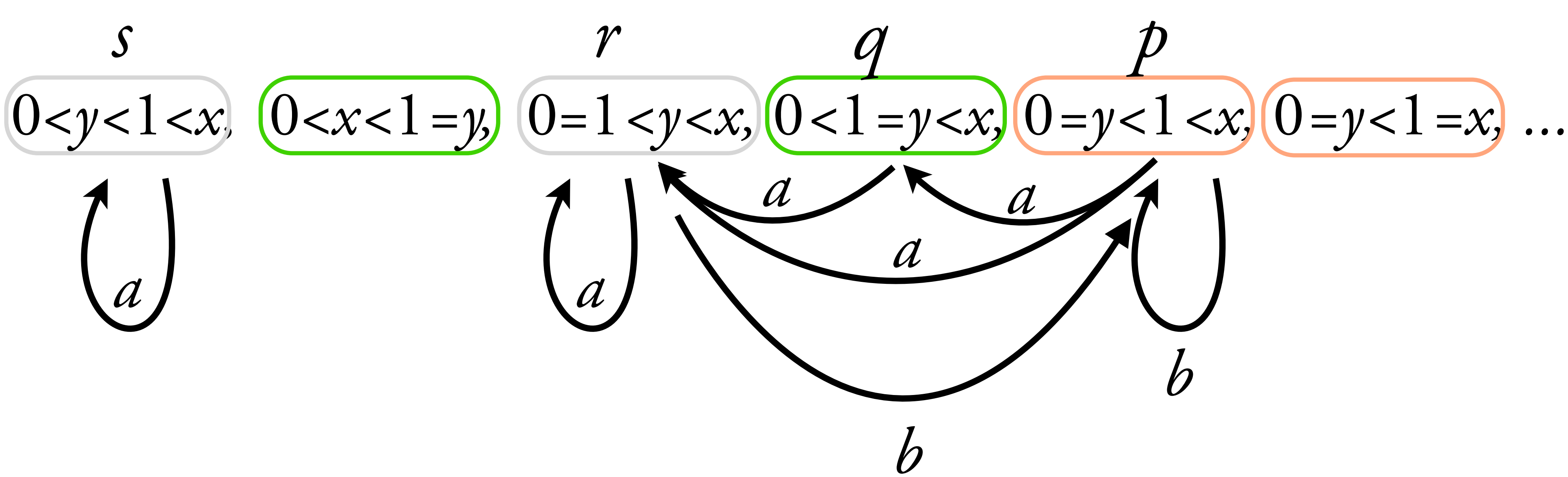

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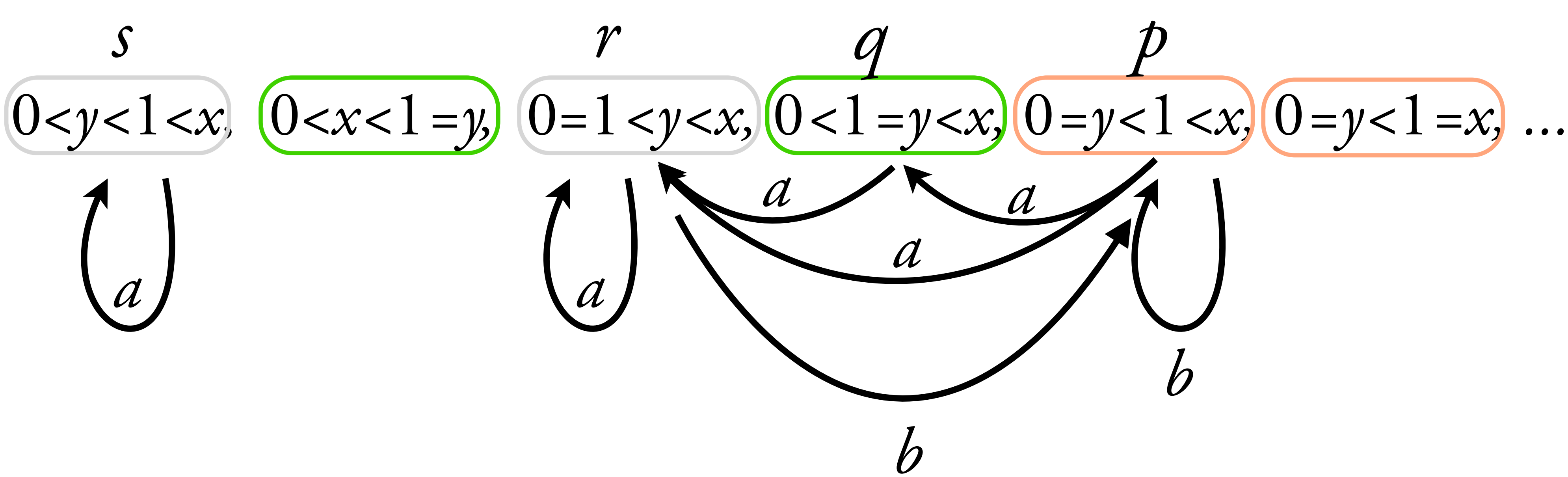

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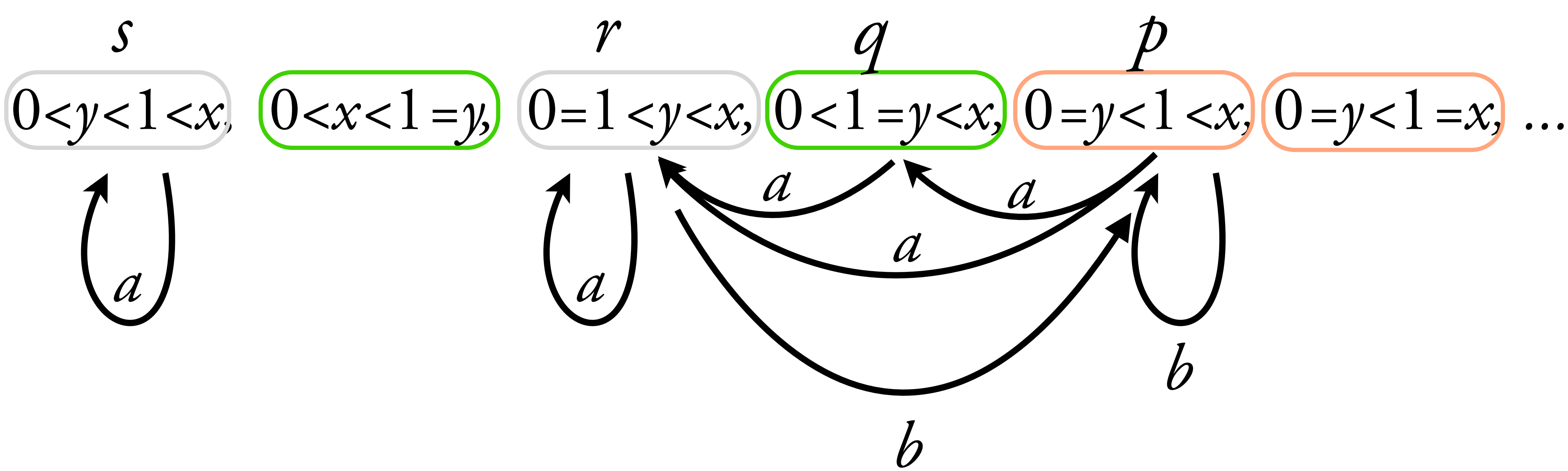

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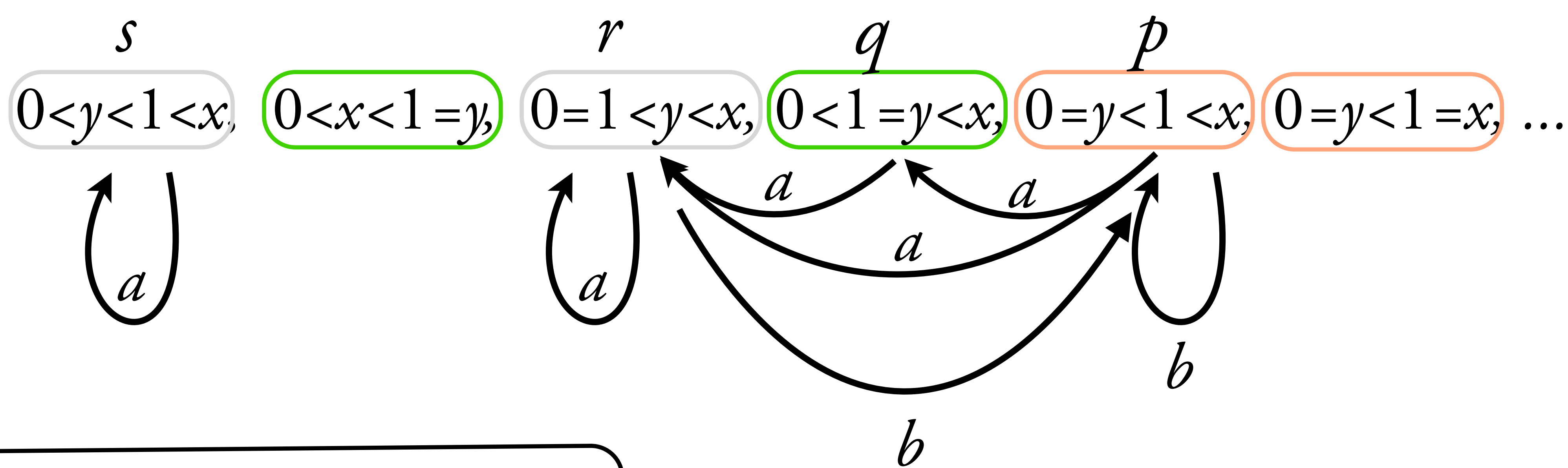

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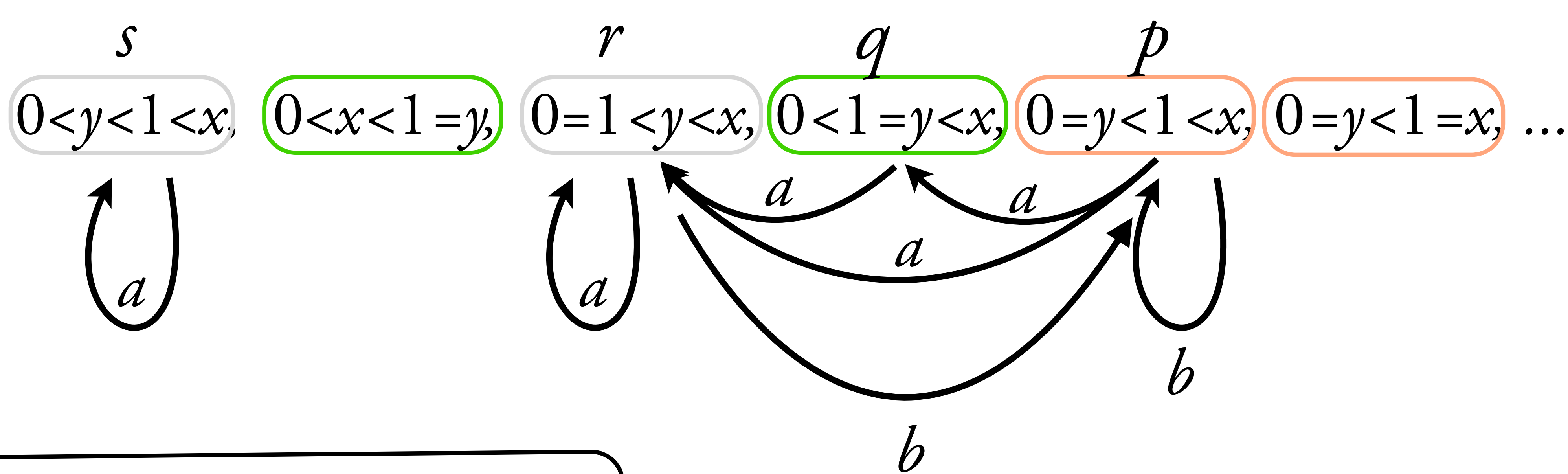

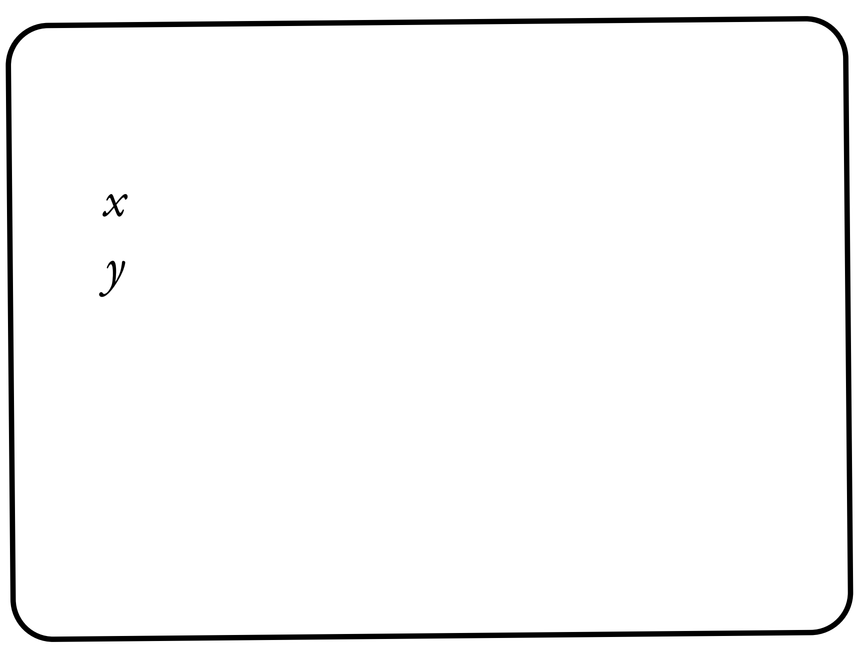

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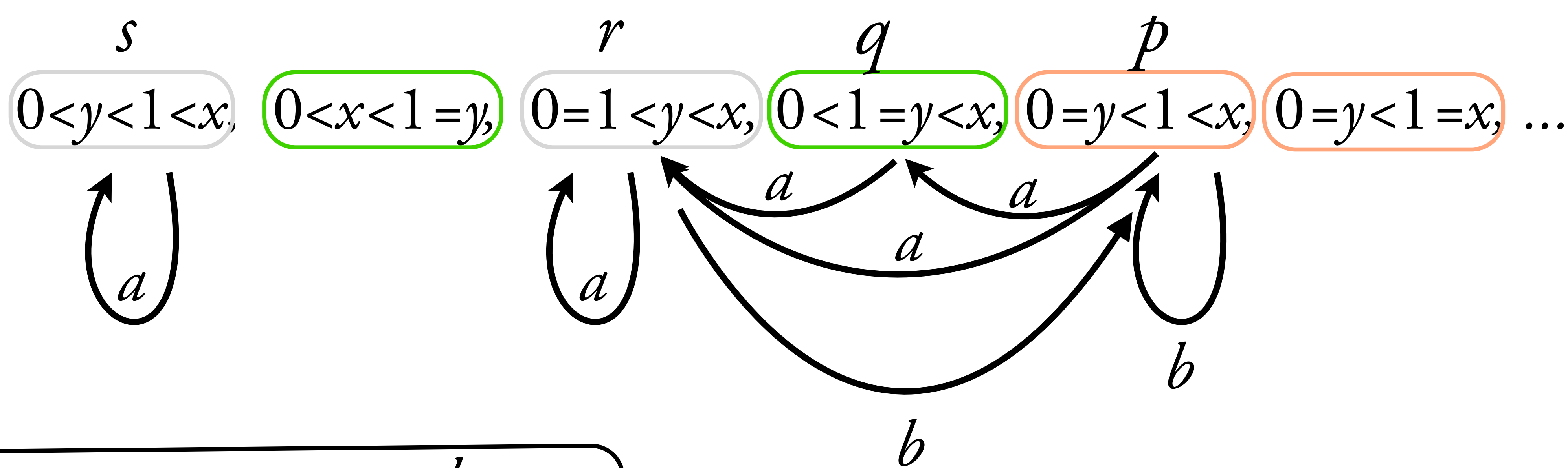

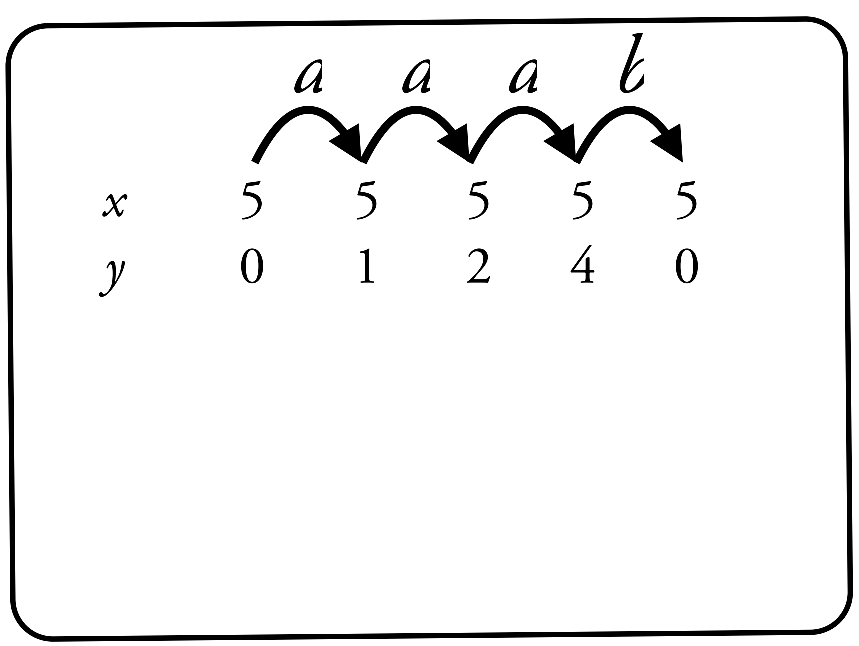

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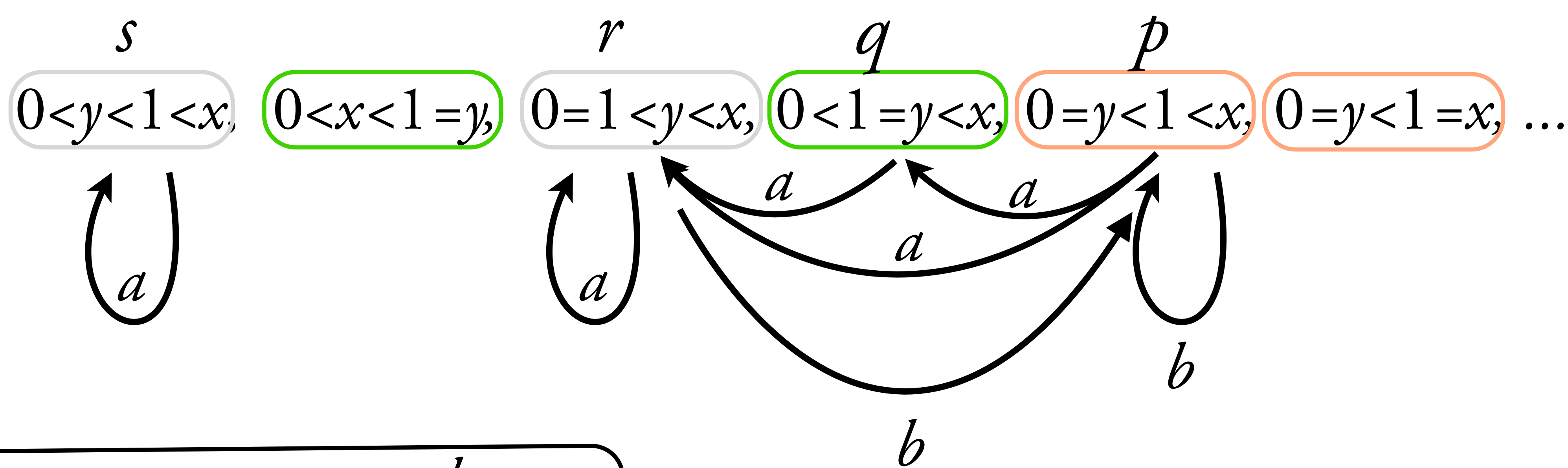

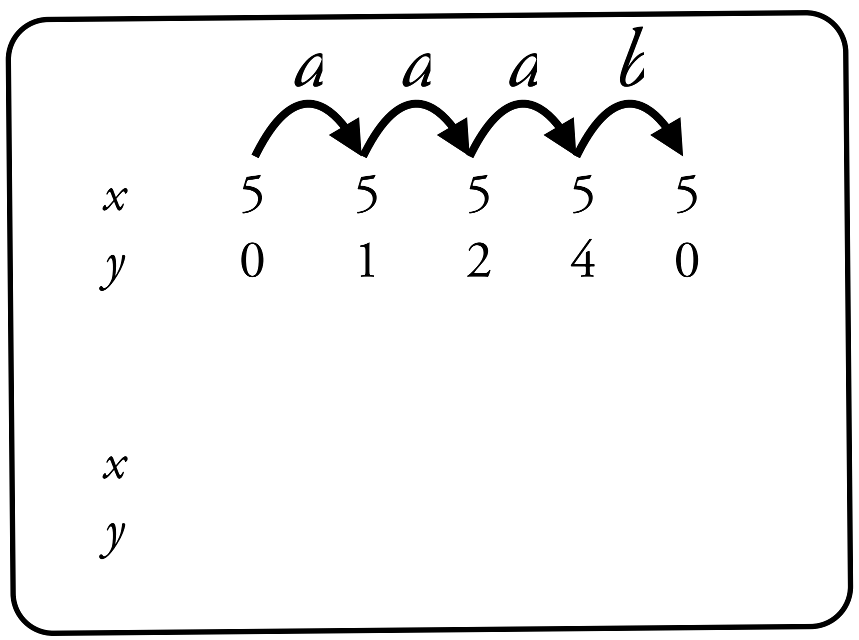

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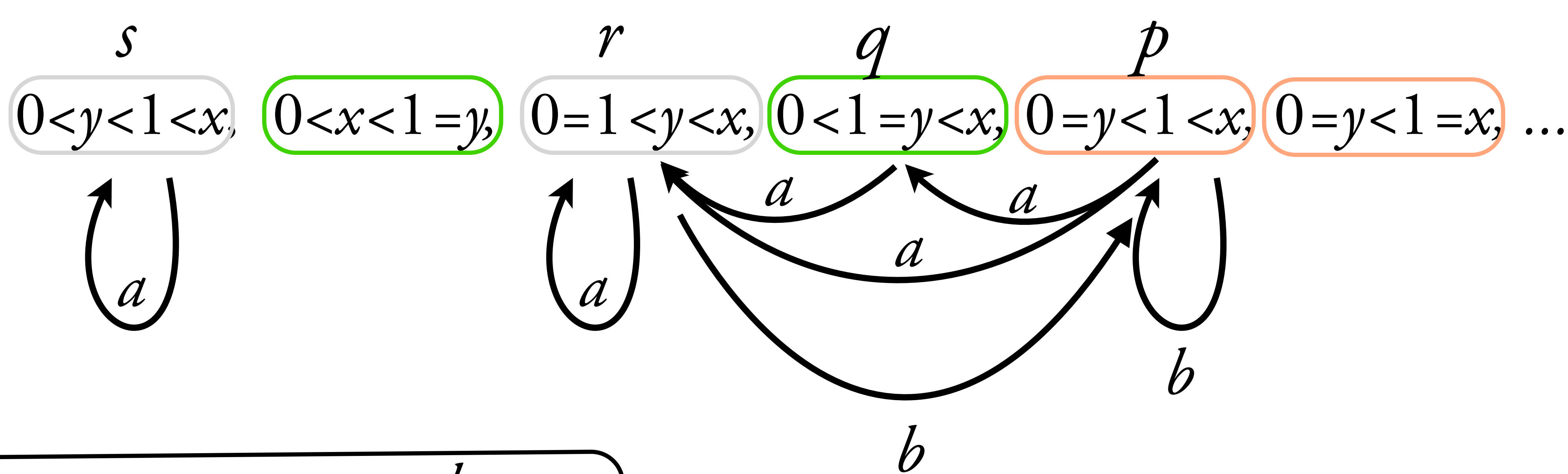

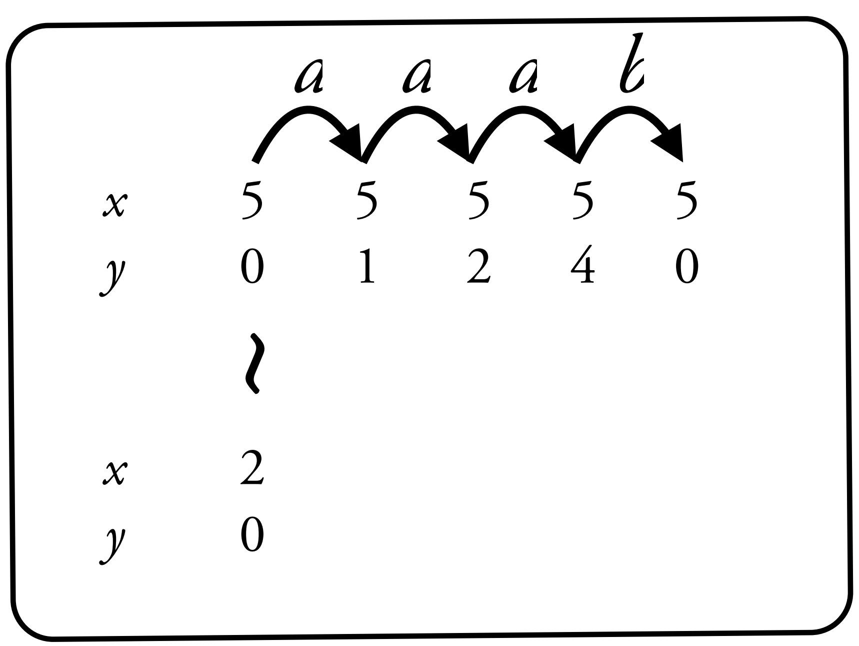

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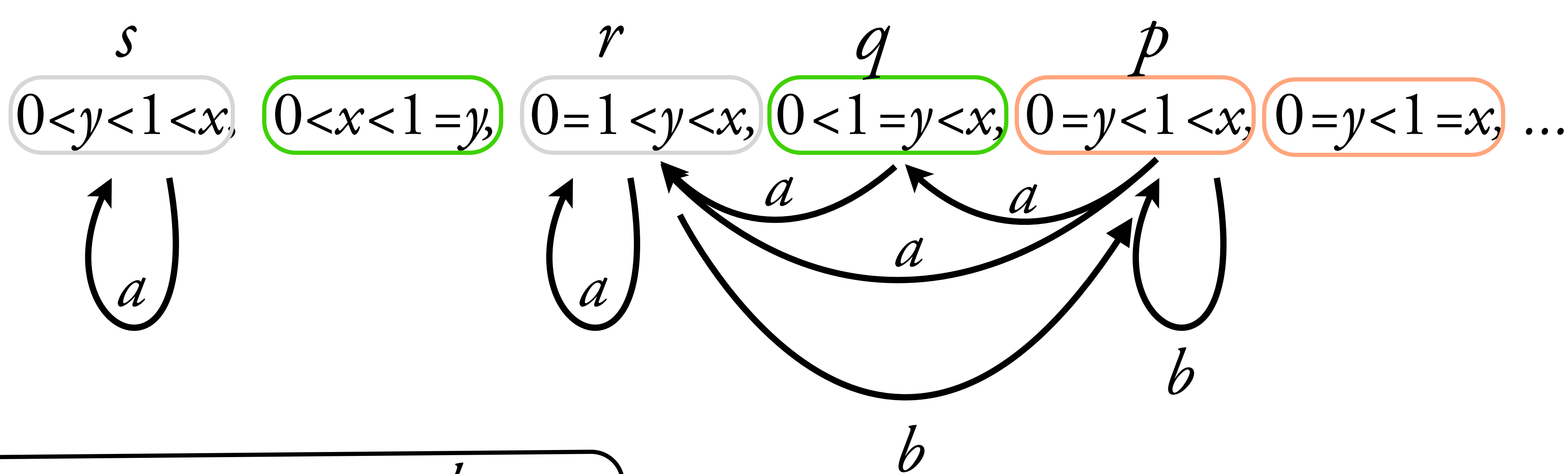

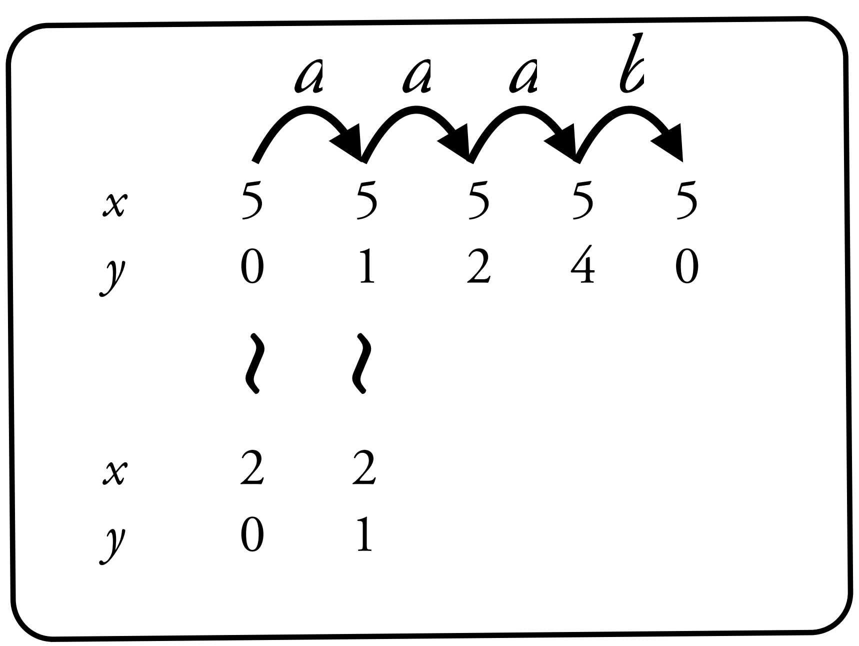

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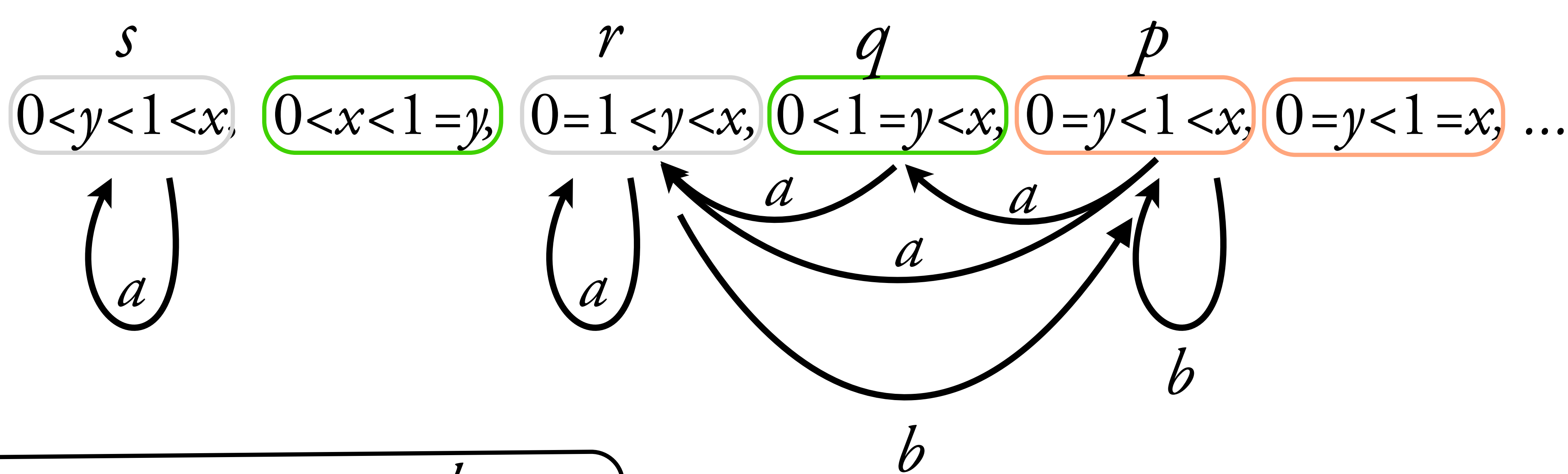

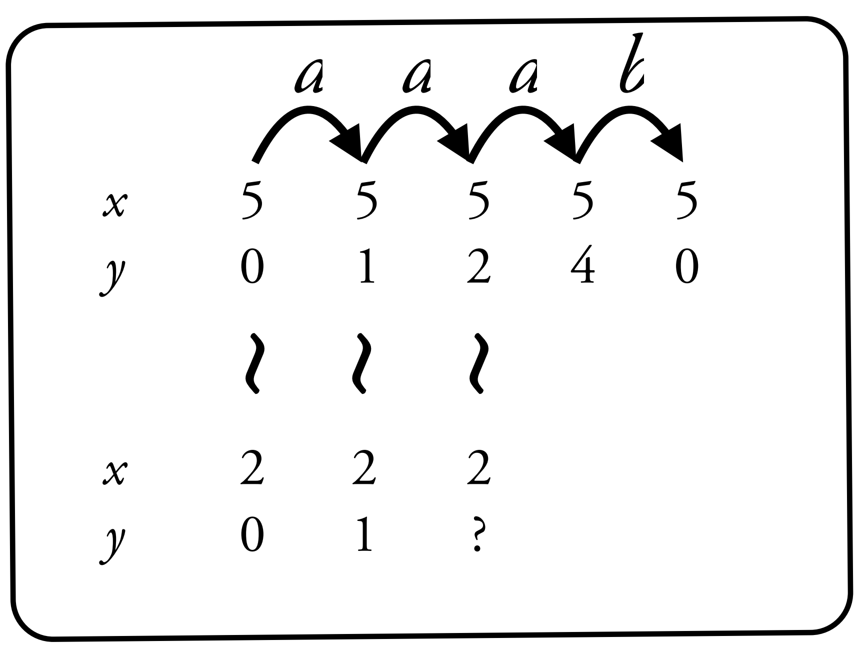

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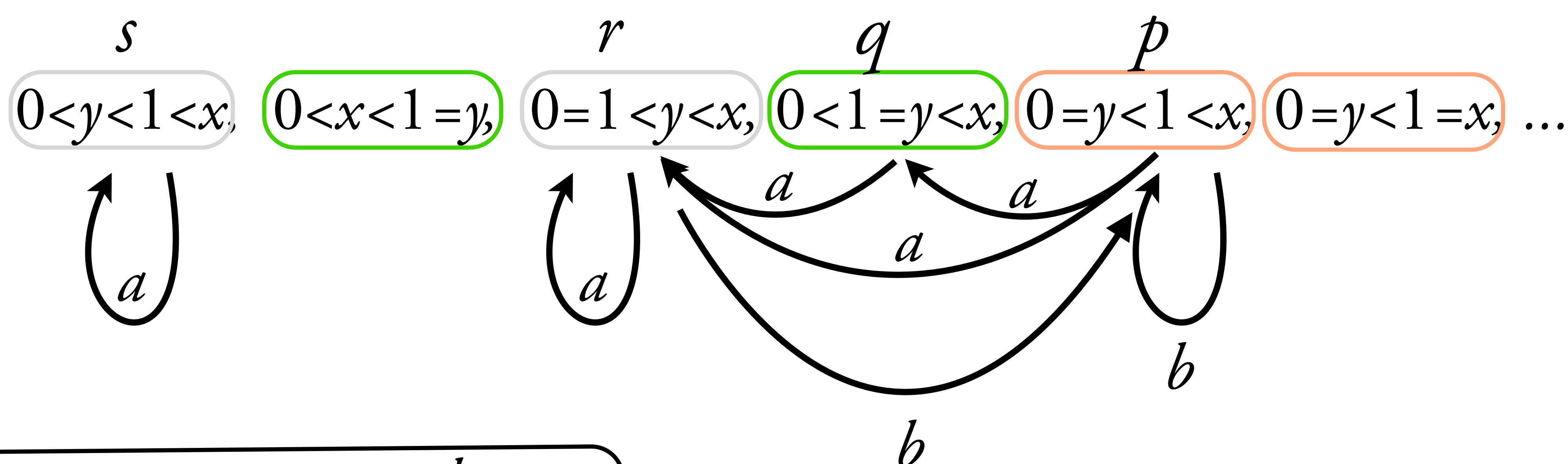

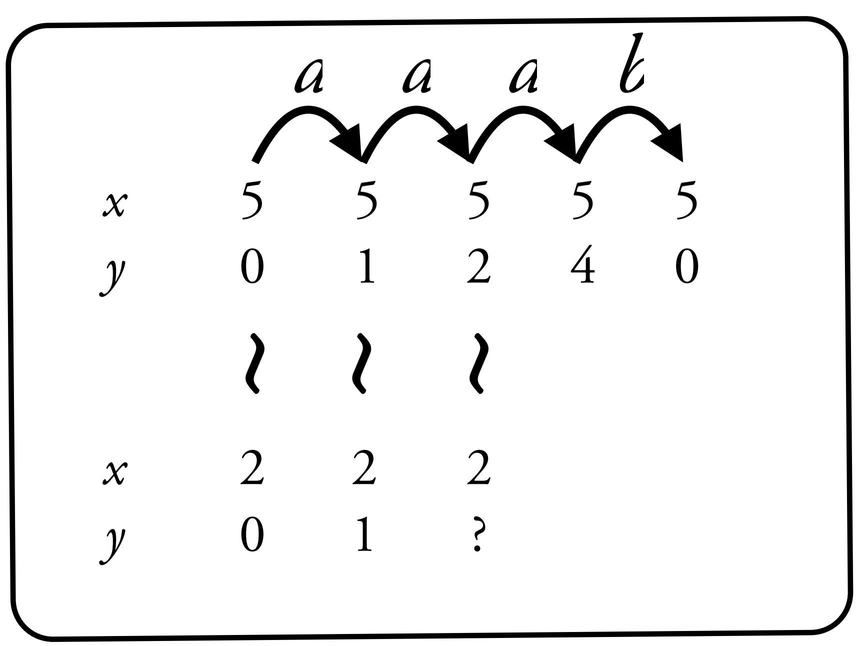

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→ *finite* runs of the region automaton correspond to runs of $\mathcal{A}$

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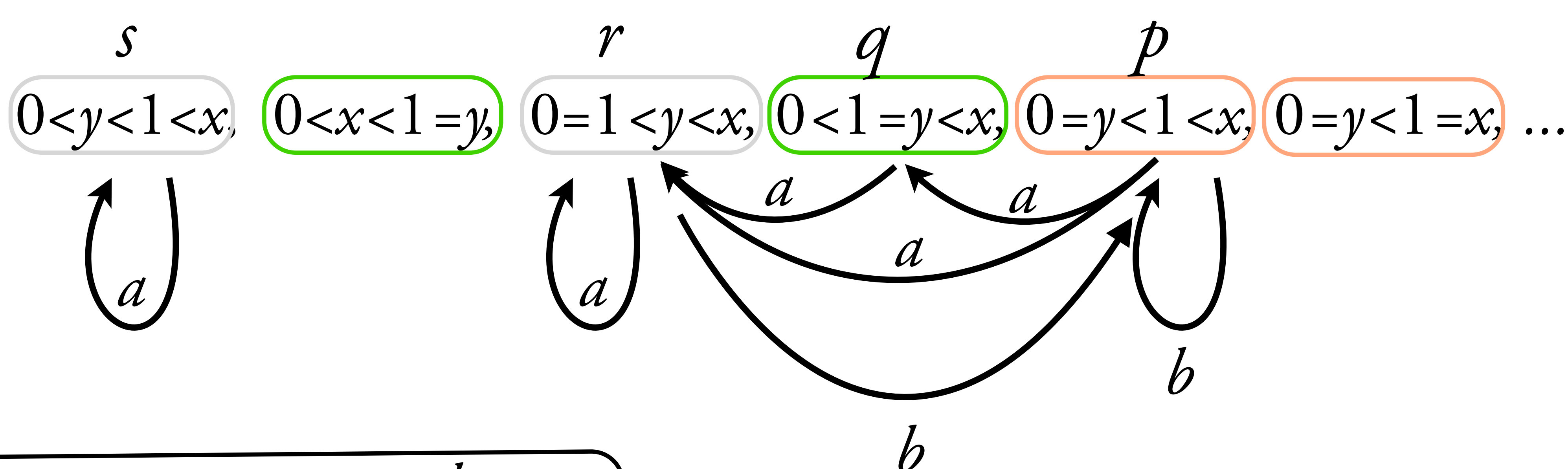

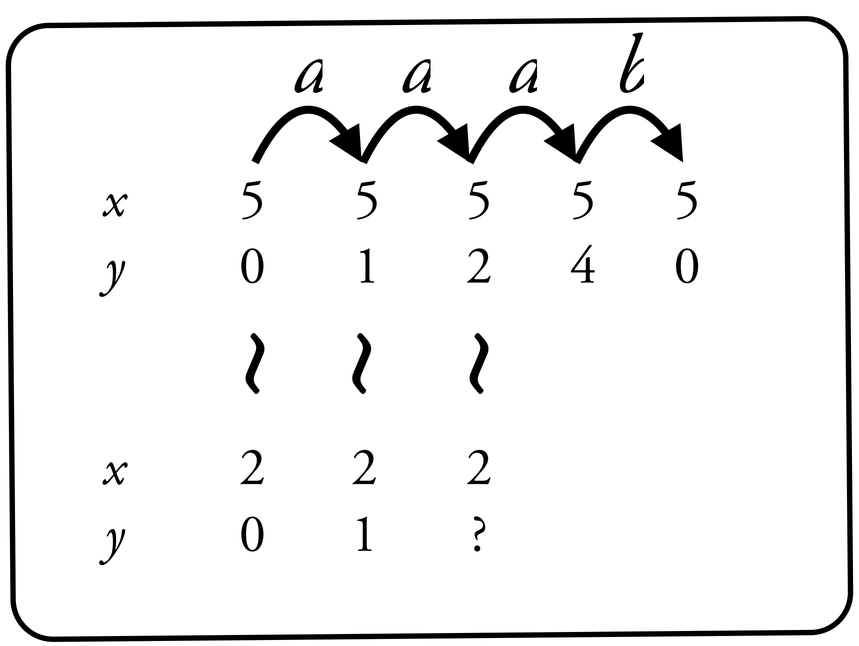

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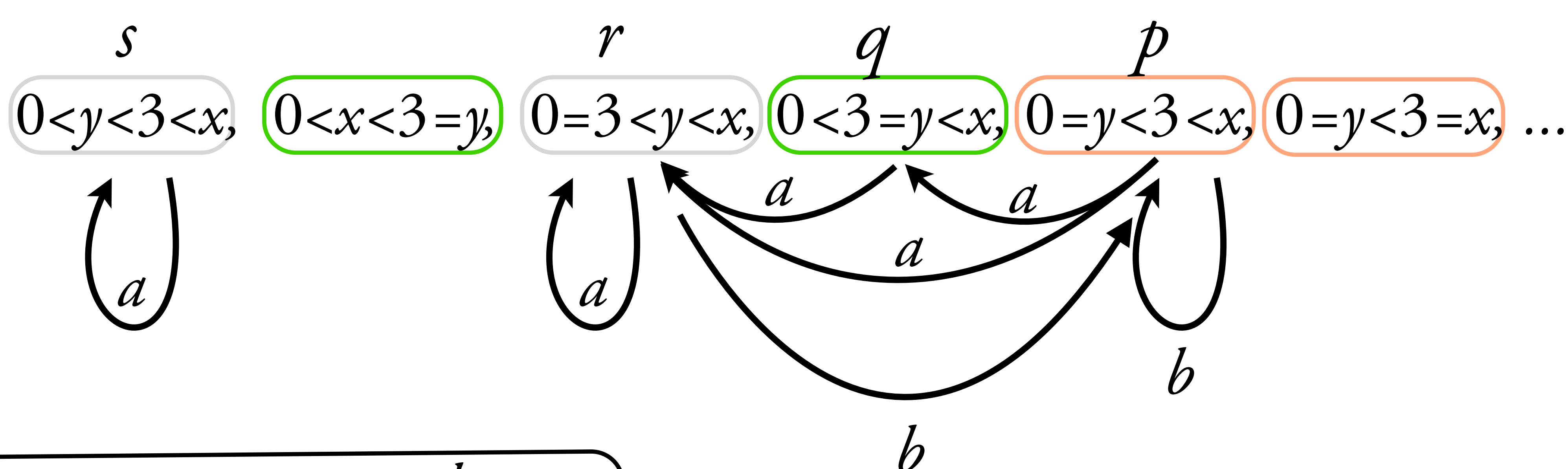

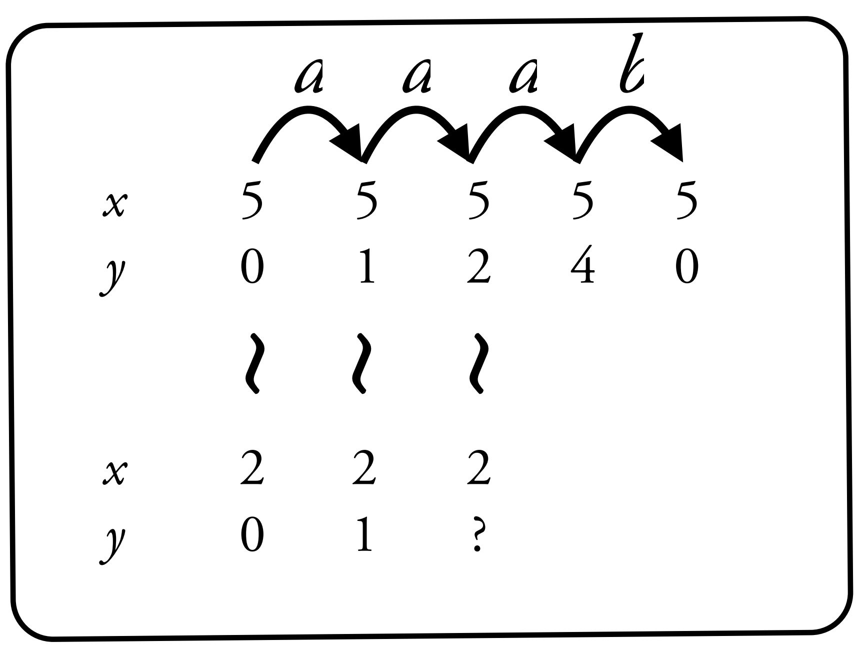

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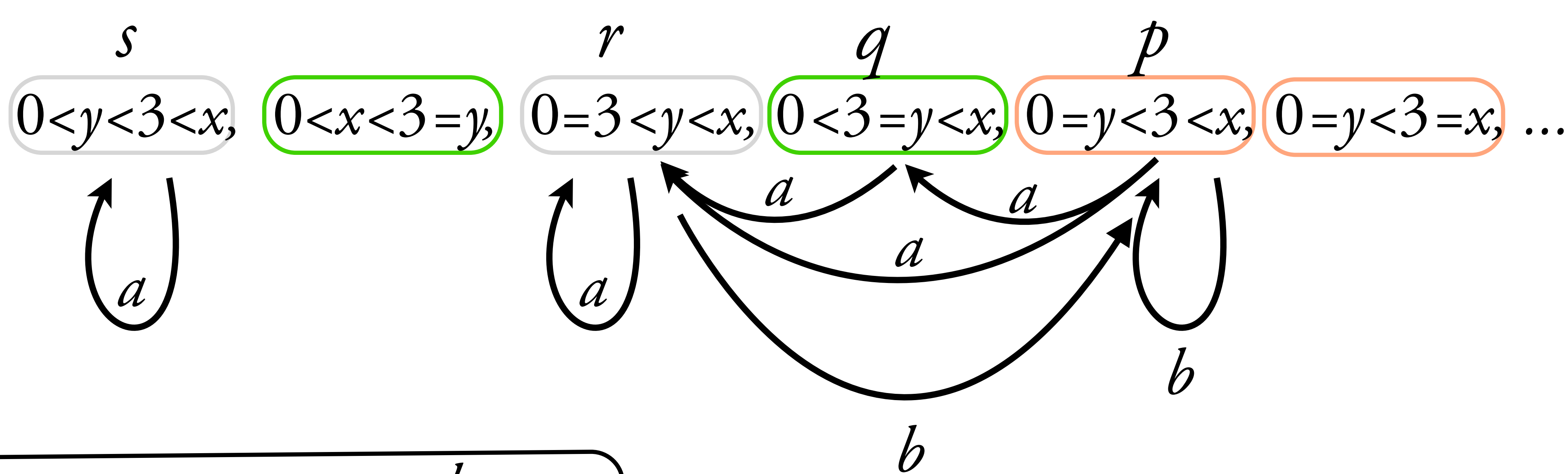

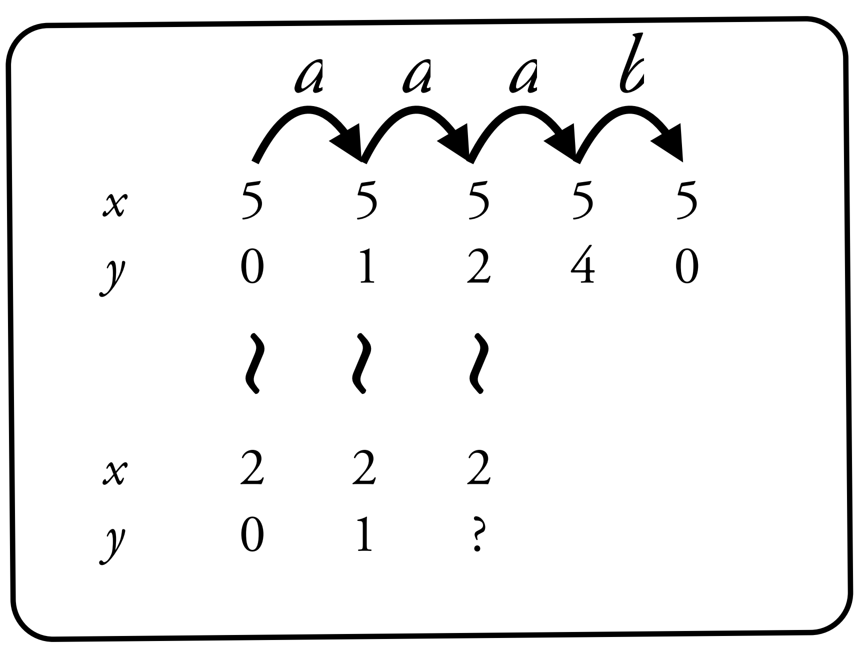

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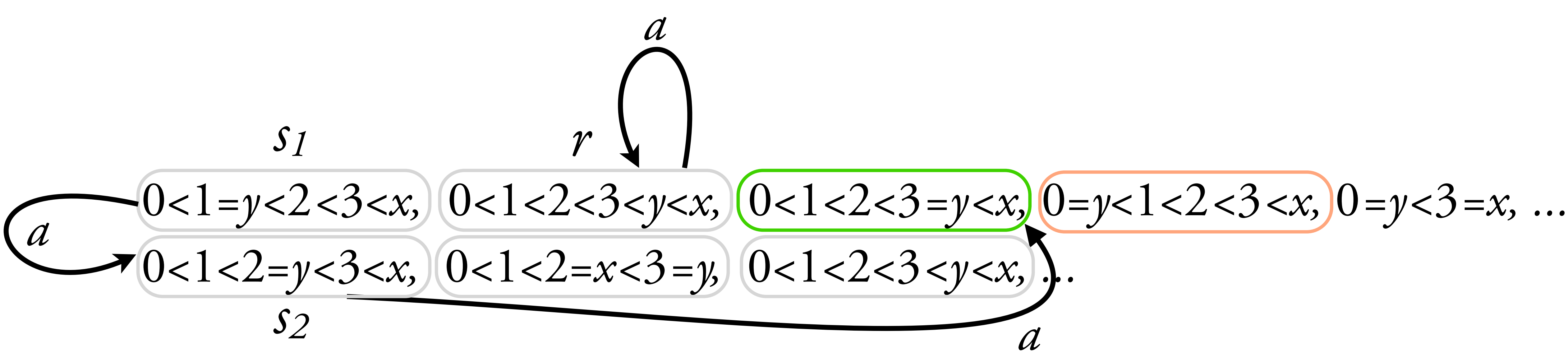

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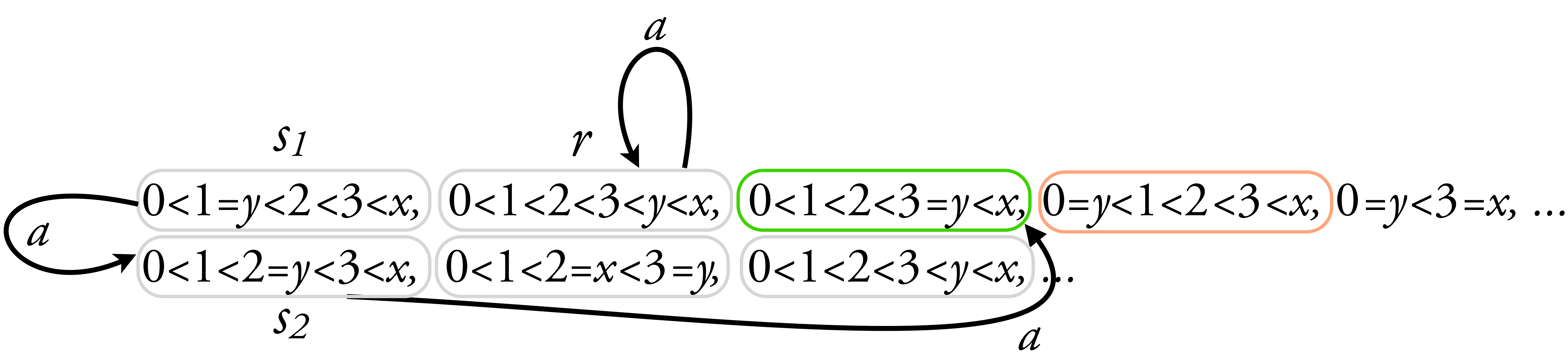

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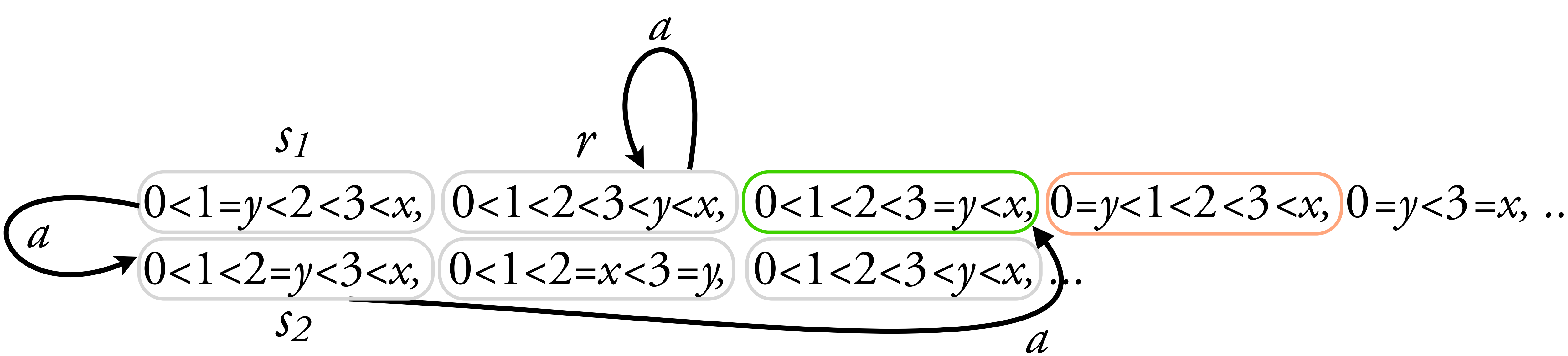

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Theorem. For any linearly ordered structure there is a finite number of special values

This solves the case of finite runs, with no database.

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Theorem. For all $\mathscr{D}$, $\mathscr{D}$-automata over finite words accept regular languages.

Theorem. Emptiness of $\mathscr{D}$-automata is decidable* in PSPACE

**For any reasonable linearly ordered structure* - there should be an efficient algorithm which can compute the special values, given some initial constants

Adding a database

τ_I: $(x>0) \wedge (y=0)$

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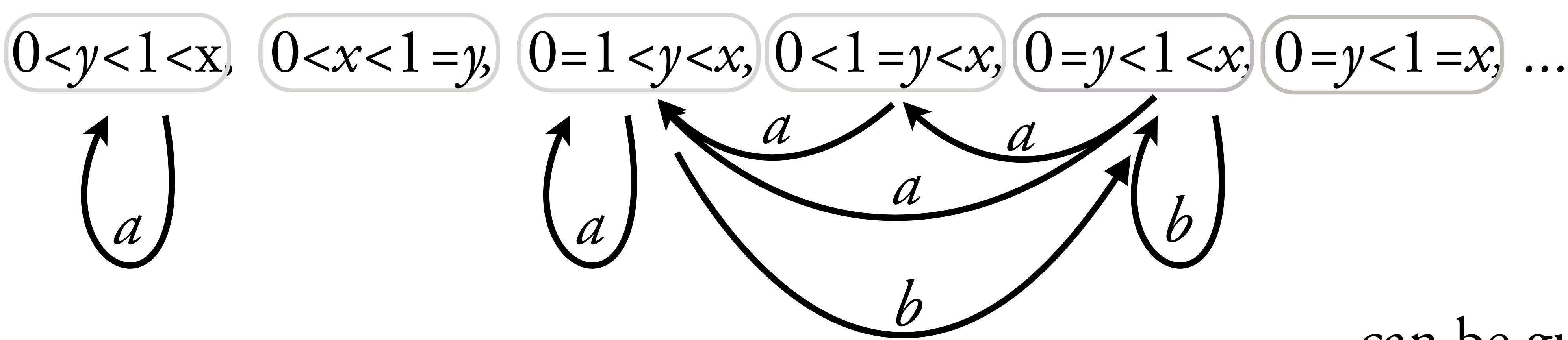

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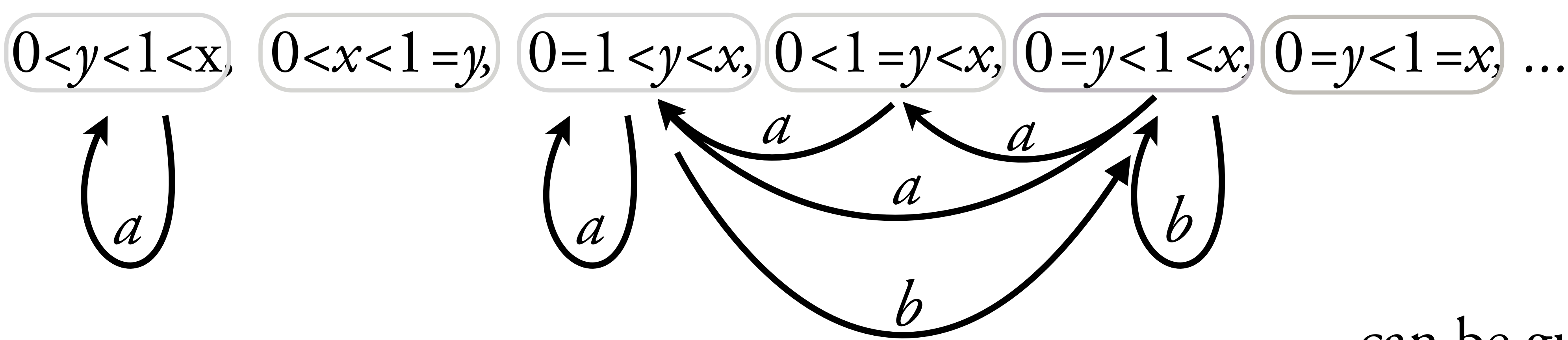

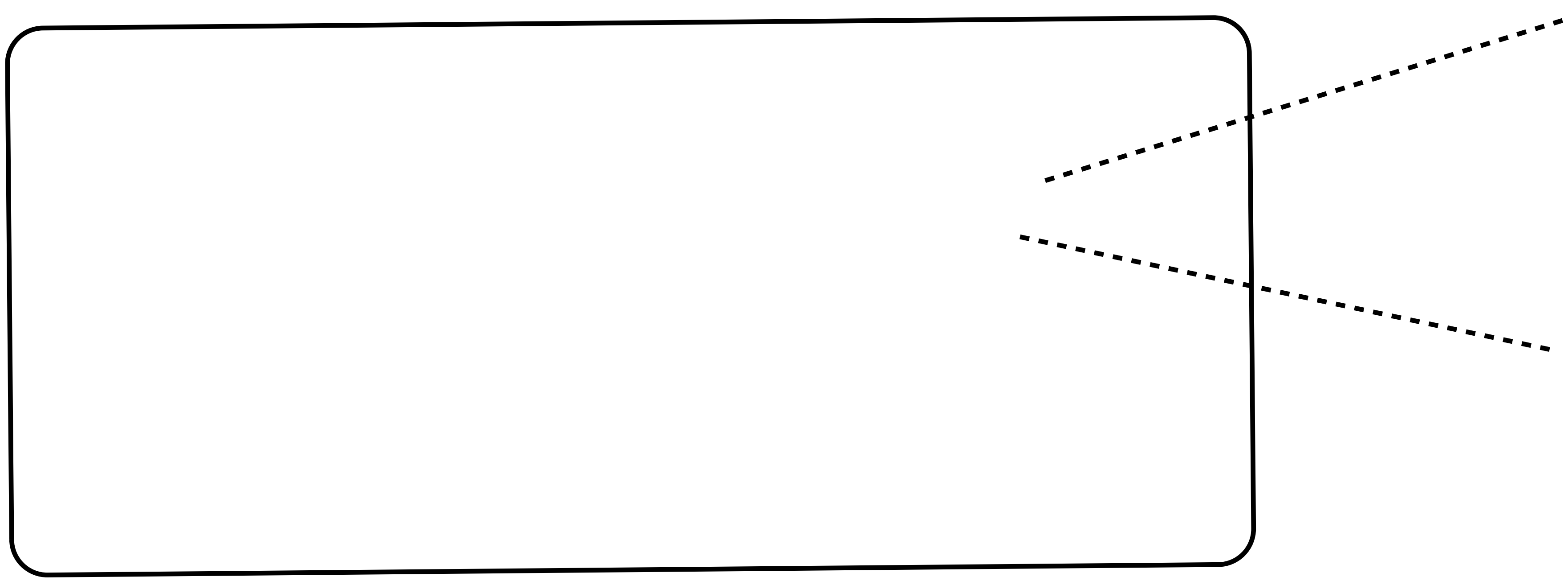

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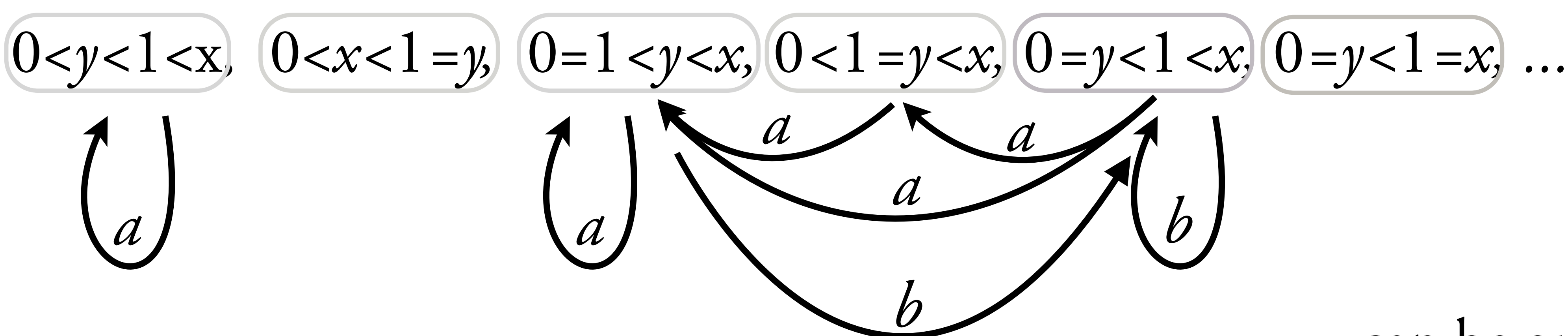

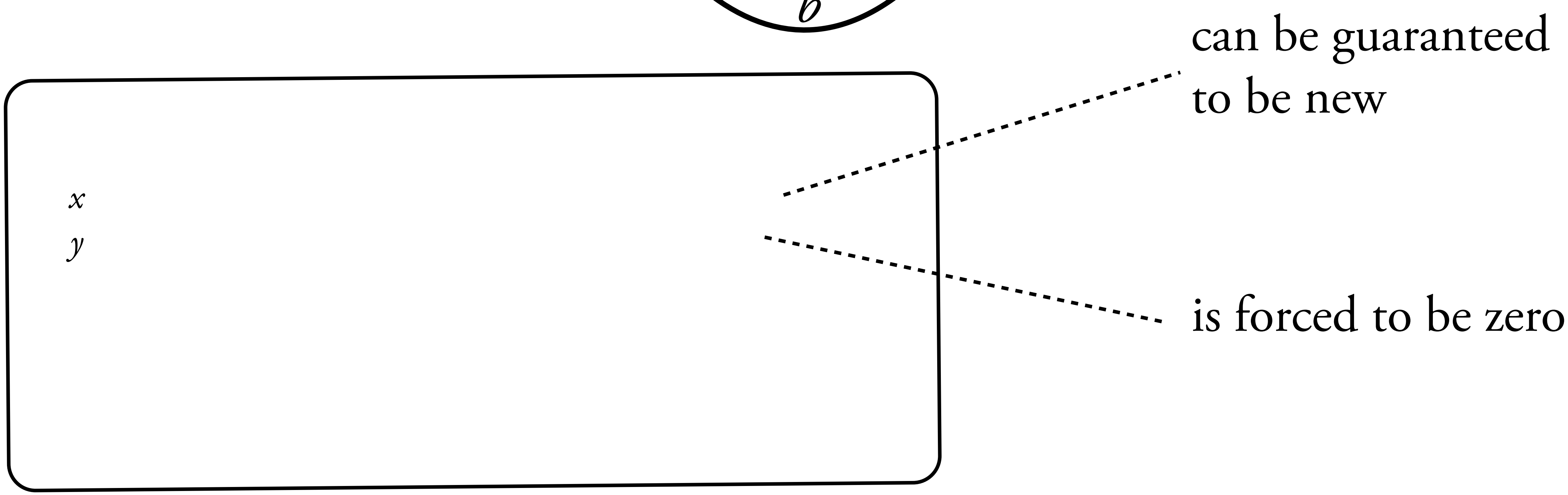

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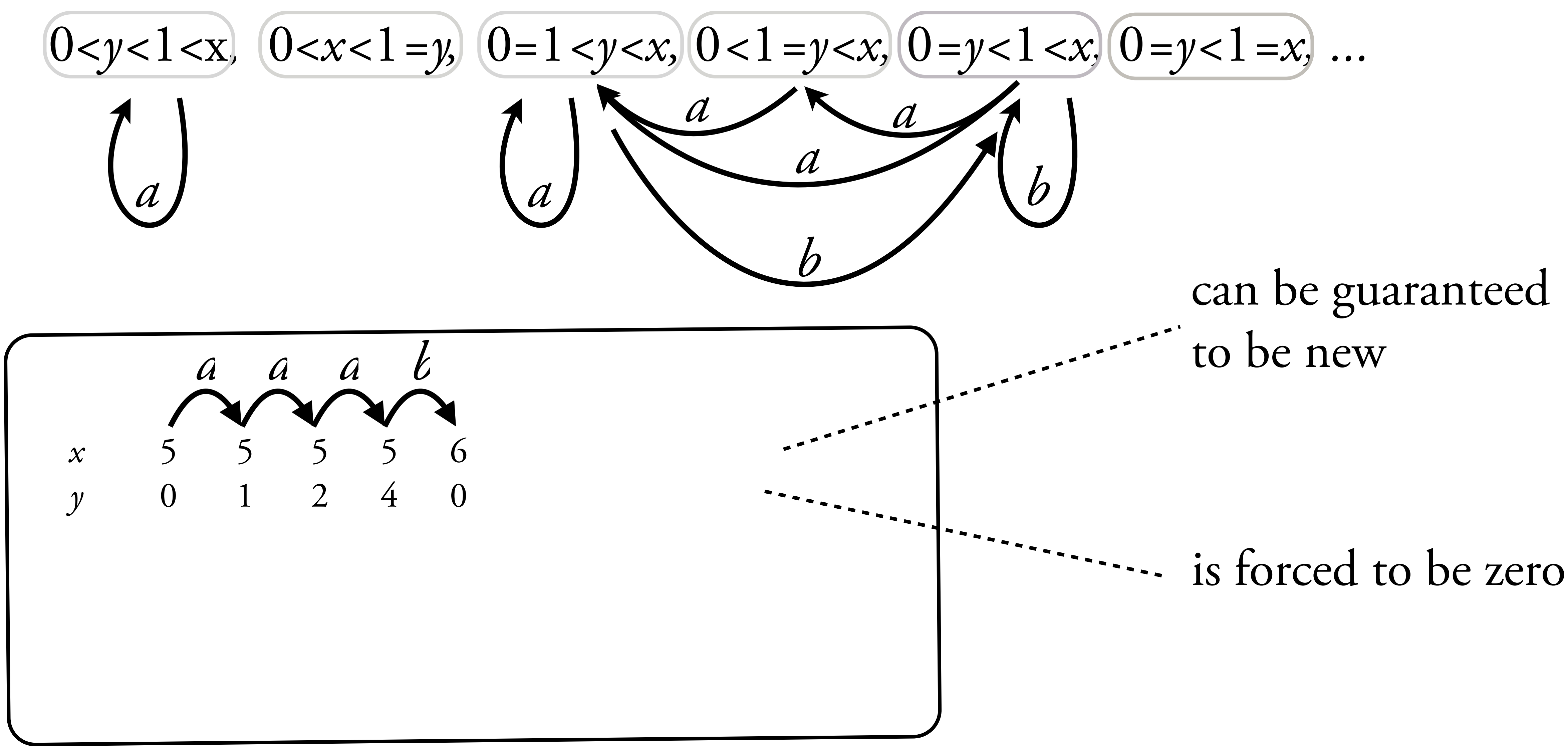

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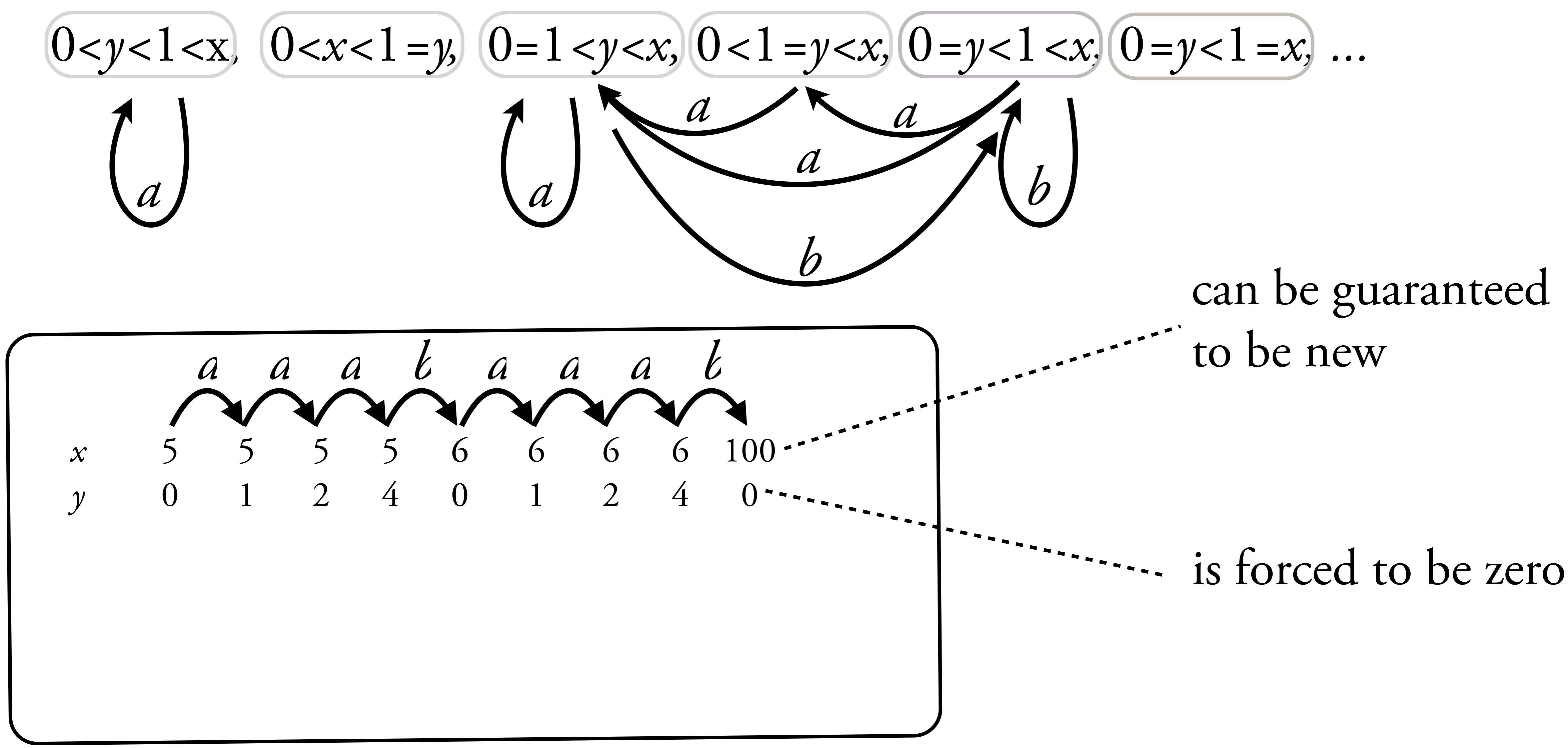

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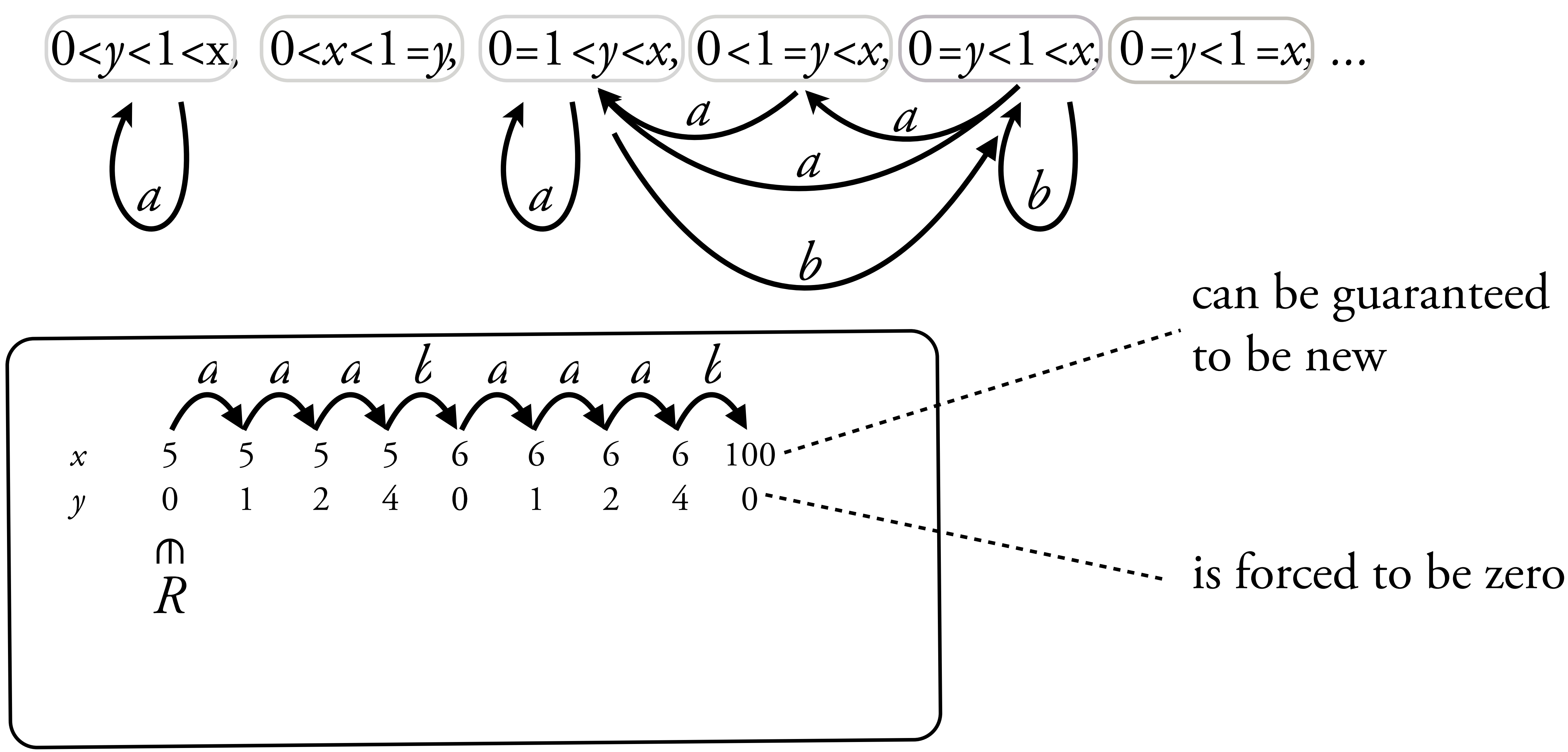

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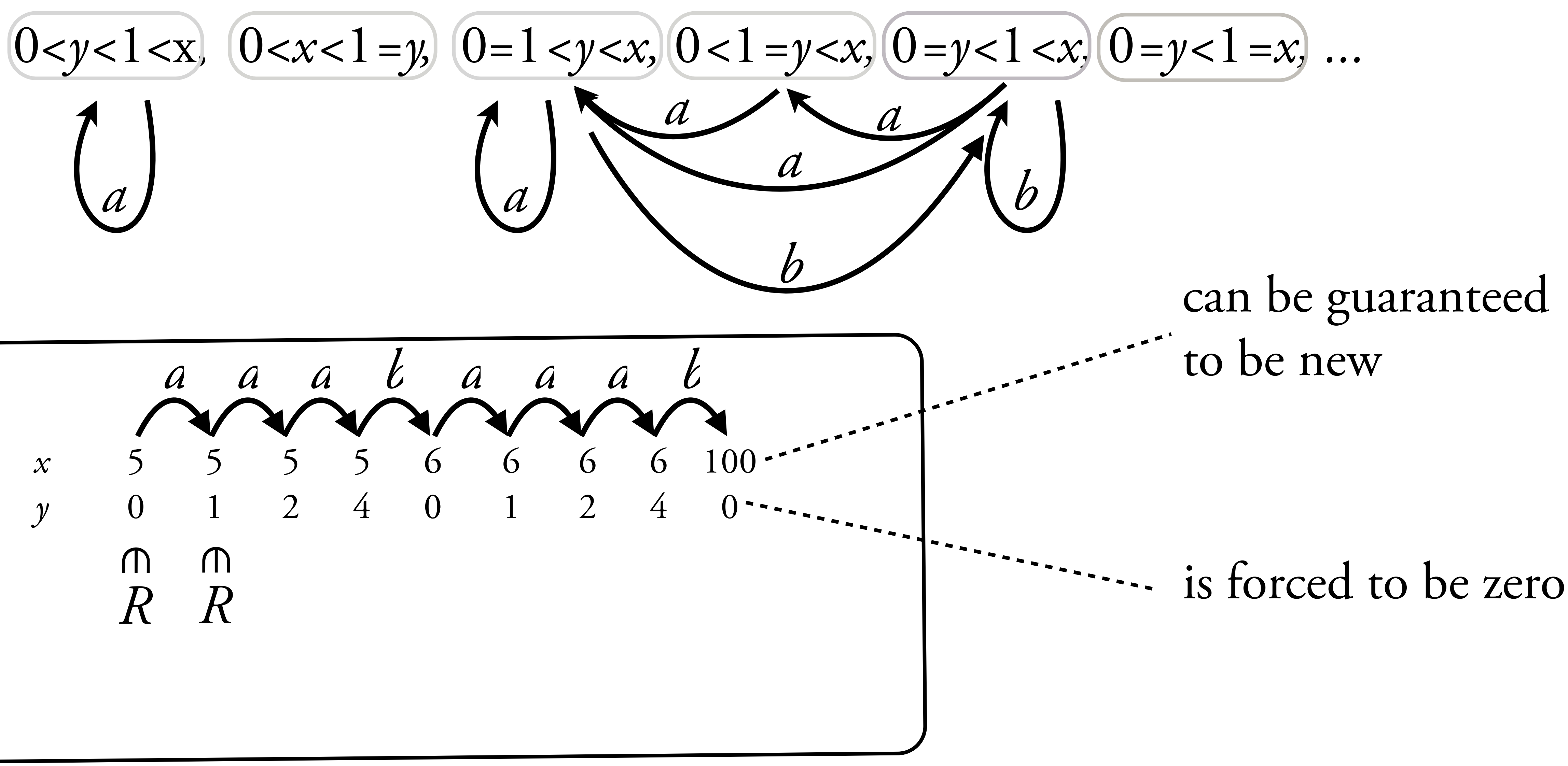

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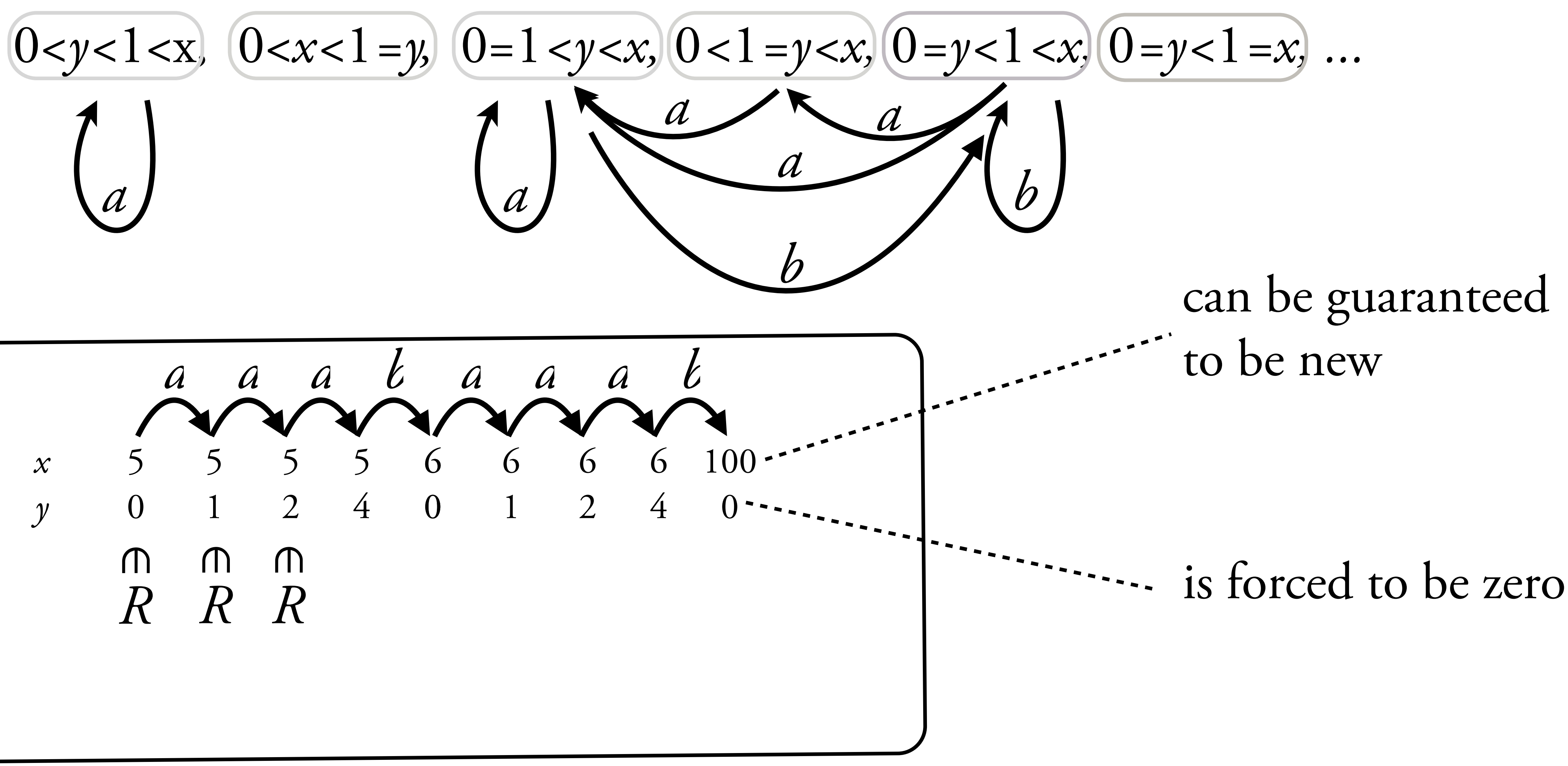

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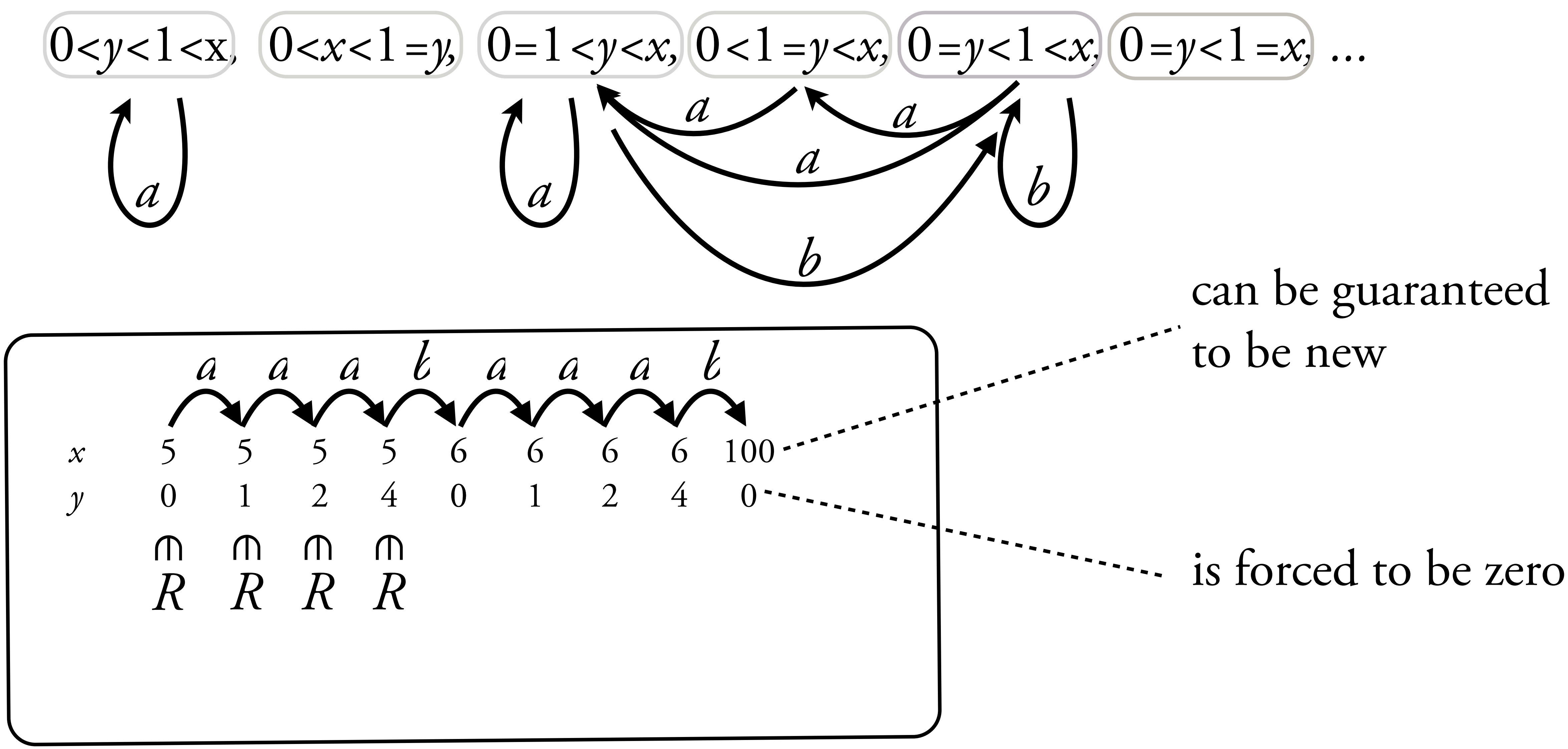

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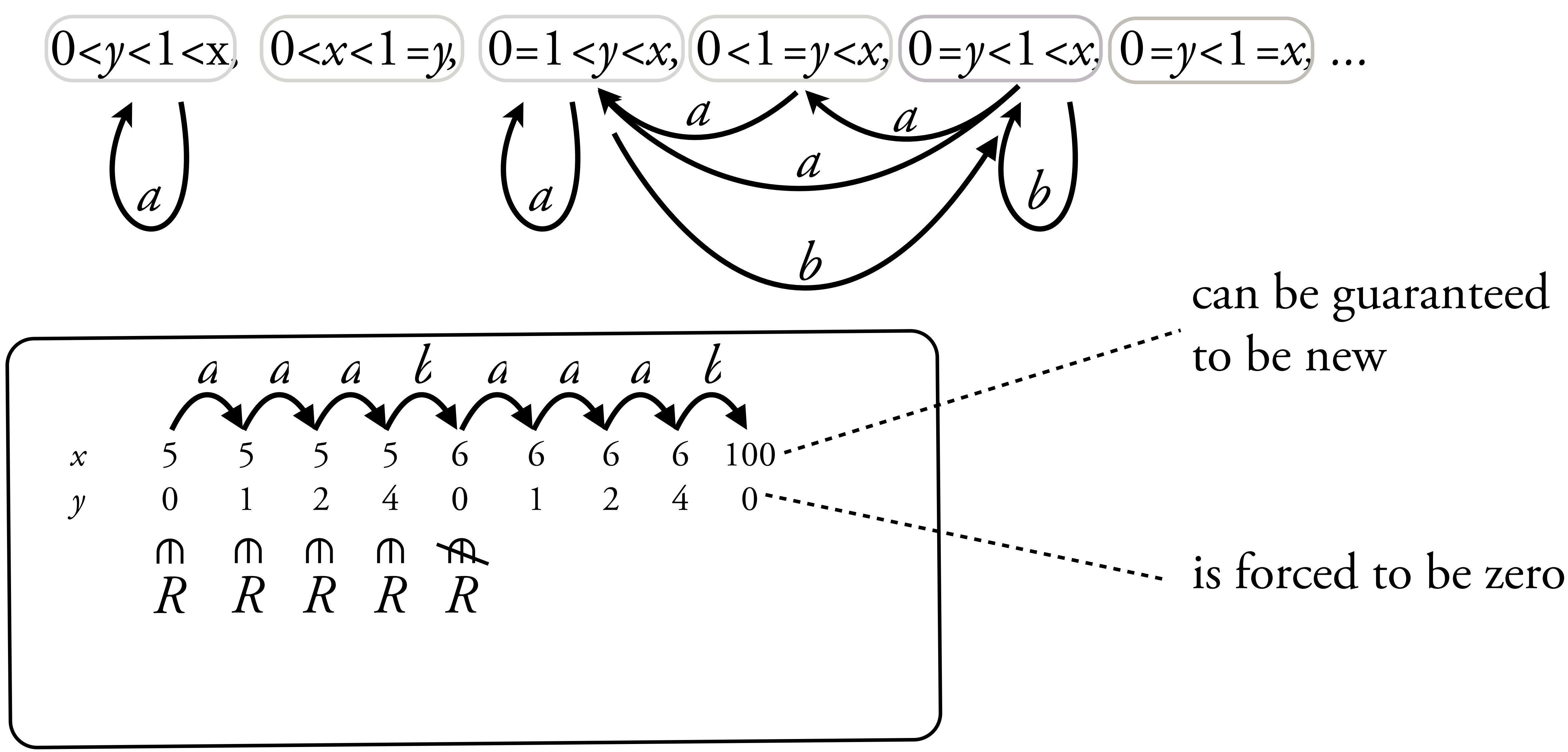

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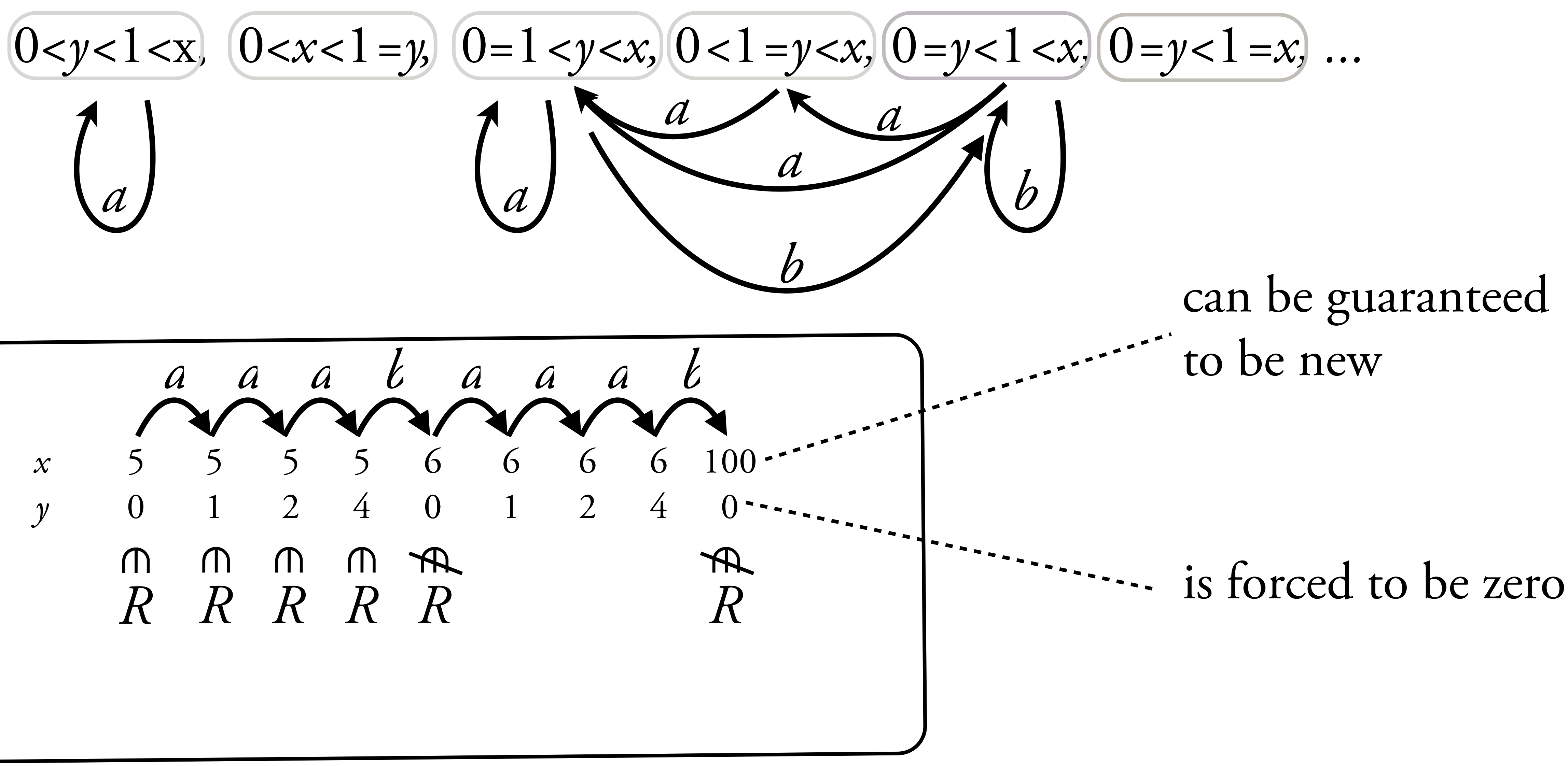

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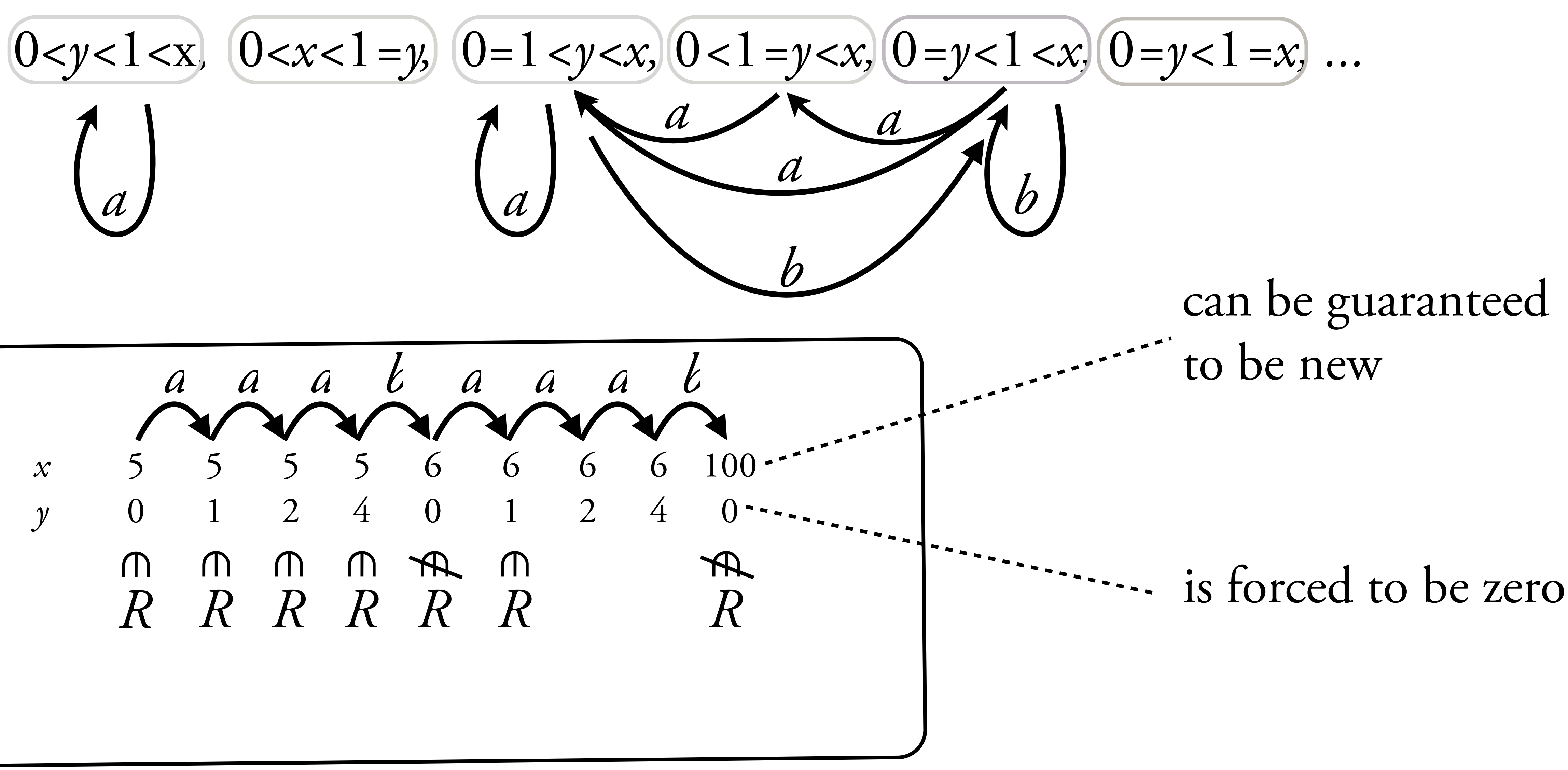

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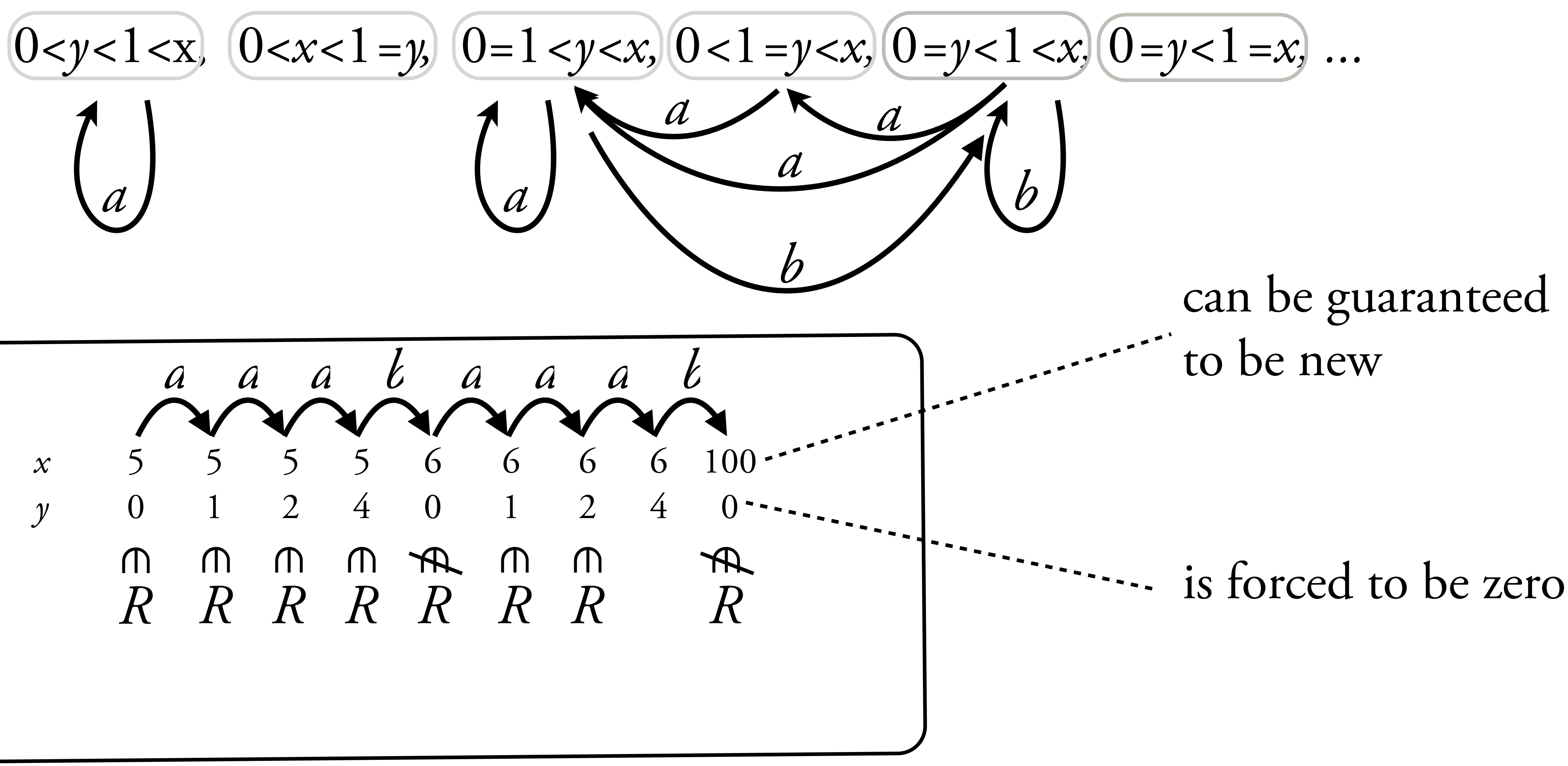

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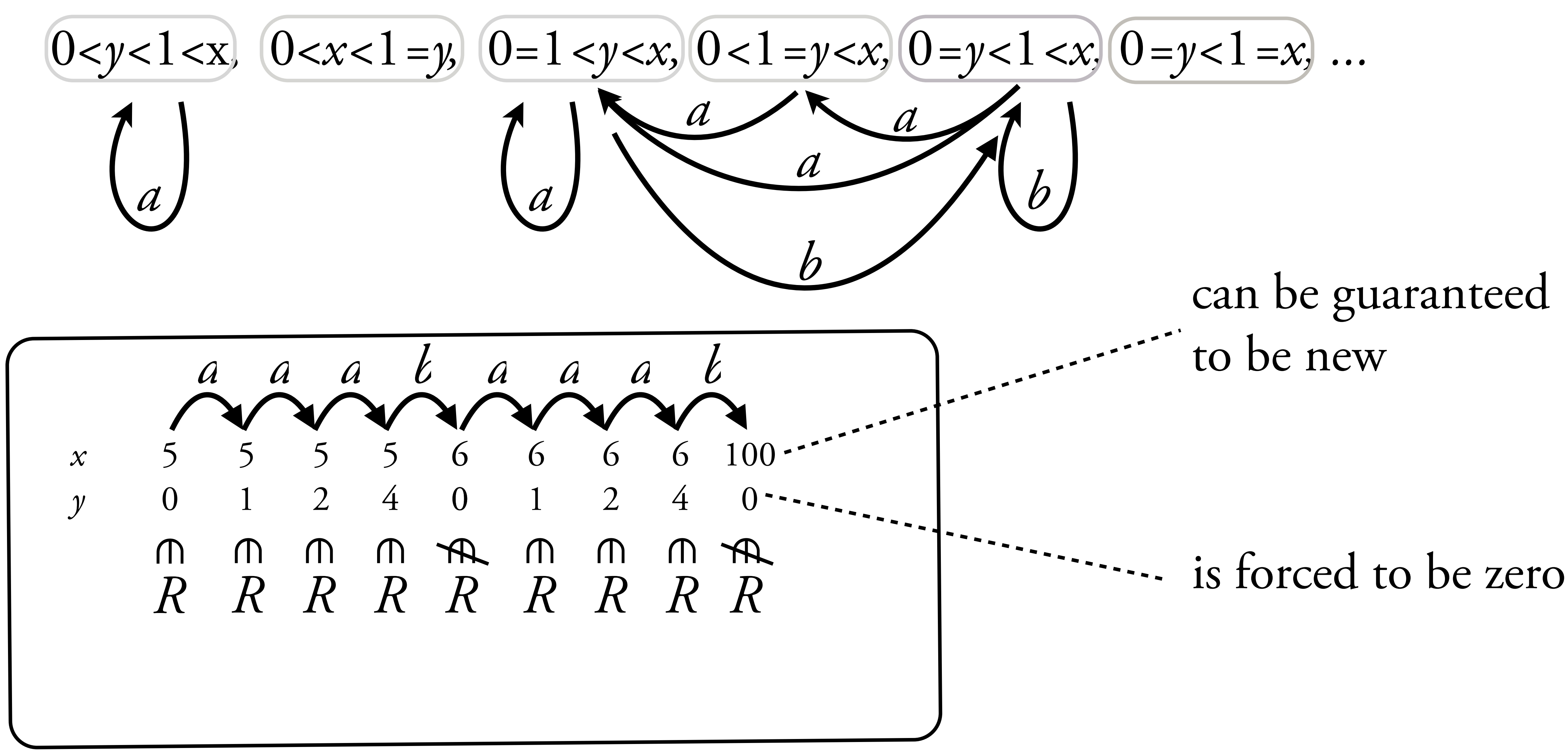

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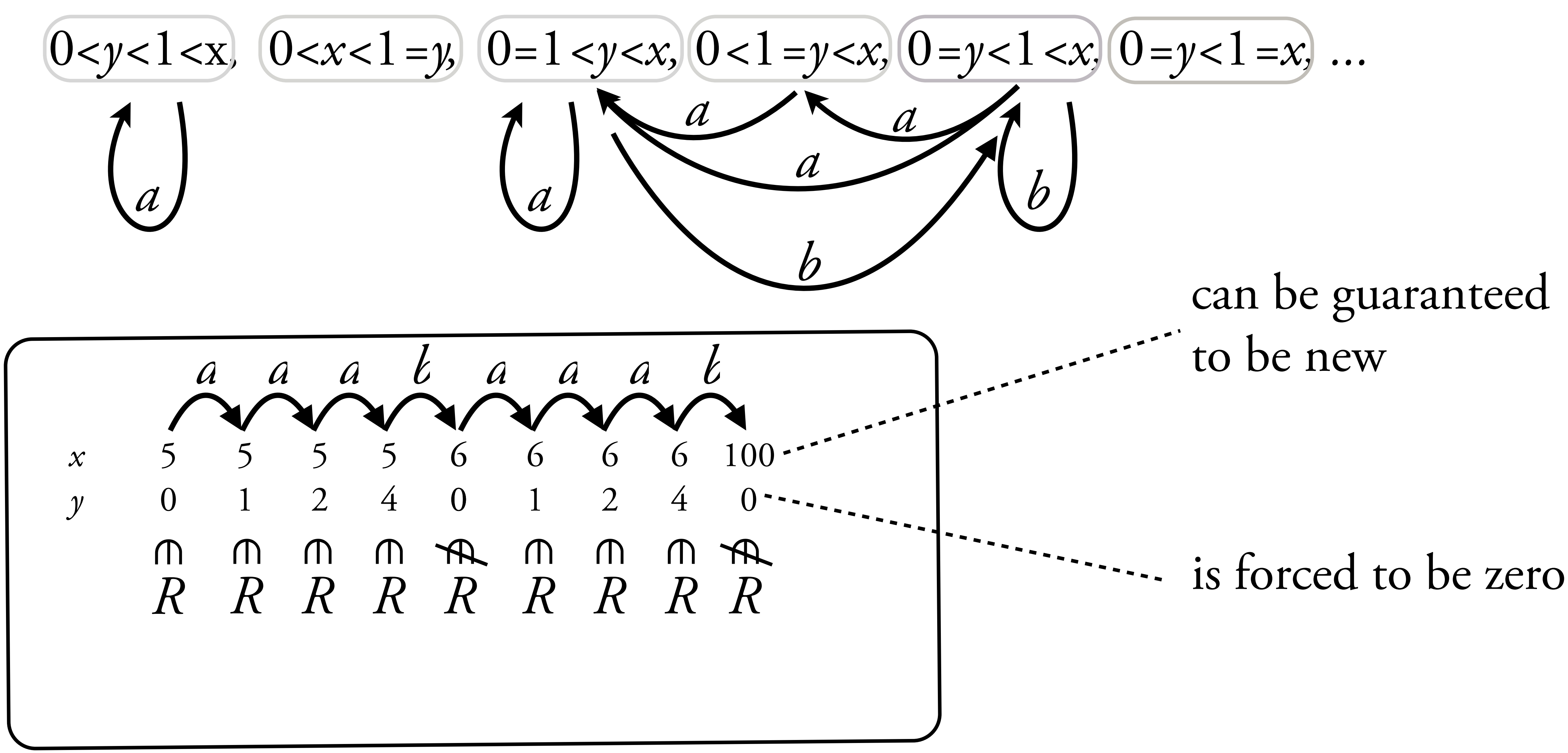

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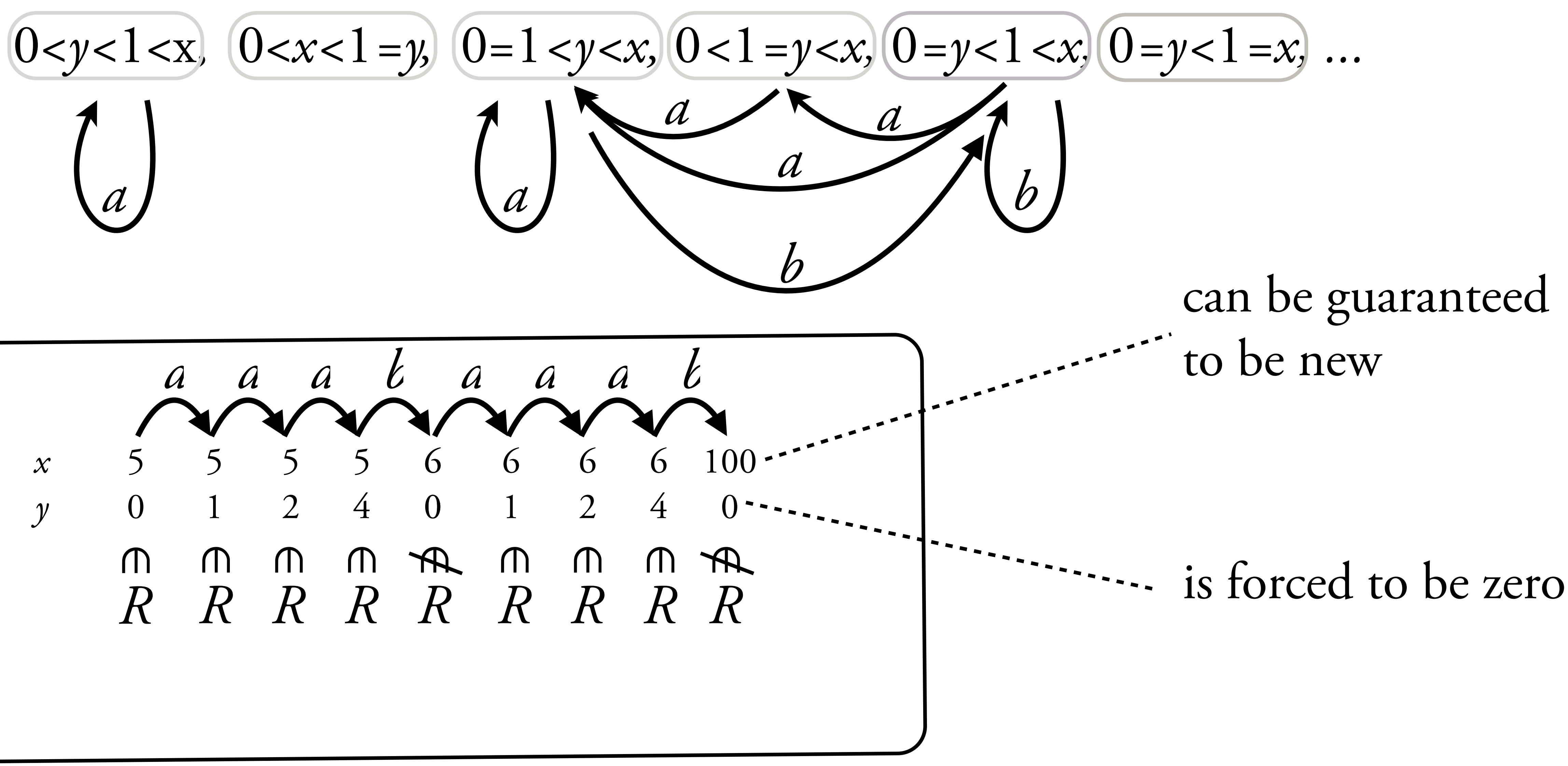

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Corollary. Deciding LTL+(data tests) properties of $\mathscr{D}$-automata is PSPACE-complete.

Thank you!