Automata based verification over linearly ordered data domains

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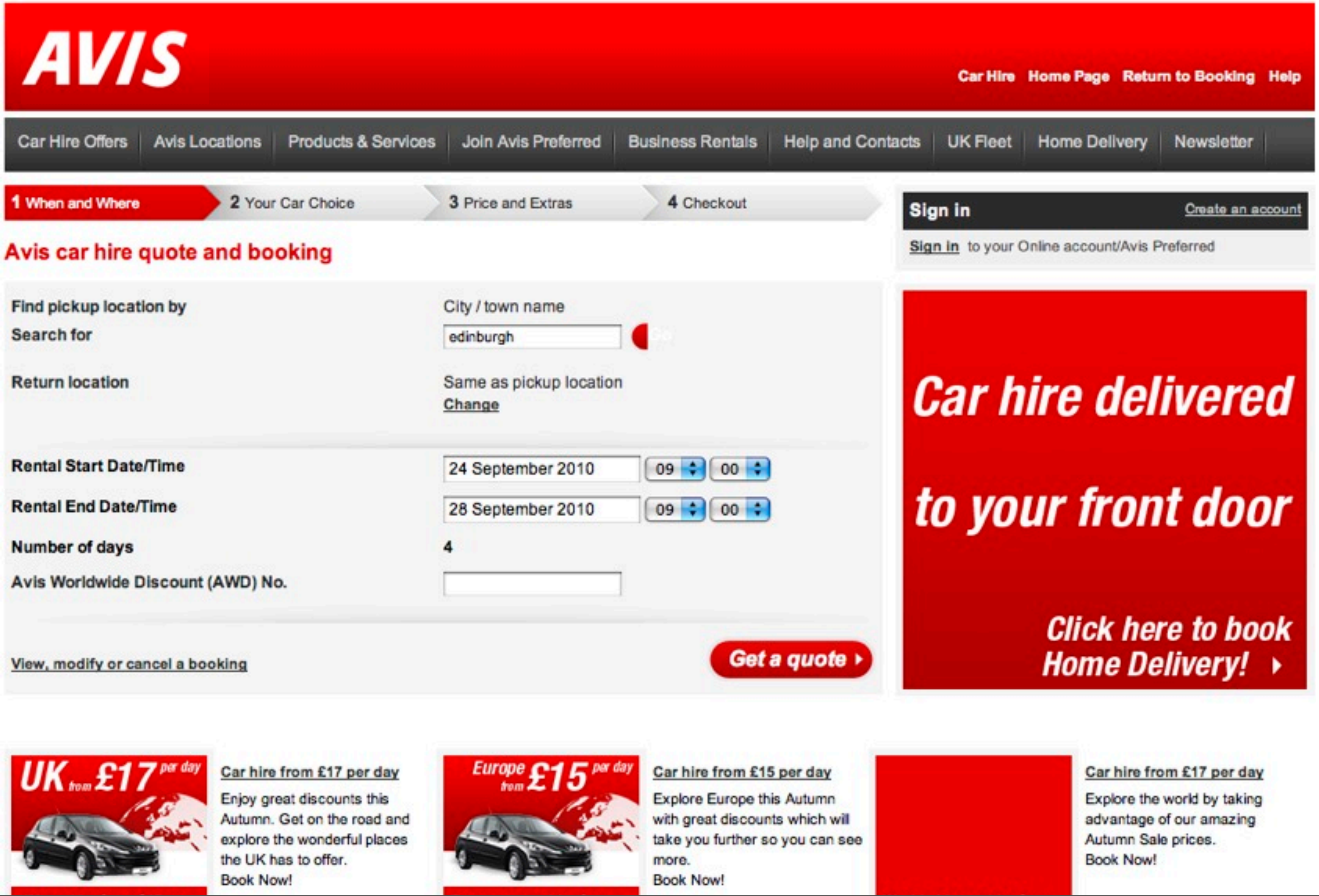
Given a system which refers to a database and data values, verify properties of the system



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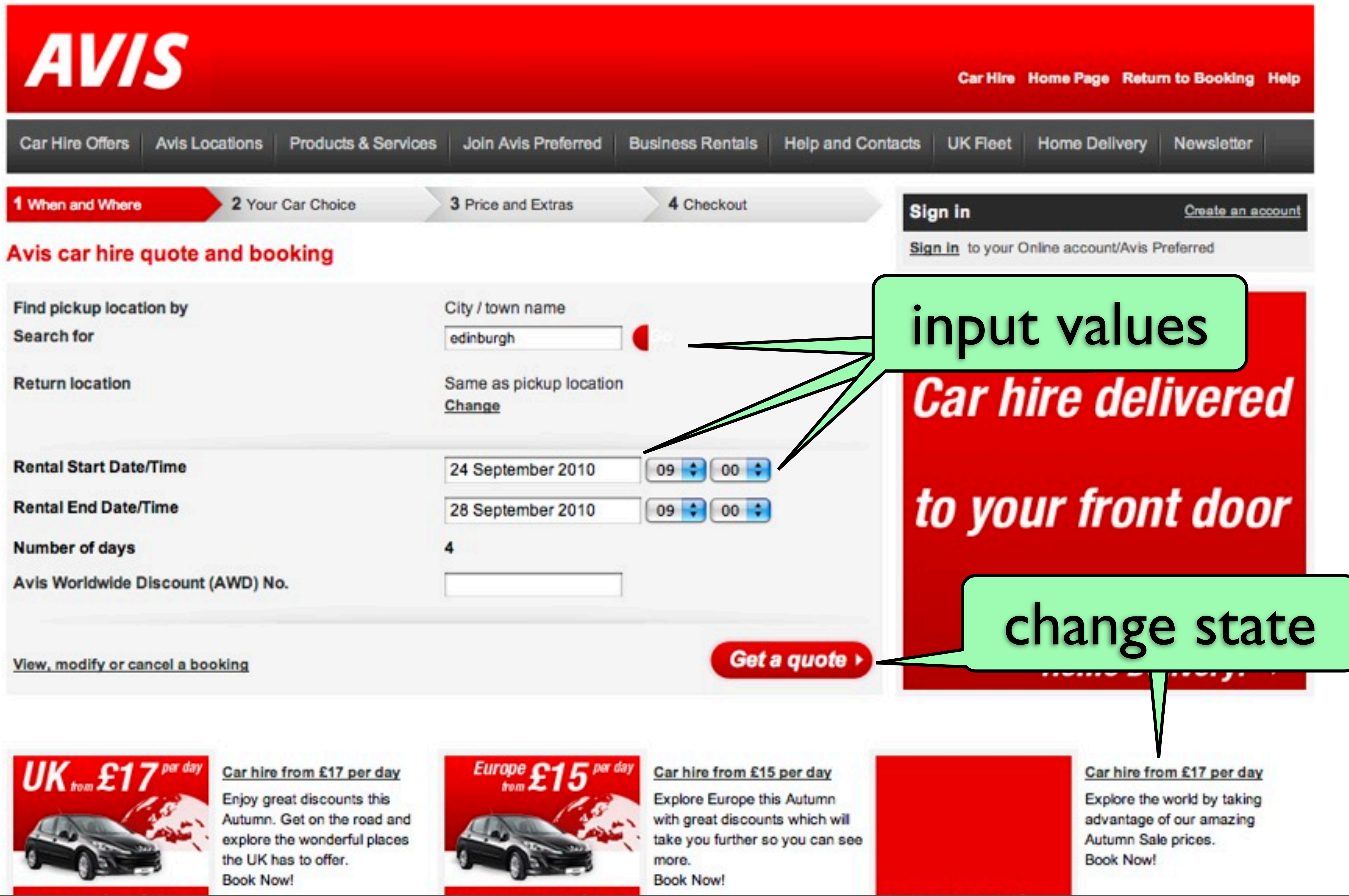

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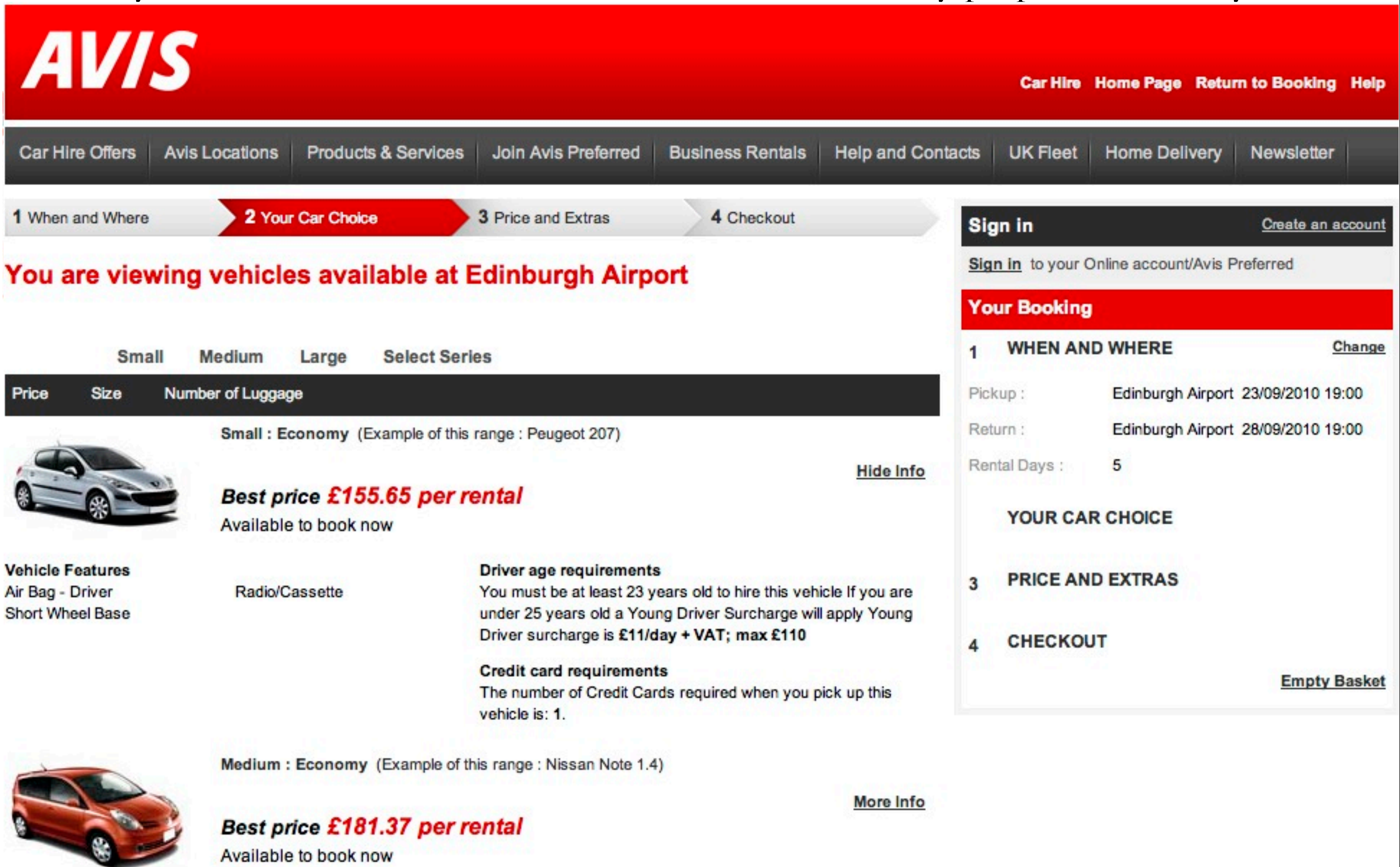

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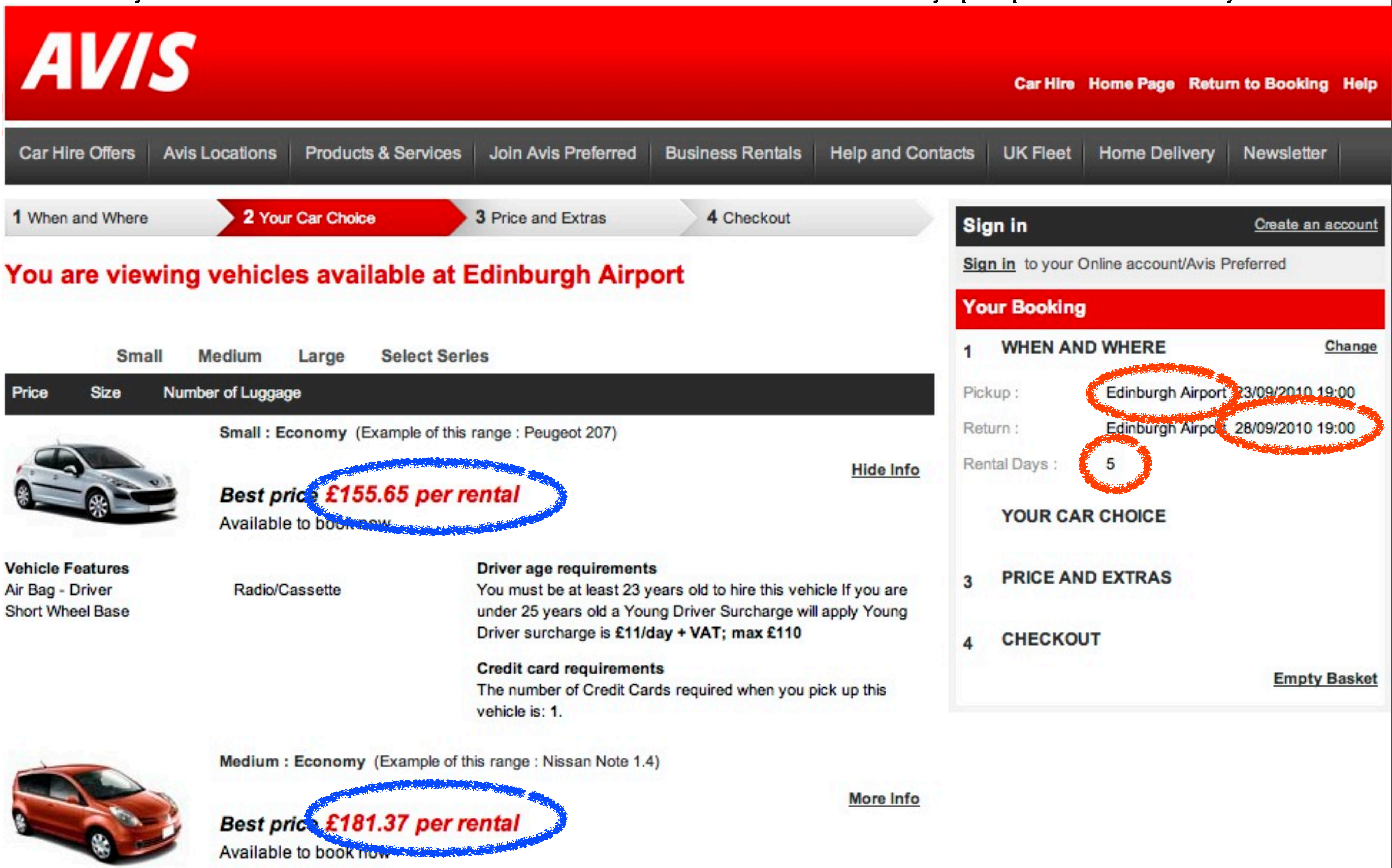

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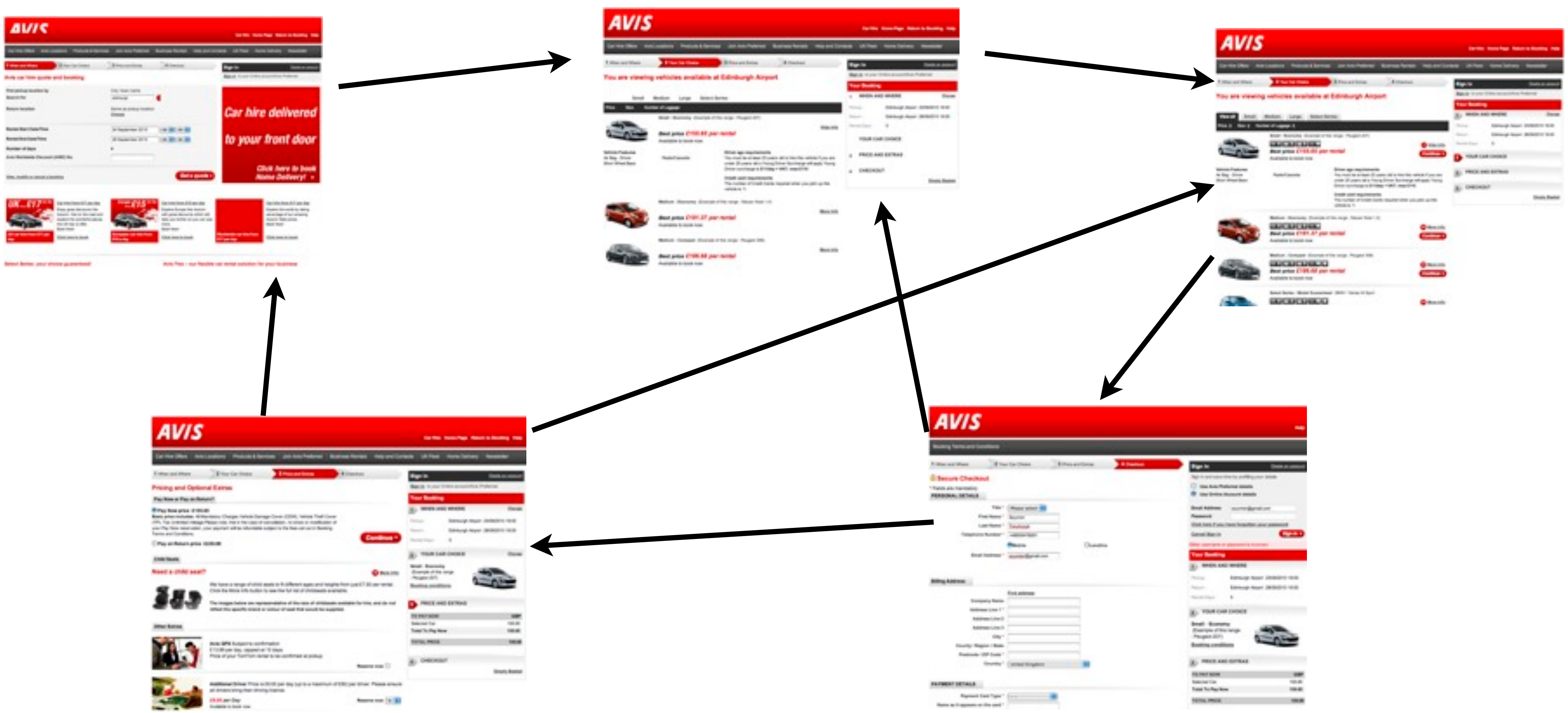

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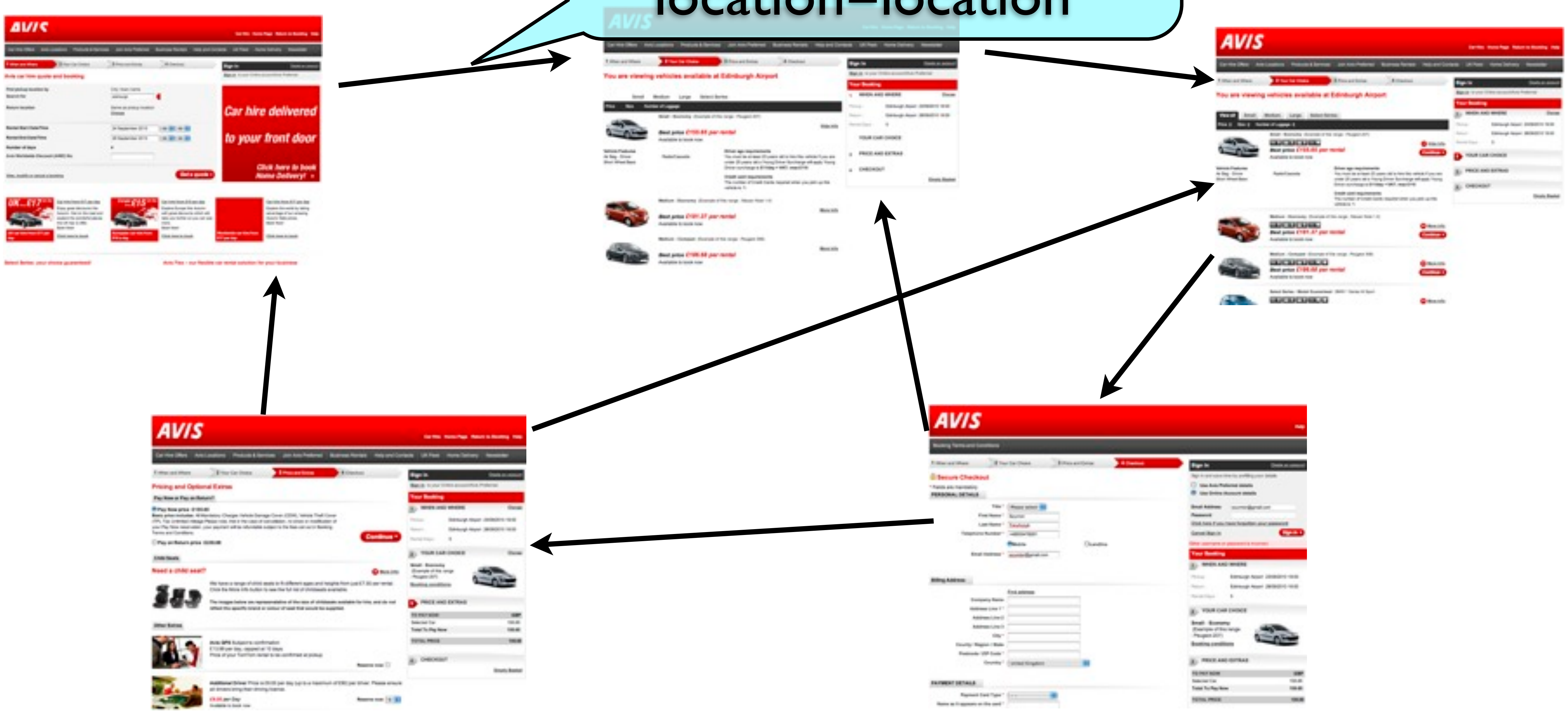

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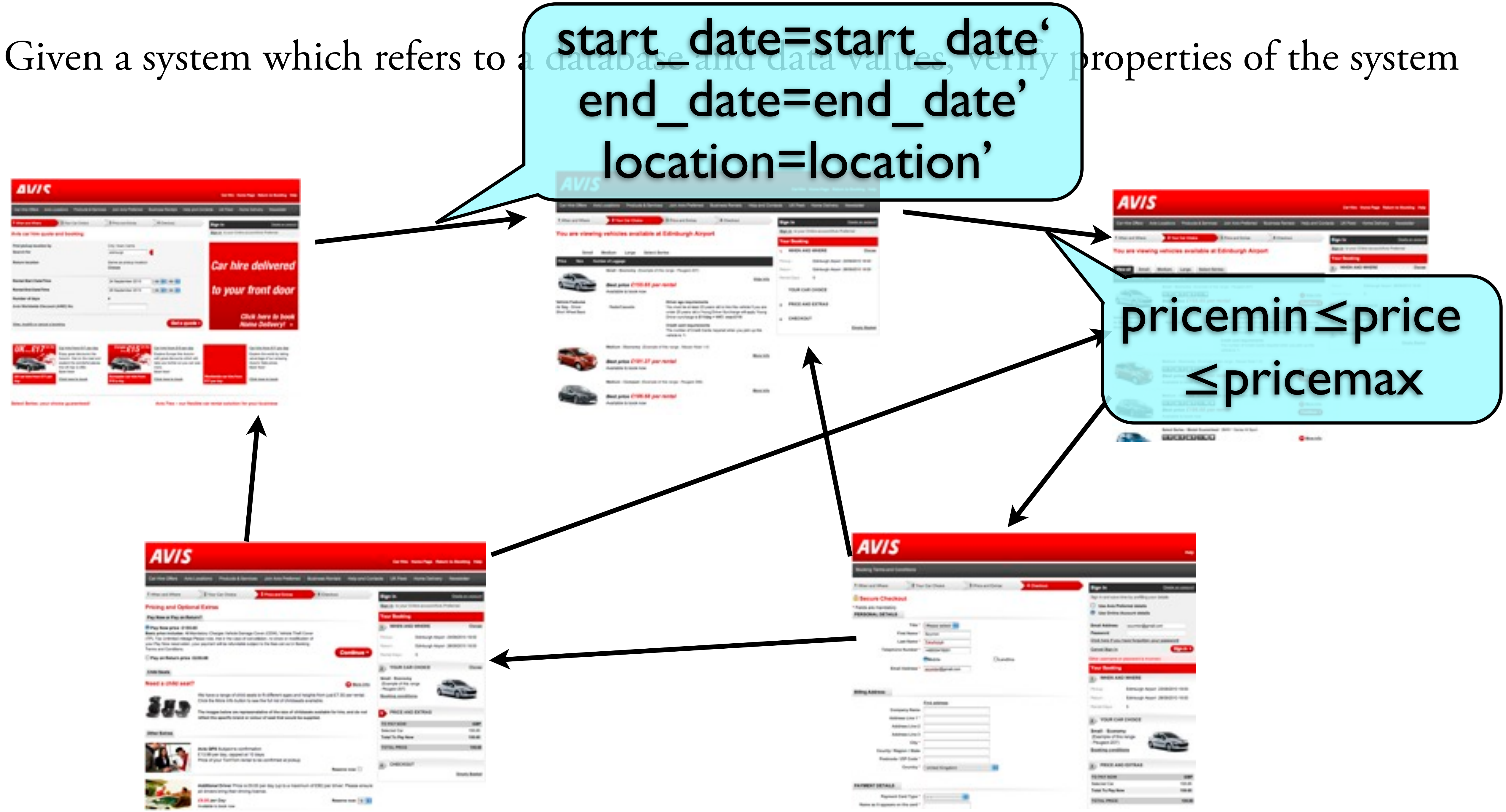

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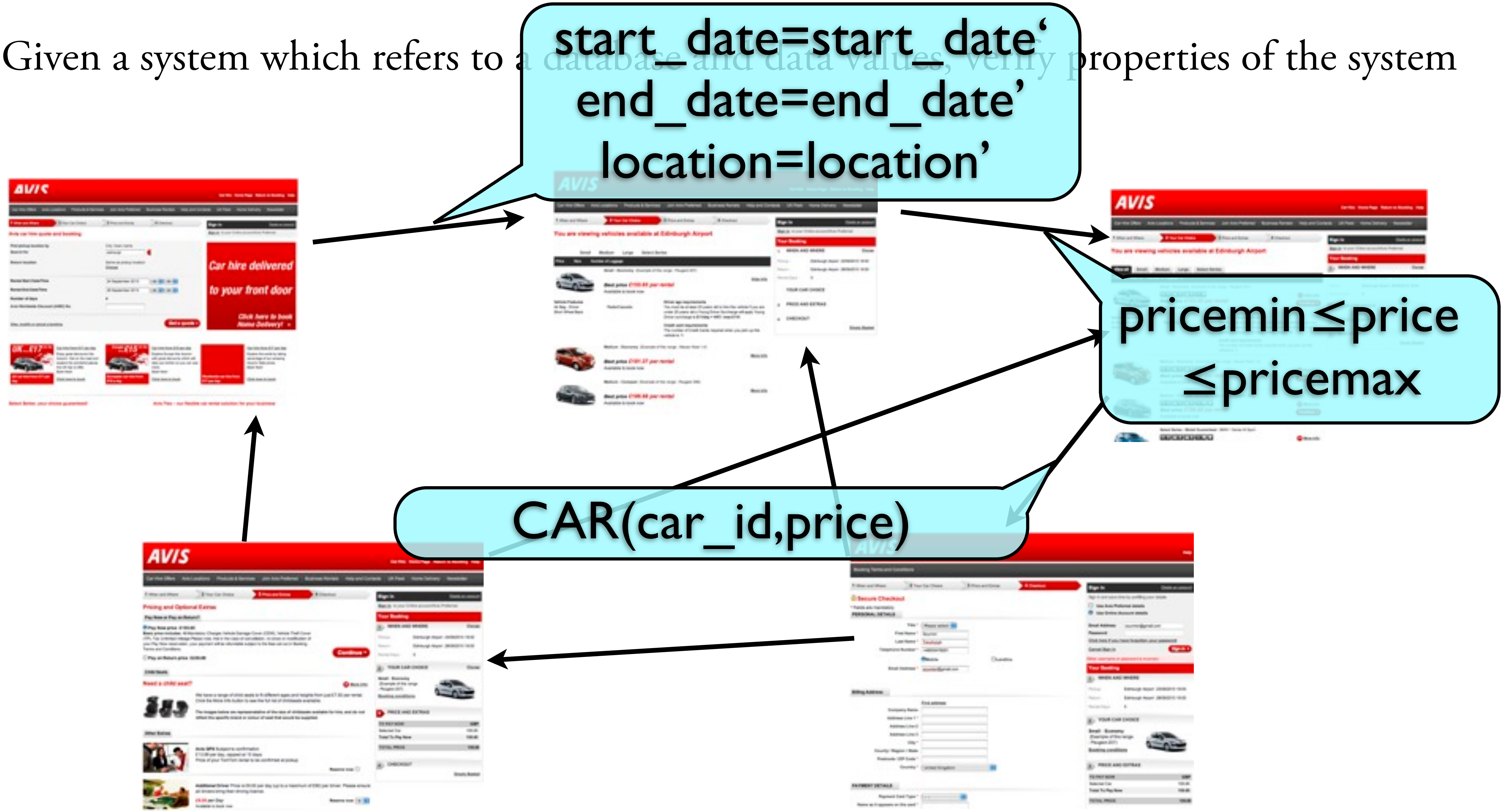

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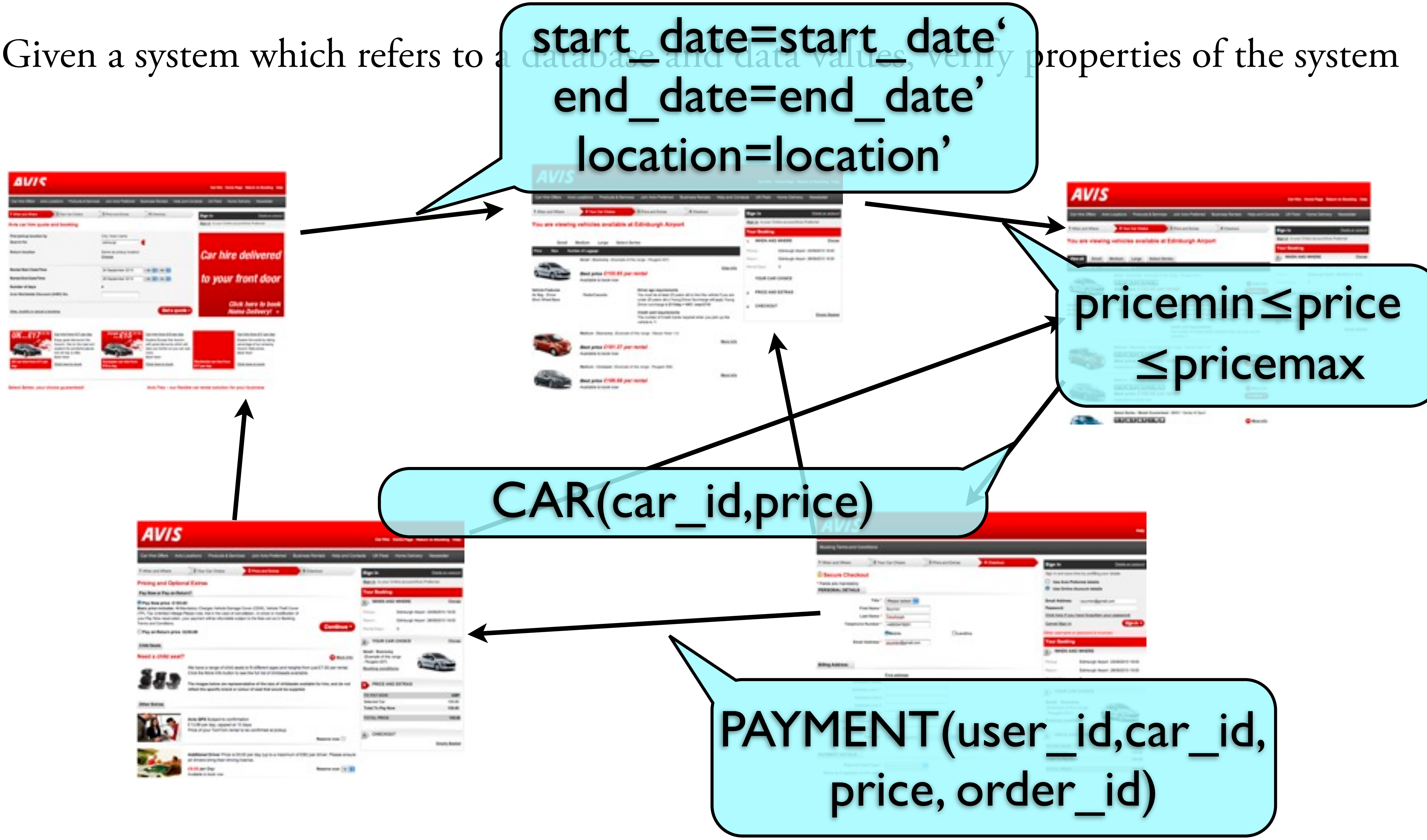

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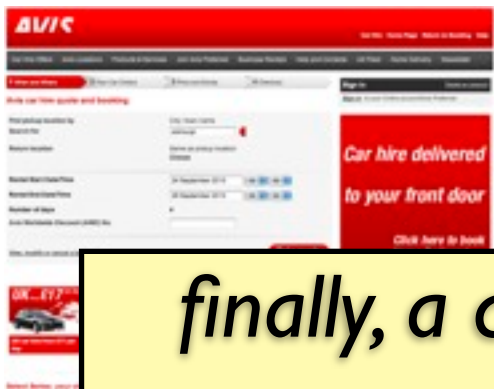
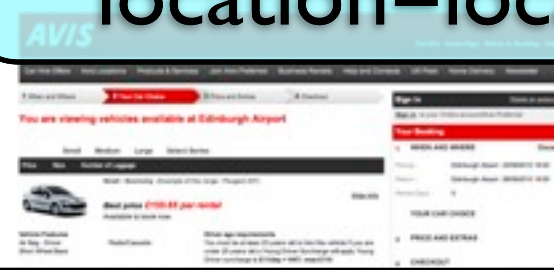
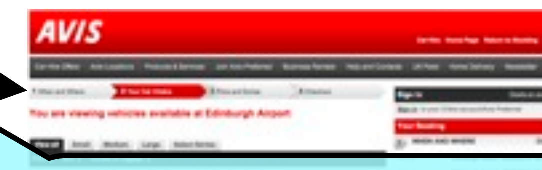


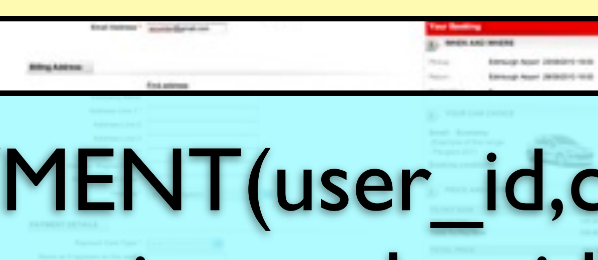




start_date=start_date'
end_date=end_date'
location=location'

pricemin≤price
x

finally, a car is rented and



PAYMENT(user_id,car_id, price, order_id)

Motivation

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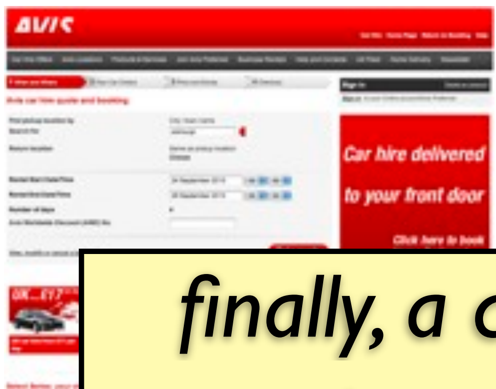
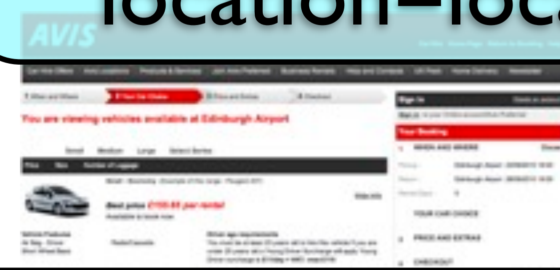
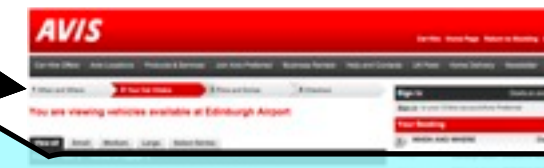


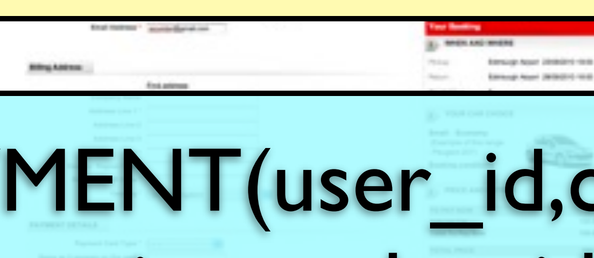




pricemin≤price

finally, a car is rented and

- *no payment was recieved*



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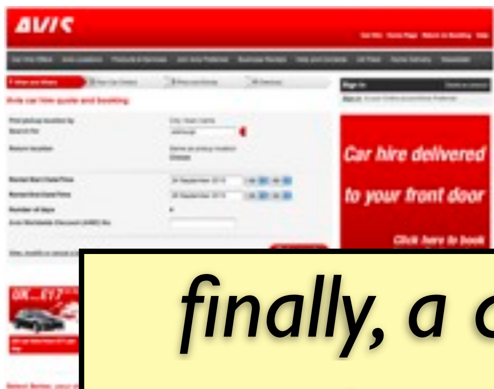
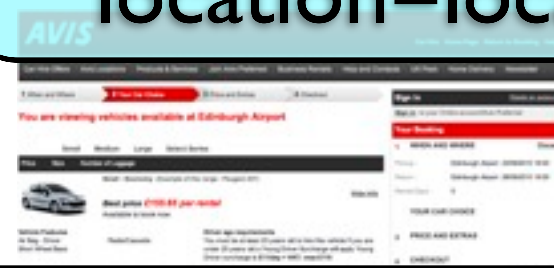
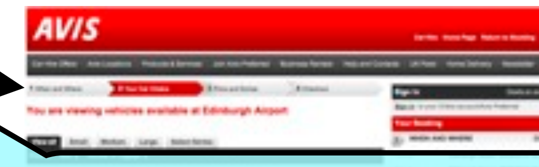




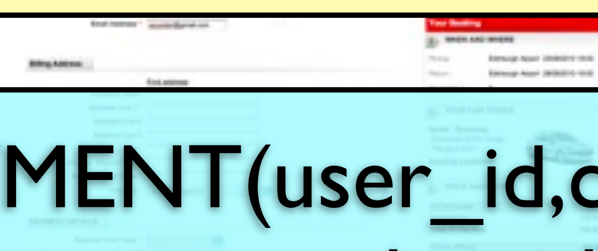


pricemin≤price

x

finally, a car is rented and

- *no payment was recieved*
- *a payment was recieved but* payed=0



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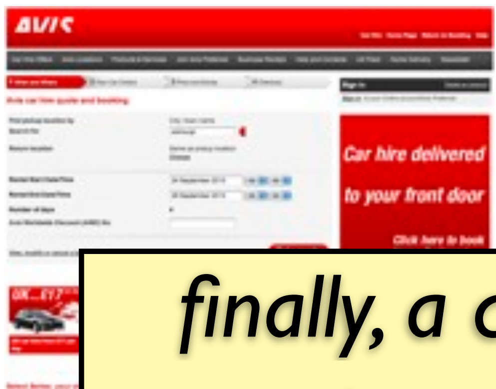
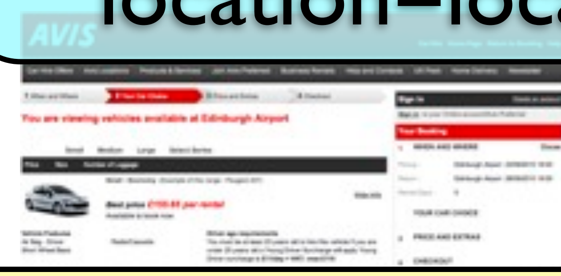
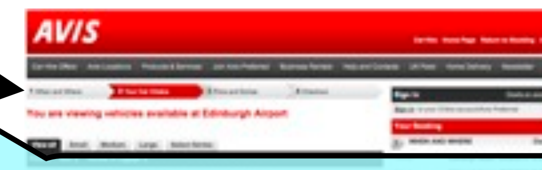




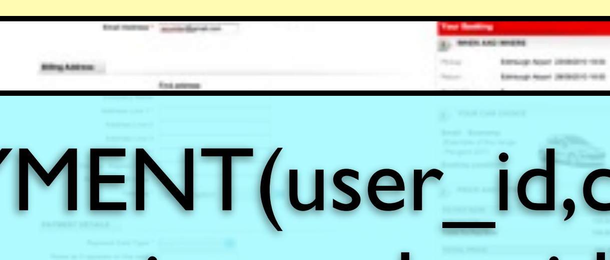


pricemin≤price

x

finally, a car is rented and

- *no payment was recieved*
- *a payment was recieved but* payed=0
- *there was a payment, but* payed<100 & end_date>2050



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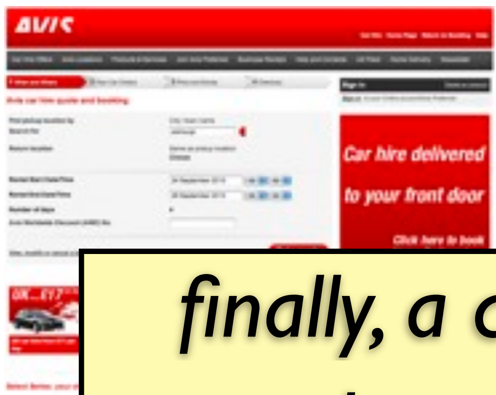

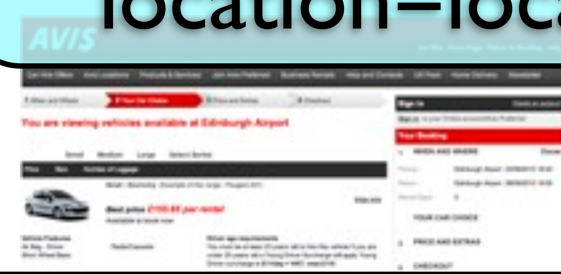

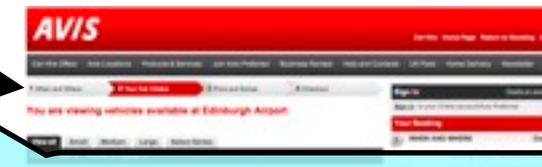

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finally, a car is rented and

- *no payment was recieved*
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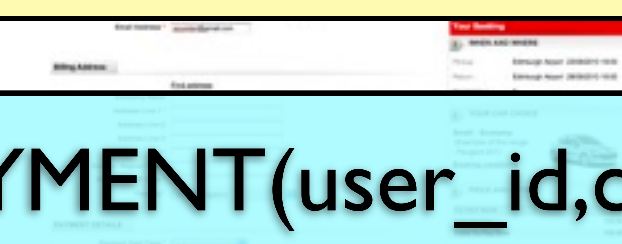
- *there was a payment, but* payed<100 & end_date>2050
- *there was a payment, but* payed<car_price



PAYMENT(user_id,car_id,
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Extended automaton

a definition that captures timed automata, or vector addition systems

D – fixed domain

Q – finite set of states

X – finite set of variables

$D^X = \{ v: \; X \to D \}$ – space of variable valuations

$Q \times D^X$ – space of configurations

restricted form

$\delta \subseteq (Q \times D^X) \times (Q \times D^X)$ – set of allowed transitions

$I \subseteq (Q \times D^X)$ – set of initial configurations

$F \subseteq (Q \times D^X)$ – set of final configurations

Examples

- *Vector Addition System*: $D = \mathbb{N}$

$$(q, v) \xrightarrow[v' = v + w]{} (q', v')$$

- *Timed Automata*: $D = \mathbb{R}$

$$(q, v) \xrightarrow[c_2' = 0]{c_1 < 2} (q', v')$$

- (*Lossy*) *channel system*: $D = \{a+b\}^*$

$$(q, v) \xrightarrow[c_1' = c_1 \cdot b]{first_a\,(c_1)} (q', v')$$

Our setting
(without the database)

The domain:

$$\mathscr{D} = \langle\ D, < ,\ P_1,\ P_2,\ P_3, \ldots, P_l\ \rangle$$

linearly ordered set

unary predicates
(subsets of D)

Examples

$\langle\ \mathbb{N}, <, 0, 100, P_{even}, P_{prime}\ \rangle$

$\langle\ \mathbb{Q}, <, 0, 100, P_{integer}, P_{<\pi}\ \rangle$

$\langle\ \{a+b\}^*, <_{lex}, P_{(ab)^*}\ \rangle$

Transitions:

$$I, F \subseteq (Q \times D^X)\ , \qquad \delta \subseteq (Q \times D^X) \times (Q \times D^X)$$

are specified by quantifier free formulas over $\mathscr{D}$

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states

$$\mathcal{A} \left\langle \begin{array}{ll} \tau_{\mathrm{I}}: & (x>0) \ \wedge \ (y=0) \\ \delta_a: & (x'=x) \ \wedge \ (y<y'<x) \\ \delta_b: & (x'=x) \ \wedge \ (y'=0) \\ \tau_{\mathrm{F}}: & (y=1) \end{array} \right.$$

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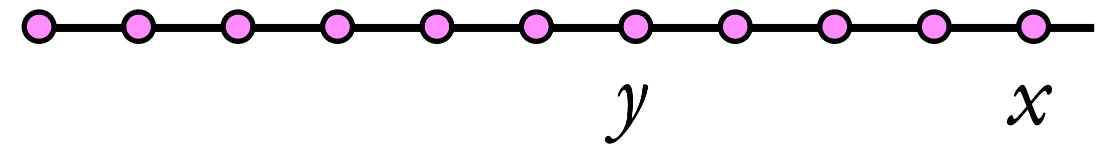

$\mathcal{A}$

τ_{I}: $(x>0) \wedge (y=0)$

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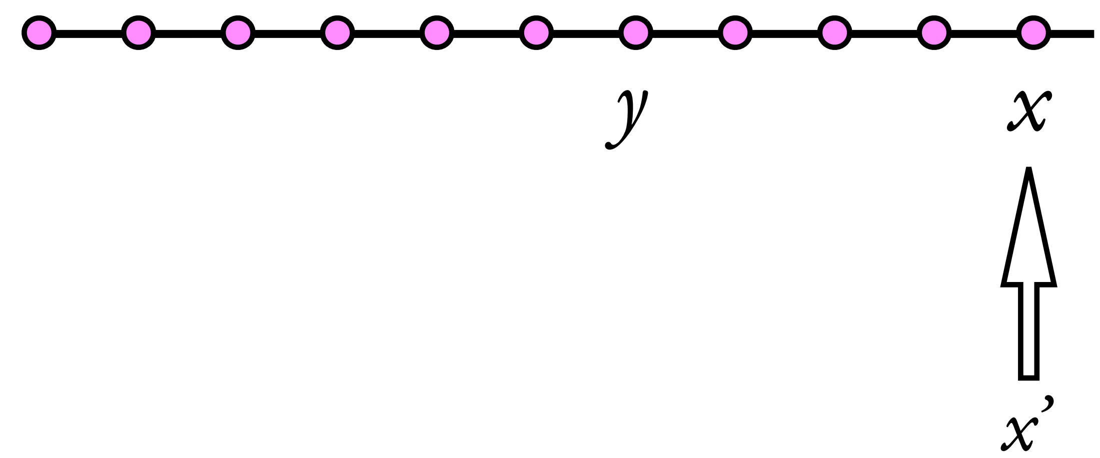

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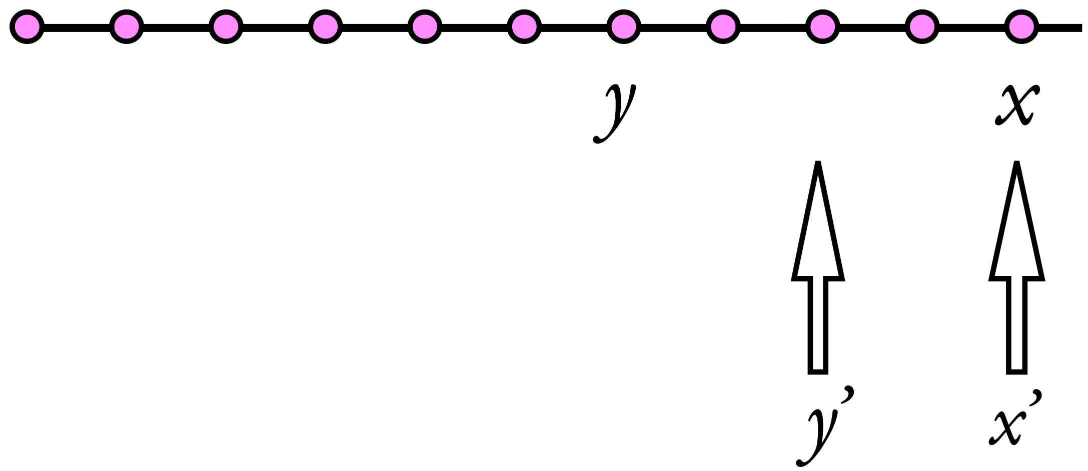

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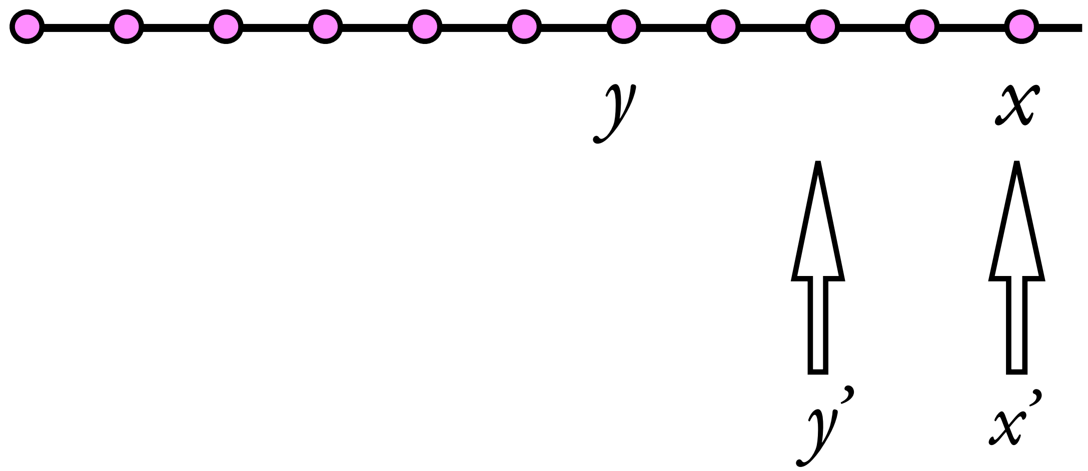

$$\mathcal{A} \begin{cases} \tau_I: & (x>0) \wedge (y=0) \\ \delta_a: & (x'=x) \wedge (y<y'<x) \\ \delta_b: & (x'=x) \wedge (y'=0) \\ \tau_F: & (y=1) \end{cases}$$

a finite run over $\mathbb{Q}$:

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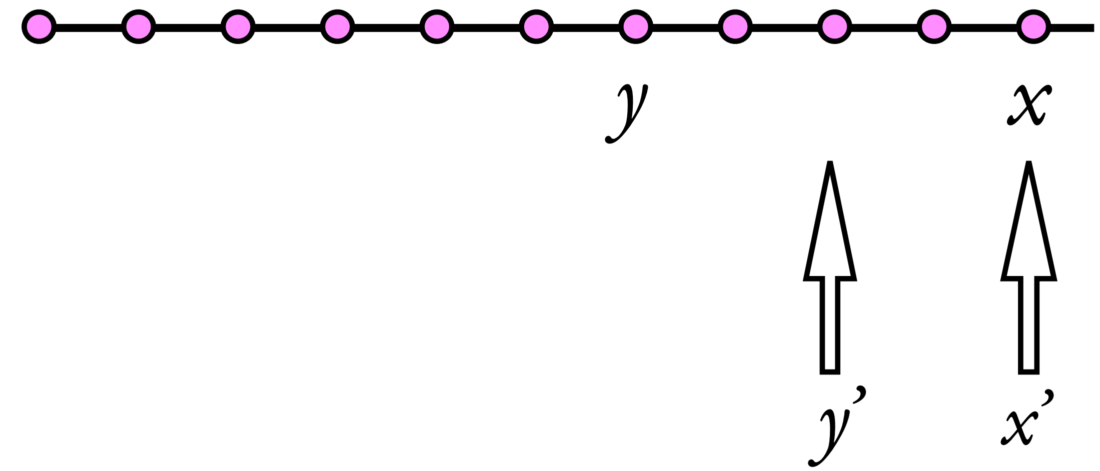
x,y – variables of $\mathcal{A}$

no states



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$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

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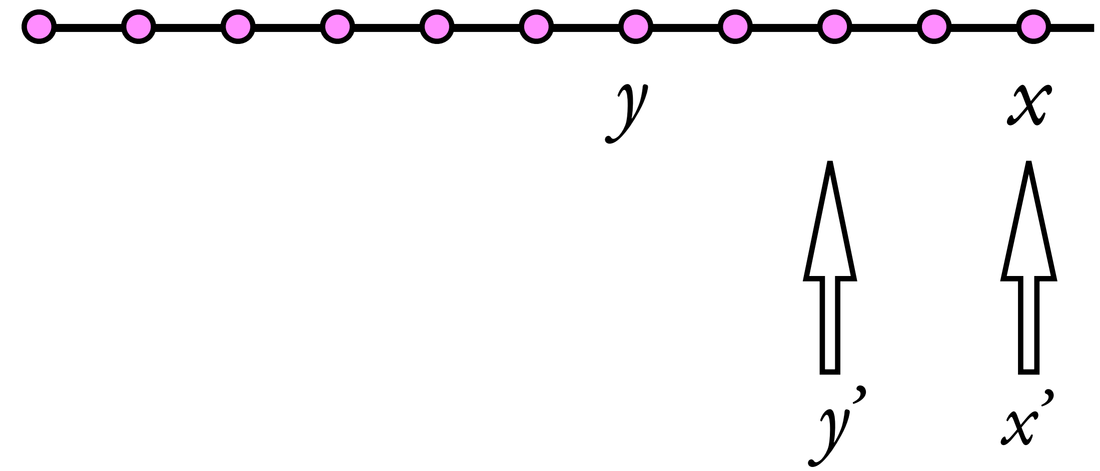

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a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x

y

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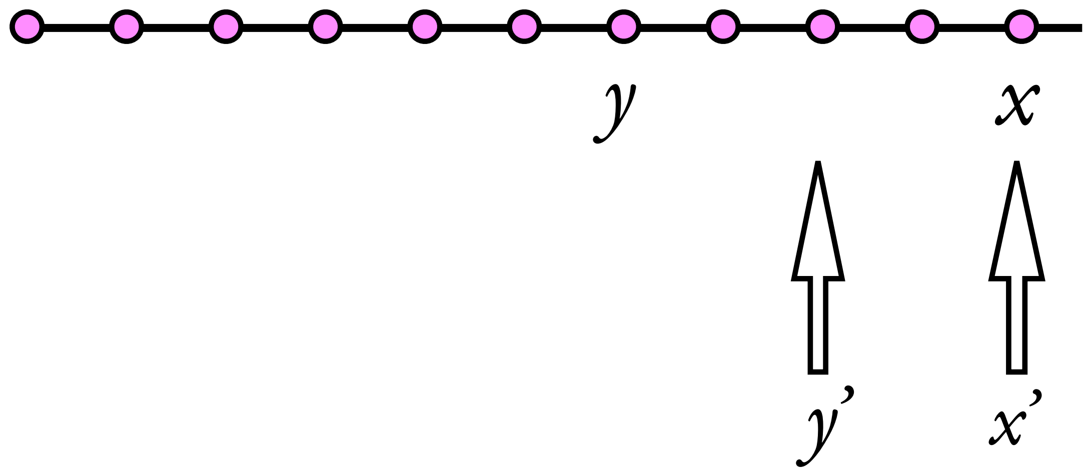

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τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

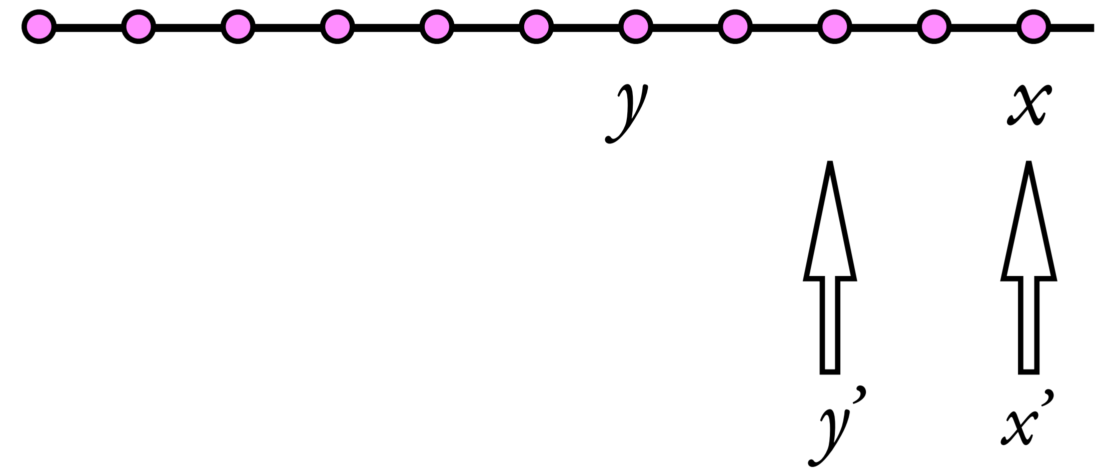
a a a b a b a a a a a

x	5
y	0

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



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a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x	5	5
y	0	1

Example D-automaton $\mathscr{A}$

x, y – variables of $\mathscr{A}$

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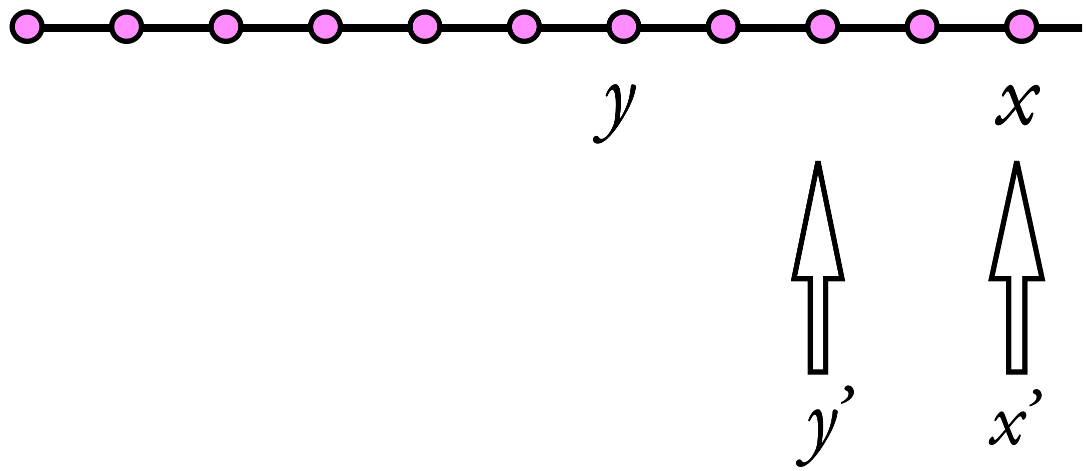

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τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

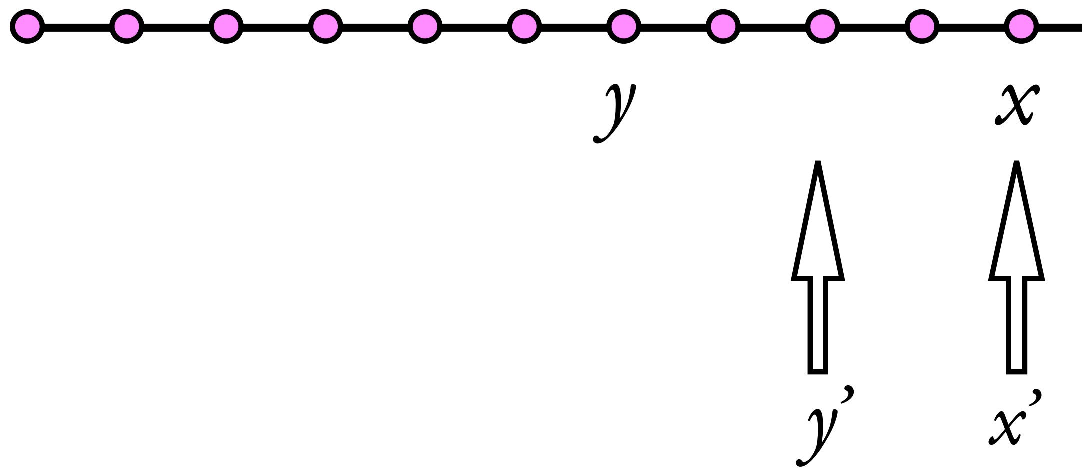
a a a b a b a a a a a

x	5	5	5
y	0	1	2

Example D-automaton $\mathcal{A}$

x, y – variables of $\mathcal{A}$

no states



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a finite run over $\mathbb{Q}$:

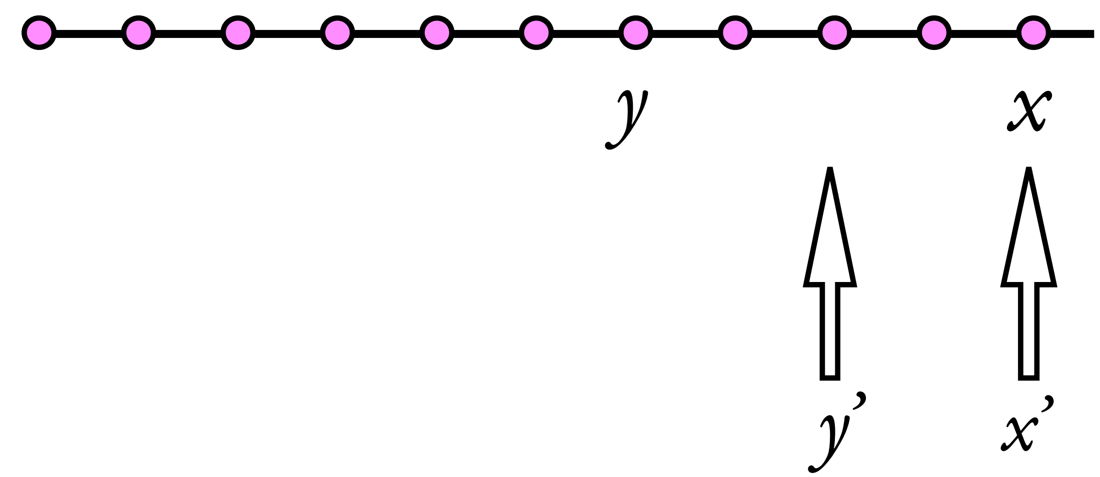
a a a b a b a a a a a

x	5	5	5	5
y	0	1	2	4

Example D-automaton $\mathcal{A}$

x, y – variables of $\mathcal{A}$

no states



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τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

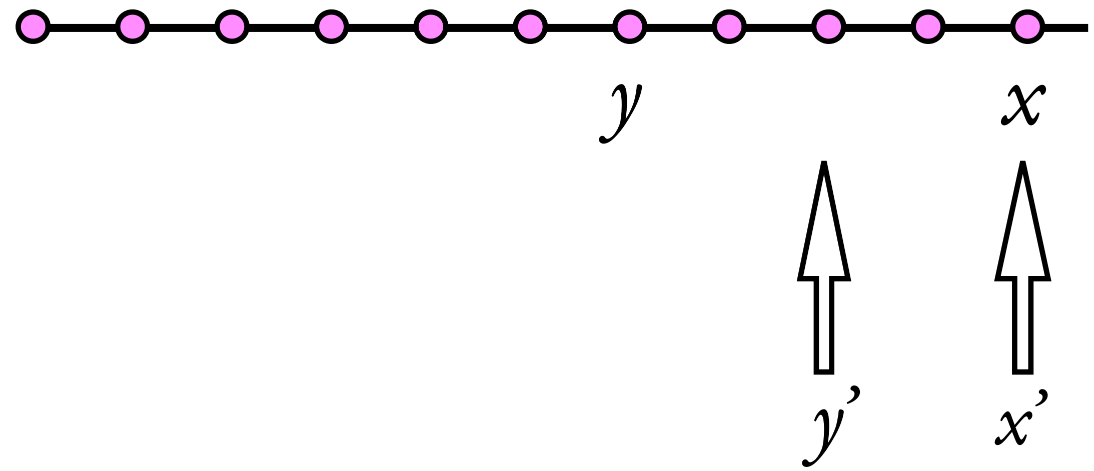
a a a b a b a a a a a

x	5	5	5	5	5
y	0	1	2	4	0

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



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a finite run over $\mathbb{Q}$:

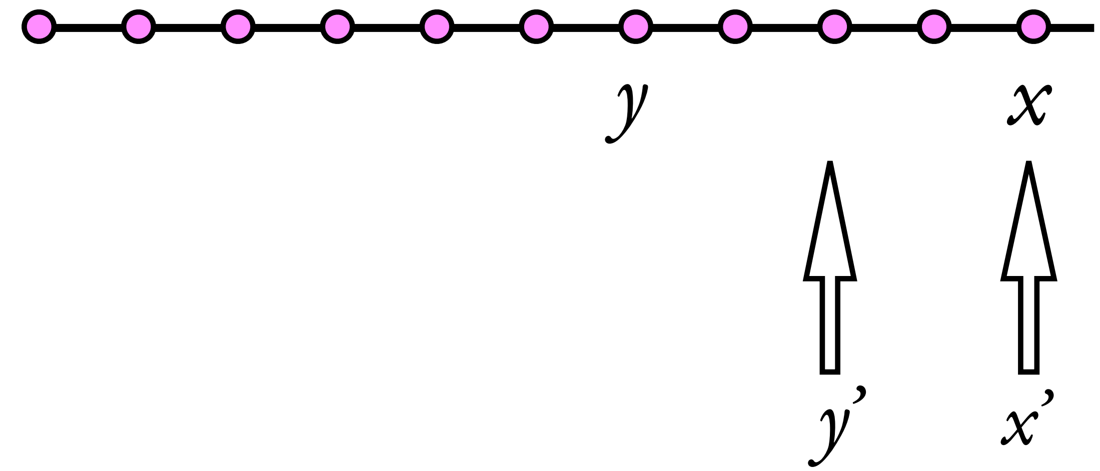
a a a b a b a a a a a

x	5	5	5	5	5	5
y	0	1	2	4	0	3

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



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x	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0

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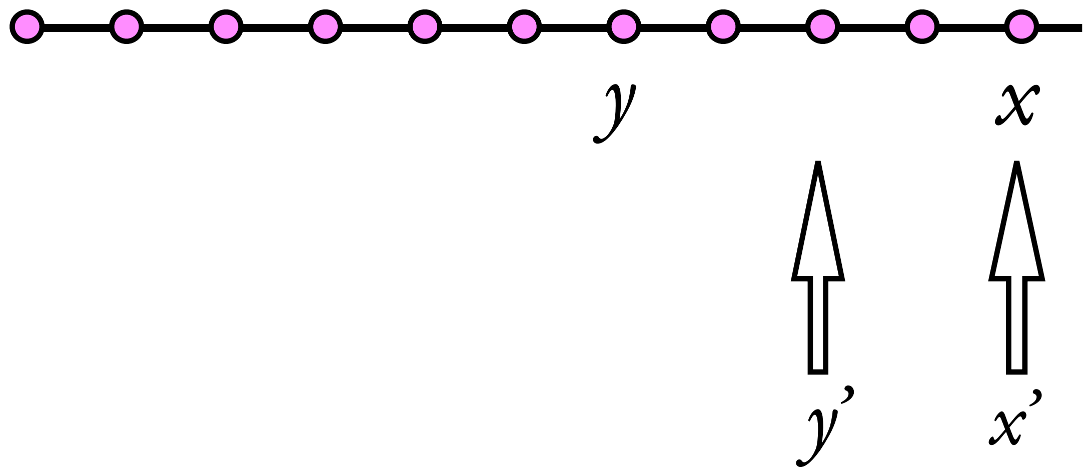
x,y – variables of $\mathcal{A}$

no states



$$
\mathcal{A} \left\langle
\begin{array}{ll}
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\delta_b: & (x'=x) \ \wedge \ (y'=0) \\
\tau_F: & (y=1)
\end{array}
\right.
$$

a finite run over $\mathbb{Q}$:

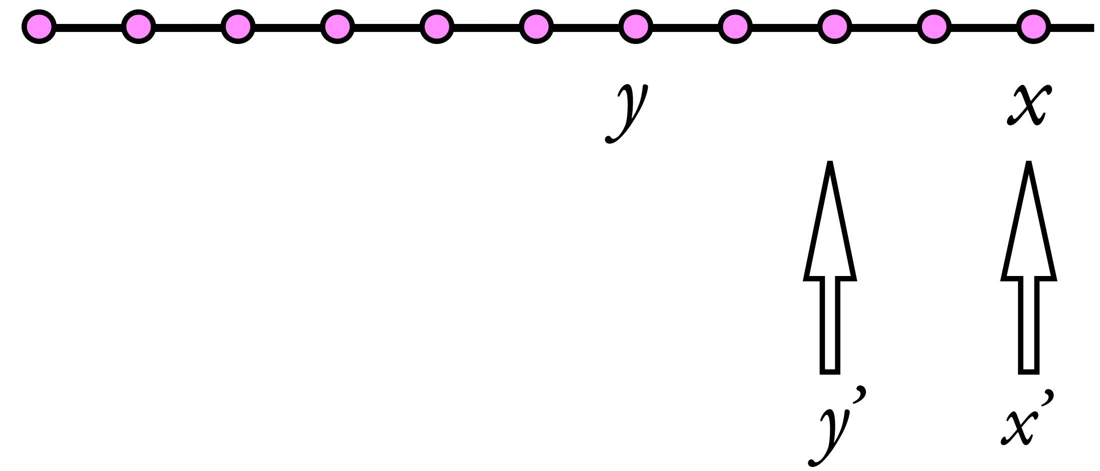
a a a b a b a a a a a

x	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$

Example D-automaton $\mathscr{A}$

x,y – variables of $\mathscr{A}$

no states



$\mathscr{A}$

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

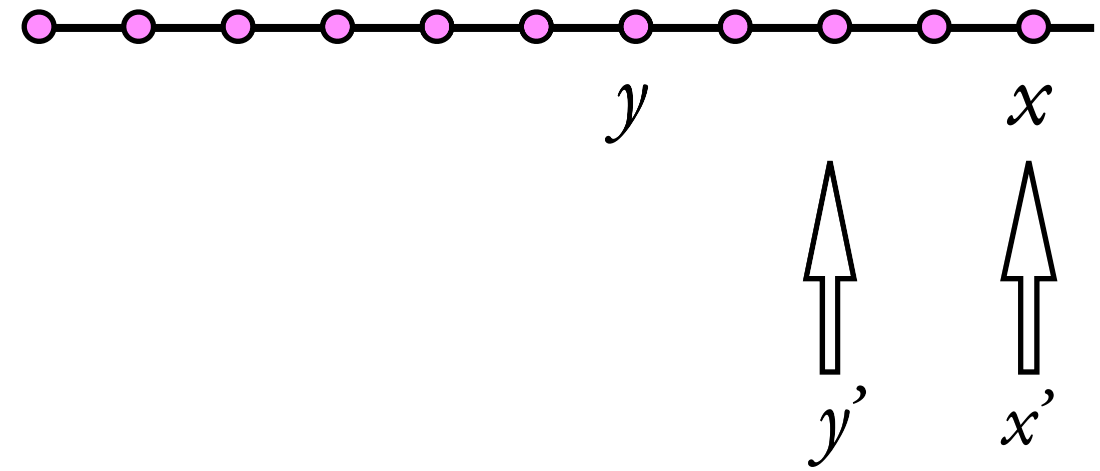
a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$\mathcal{A}$

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

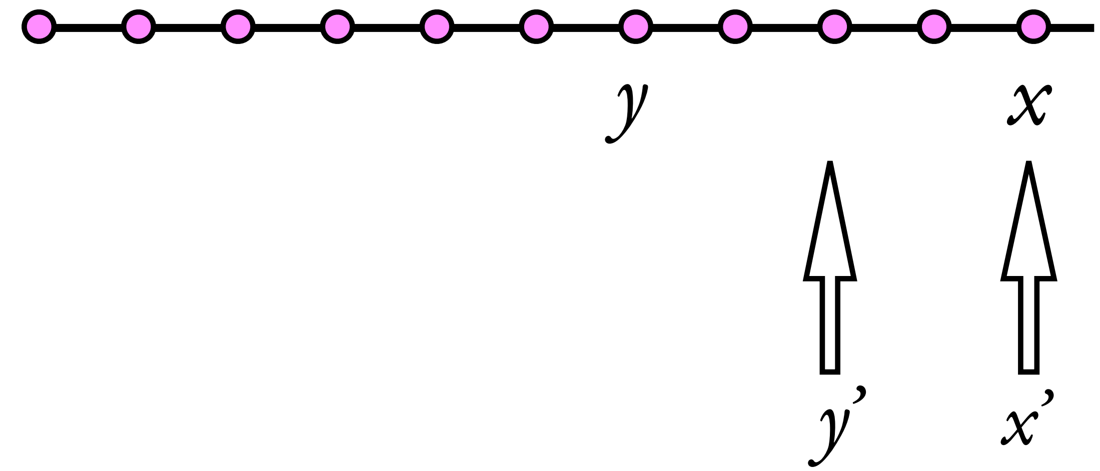
a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$\mathcal{A}$

τ_I: $(x>0) \wedge (y=0)$

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δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

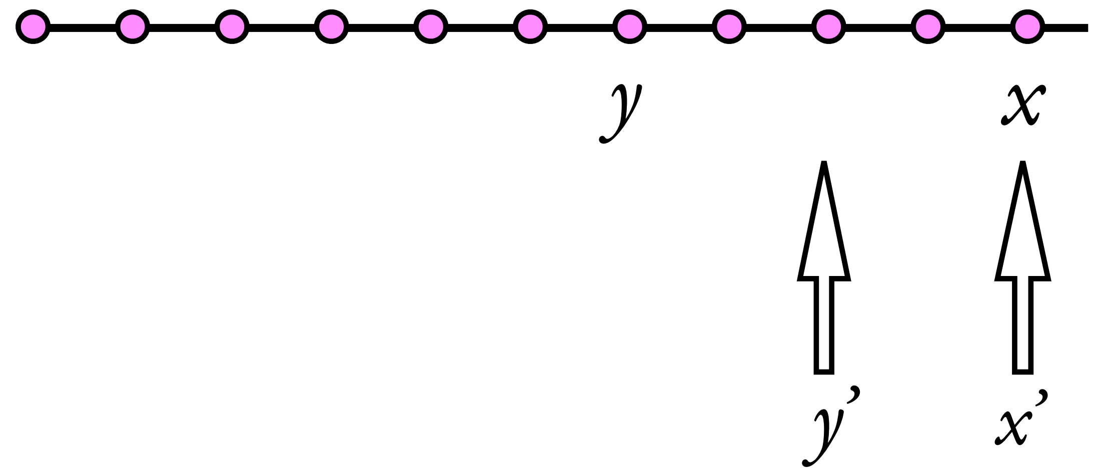
a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$

Example D-automaton $\mathscr{A}$

x,y – variables of $\mathscr{A}$

no states



$$\mathscr{A} \quad \begin{cases} \tau_I: & (x>0) \ \wedge \ (y=0) \\ \delta_a: & (x'=x) \ \wedge \ (y<y'<x) \\ \delta_b: & (x'=x) \ \wedge \ (y'=0) \\ \tau_F: & (y=1) \end{cases}$$

a finite run over $\mathbb{Q}$:

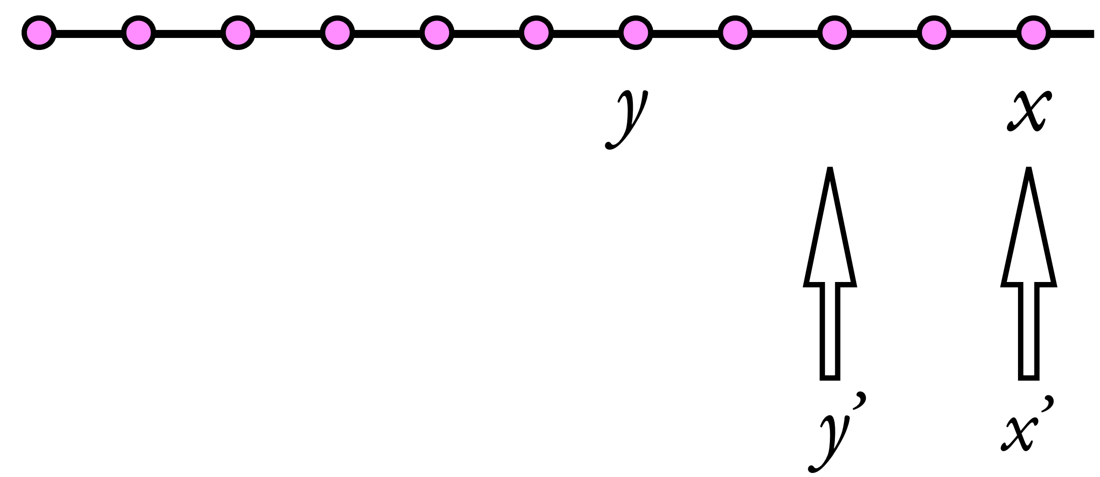
a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$\mathcal{A}$

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

$\boxed{\tau_F\text{: } (y=1)}$

a finite run over $\mathbb{Q}$:

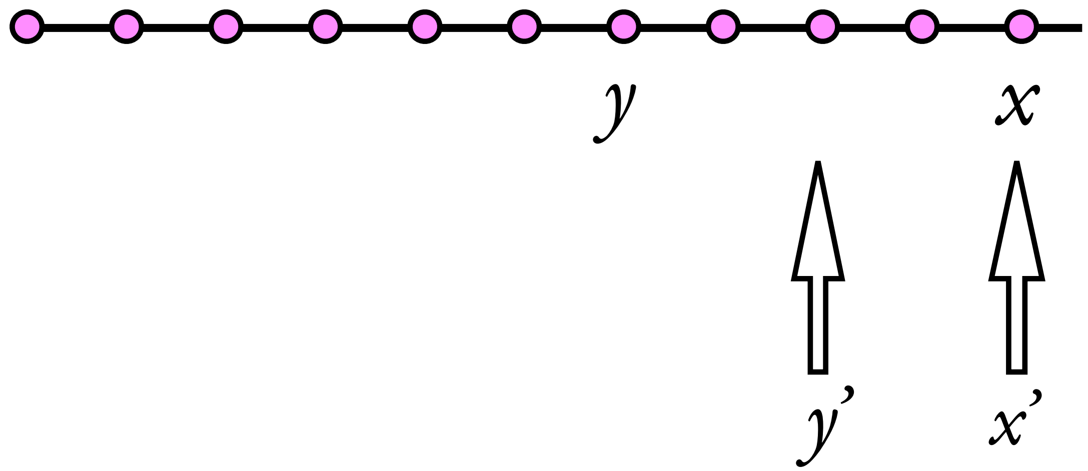
a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$\mathcal{A}$

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

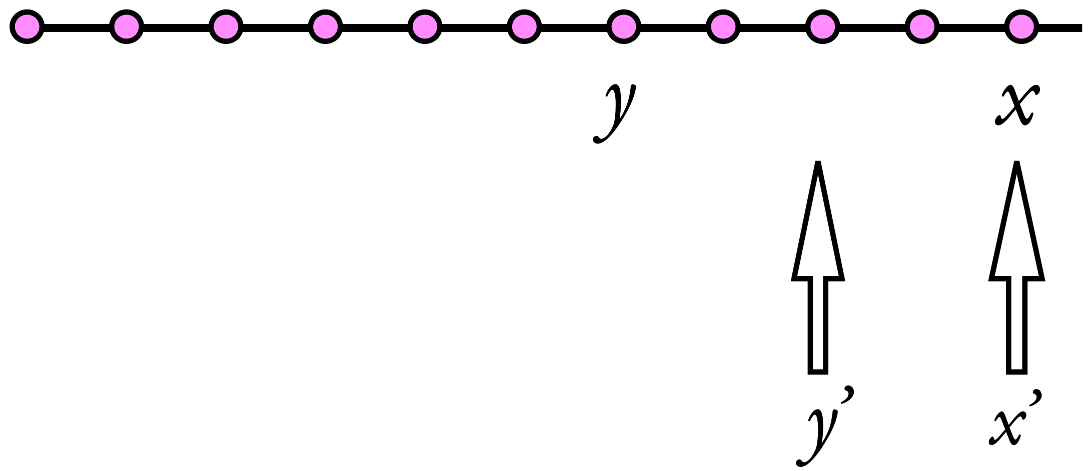
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

Example D-automaton $\mathcal{A}$

x, y – variables of $\mathcal{A}$

no states



$\mathcal{A}$

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

$\boxed{\tau_F: (y=1)}$

a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

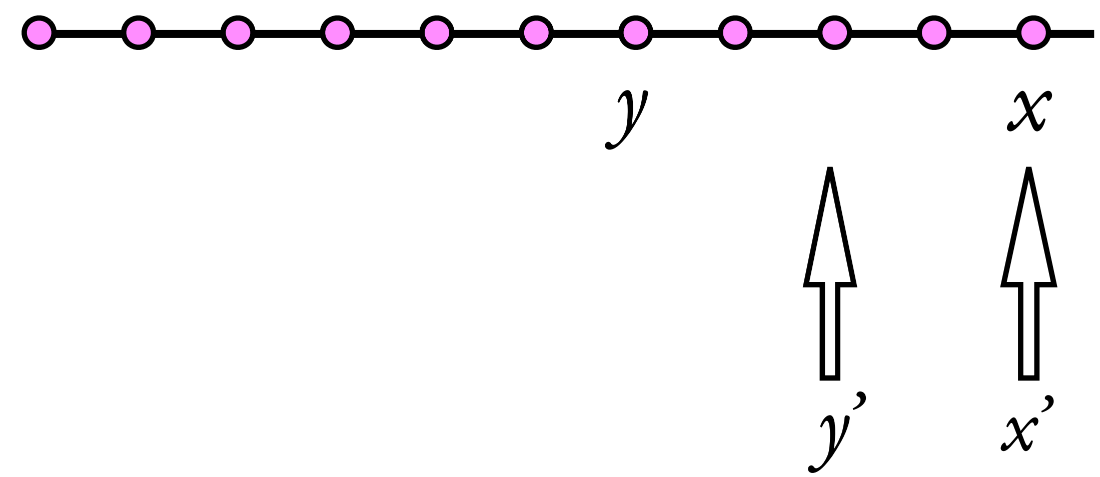
accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$$\mathcal{A} \begin{cases} \tau_I: & (x>0) \wedge (y=0) \\ \delta_a: & (x'=x) \wedge (y<y'<x) \\ \delta_b: & (x'=x) \wedge (y'=0) \\ \tau_F: & \boxed{(y=1)} \end{cases}$$

a finite run over $\mathbb{Q}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

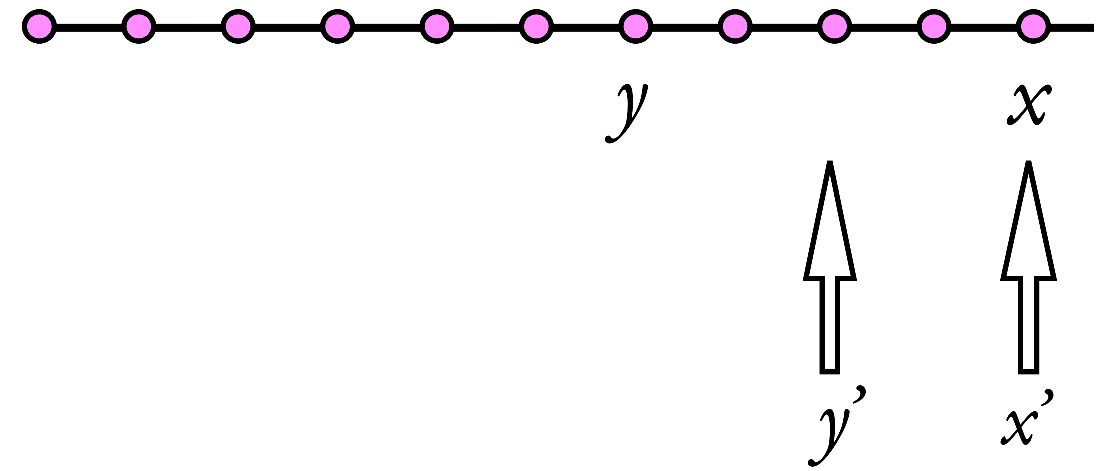
a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a$

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$$\mathcal{A} \left\langle \begin{array}{ll} \tau_I: & (x>0) \wedge (y=0) \\ \delta_a: & (x'=x) \wedge (y<y'<x) \\ \delta_b: & (x'=x) \wedge (y'=0) \\ \tau_F: & (y=1) \end{array} \right.$$

a finite run over $\mathbb{Q}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a$

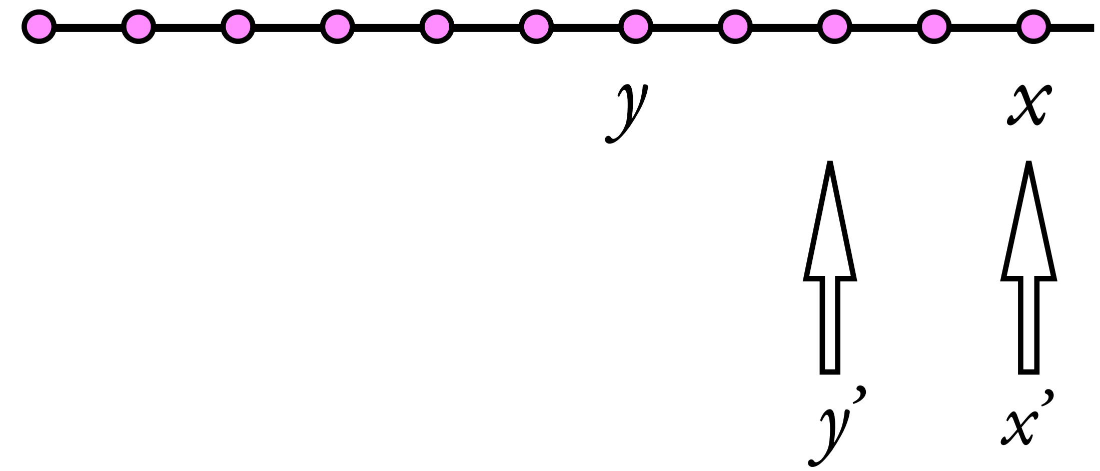
x

y

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$$\mathcal{A} \begin{cases} \tau_I: & (x>0) \wedge (y=0) \\ \delta_a: & (x'=x) \wedge (y<y'<x) \\ \delta_b: & (x'=x) \wedge (y'=0) \\ \boxed{\tau_F: \quad (y=1)} \end{cases}$$

a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

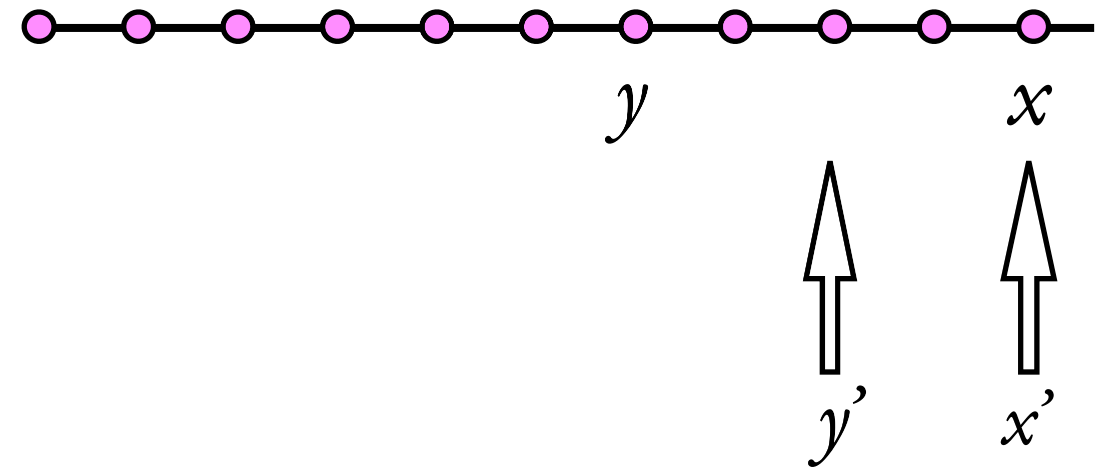
a a a b a b a a a a

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$\mathcal{A}$

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

$\boxed{\tau_F\text{: } (y=1)}$

a finite run over $\mathbb{Q}$:

		a	a	a	b	a	b	a	a	a	a	a
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

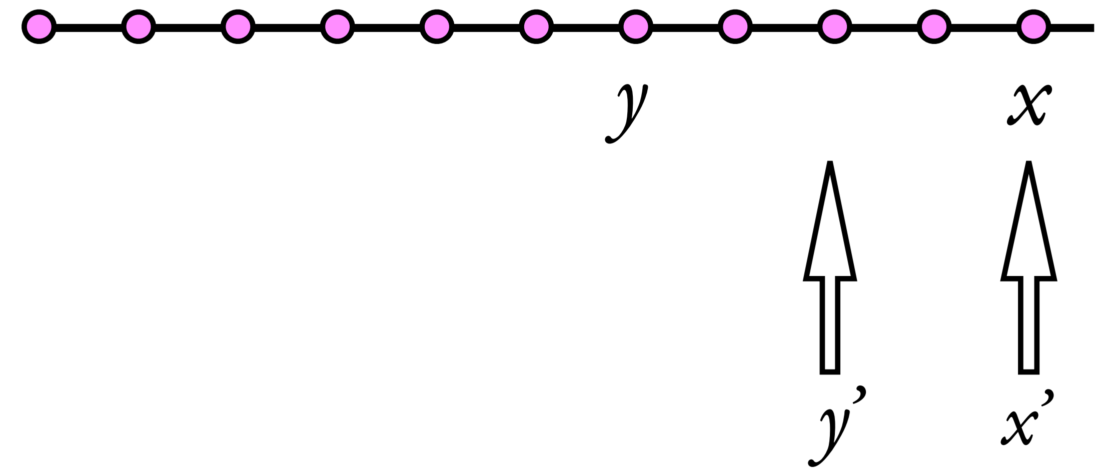
		a	a	a	b	a	b	a	a	a	a
x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

accepted language: $(a^*b)^*a$

Example D-automaton $\mathcal{A}$

x, y – variables of $\mathcal{A}$

no states



$$\mathcal{A} \left\langle \begin{array}{ll} \tau_I: & (x>0) \wedge (y=0) \\ \delta_a: & (x'=x) \wedge (y<y'<x) \\ \delta_b: & (x'=x) \wedge (y'=0) \\ \tau_F: & (y=1) \end{array} \right.$$

a finite run over $\mathbb{Q}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a$

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

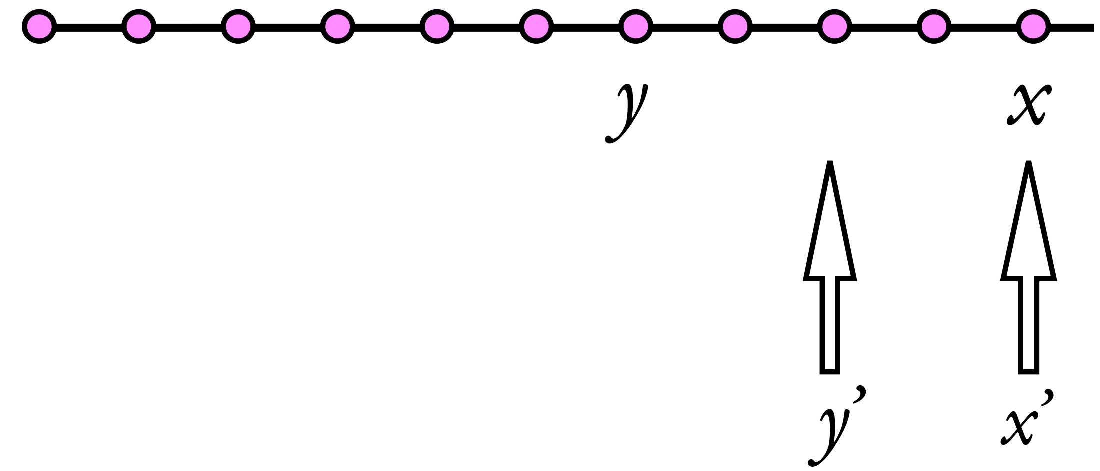
$a\ a\ \ a\ b\ a\ b\ a\ \ a\ a\ a$

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

Example D-automaton $\mathcal{A}$

x,y – variables of $\mathcal{A}$

no states



$$\mathcal{A} \begin{cases} \tau_I: & (x>0) \wedge (y=0) \\ \delta_a: & (x'=x) \wedge (y<y'<x) \\ \delta_b: & (x'=x) \wedge (y'=0) \\ \tau_F: & \boxed{(y=1)} \end{cases}$$

a finite run over $\mathbb{Q}$:

a a a b a b a a a a a

x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

a finite run over $\mathbb{N}$:

a a a b a b a a a a

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

a a a b a b a a a a

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4

accepted language: $(a^{\mathrm{B}}b)^\omega$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_{I}: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_{F}: $(x=1)$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
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δ_b: $(x'=0)$

τ_{F}: $(x=1)$

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_{I}: $(x=0)$

δ_a: $(x' > x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_{F}: $(x=1)$

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

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$R,S,\ldots$ – a finite set of relational symbols

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δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

$\boxed{\tau_F: (x=1)}$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

a a a b a b a

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={}

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x, y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={}

accepted language: $(a^*b)^*a$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_{I}: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

$\boxed{\tau_{\mathrm{F}}: \ (x=1)}$

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

x 0 2 3 4 0 3 0 1

R={(0),(1),(2),(3),(4)} S={}

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_{I}: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

$\boxed{\tau_{\mathrm{F}}: \ (x=1)}$

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={}

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ b\ a\ a\ldots$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x' > x) \wedge R(x) \wedge \neg S(x, y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

	a	a	a	b	a	b	a	
x	0	2	3	4	0	3	0	1

$R = \{(0),(1),(2),(3),(4)\}$ S={}

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

	a	a	a	b	a	b	a	a	b	a	$a\ldots$	
x	0	1	3	4	0	3	0	1	4	0	1	3

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

$\boxed{\tau_F: \ (x=1)}$

EMPTINESS: is there a *finite* database *M*, a word *w*, and an accepting run over *w* consistent with *M*?

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

x	0	2	3	4	0	3	0	1

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ b\ a\ a\ldots$

x	0	1	3	4	0	3	0	1	4	0	1	3

$R=\{(0),(1),(2),(3),(4)\}$ $S=\{\}$

Adding database constraints

$x, y, z, \ldots$ – a finite set of variables
$R, S, \ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

EMPTINESS: is there a *finite* database *M,* a word *w,* and an accepting run over *w* consistent with *M?*

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={}

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ b\ a\ a\ldots$

x 0 1 3 4 0 3 0 1 4 0 1 3

$R=\{(0),(1),(2),(3),(4)\}$ S={}

accepted language: $(a^{\mathrm{B}}b)^\omega$

Adding database constraints

$x,y,z,\ldots$ – a finite set of variables
$R,S,\ldots$ – a finite set of relational symbols

τ_I: $(x=0)$

δ_a: $(x'>x) \wedge R(x) \wedge \neg S(x,y')$

δ_b: $(x'=0)$

τ_F: $(x=1)$

Decide emptiness of D-automata over ω-words, with databases

EMPTINESS: is there a *finite* database M, a word w, and an accepting run over w consistent with M?

a finite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a$

x 0 2 3 4 0 3 0 1

$R=\{(0),(1),(2),(3),(4)\}$ S={}

accepted language: $(a^*b)^*a$

an infinite run over $\mathbb{N}$:

$a\ a\ a\ b\ a\ b\ a\ a\ b\ a\ a\ldots$

x 0 1 3 4 0 3 0 1 4 0 1 3

$R=\{(0),(1),(2),(3),(4)\}$ S={}

accepted language: $(a^B b)^\omega$

Known results
deciding emptiness

	no database	with database
$\mathbb{N}$,=	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q}$,<	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
$\mathbb{N}$,<	Čerans (94) NONPRIMITIVE	?

Known results
deciding emptiness

	no database	with database
$\mathbb{N}$,=	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q}$,<	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
$\mathbb{N}$,<	Čerans (94) NONPRIMITIVE	?
$\mathscr{D}$,<	PSPACE	PSPACE

Known results

deciding emptiness

	no database	with database
$\mathbb{N}$,=	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q}$,<	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
$\mathbb{N}$,<	Čerans (94) NONPRIMITIVE	PSPACE
$\mathscr{D}$,<	PSPACE	PSPACE

Known results
deciding emptiness

	no database	with database
$\mathbb{N},=$	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q},<$	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
$\mathbb{N},<$	PSPACE	PSPACE
$\mathscr{D},<$	PSPACE	PSPACE

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

Dense case

- D = $\mathbb{Q}$
- finite words
- no database

τ_I: $\boxed{(x>0) \wedge (y=0)}$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $\boxed{(y=1)}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

τ_{I}: $\boxed{(x>0) \wedge (y=0)}$

δ_{a}: $(x'=x) \wedge (y<y'<x)$

δ_{b}: $(x'=x) \wedge (y'=0)$

τ_{F}: $\boxed{(y=1)}$

The region construction

$\boxed{0<y<1<x}$, $\boxed{0<x<1=y}$, $\boxed{0<1<y<x}$, $\boxed{0<1=y<x}$, $\boxed{0=y<1<x}$, $\boxed{0=y<1=x}$, …

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction

$0<y<1<x$, $0<x<1=y$, $0=1<y<x$, $0<1=y<x$, $0=y<1<x$, $0=y<1=x$, …

a

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction

$0<y<1<x,\ 0<x<1=y,\ 0=1<y<x,\ 0<1=y<x,\ 0=y<1<x,\ 0=y<1=x,\ \ldots$

a

⋓

$\exists(x,y)$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

τ_{I}: $\boxed{(x>0) \wedge (y=0)}$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_{F}: $\boxed{(y=1)}$

The region construction

$0<y<1<x$, $\boxed{0<x<1=y}$, $0=1<y<x$, $\boxed{0<1=y<x}$, $\boxed{0=y<1<x}$, $\boxed{0=y<1=x}$, …

$\boxed{0<1=y<x} \xleftarrow{a} \boxed{0=y<1<x}$

$\exists(x',y') \qquad \exists(x,y)$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction

$0<y<1<x$, $0<x<1=y$, $0=1<y<x$, $0<1=y<x$, $0=y<1<x$, $0=y<1=x$, ...

$$\overset{\rotatebox{90}{$\in$}}{\exists(x',y')} \quad \overset{\rotatebox{90}{$\in$}}{\exists(x,y)}: \quad \delta_a(x, y, x', y')$$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction

$0<y<1<x$, $0<x<1=y$, $0=1<y<x$, $0<1=y<x$, $0=y<1<x$, $0=y<1=x$, …

a

$$\overset{\Cup}{\exists(x',y')} \quad \overset{\Cup}{\exists(x,y)}: \quad \delta_a(x, y, x', y')$$

$(5,0)$

Dense case

- D = $\mathbb{Q}$
- finite words
- no database

τ_{I}: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_{F}: $(y=1)$

The region construction

$0<y<1<x$, $0<x<1=y$, $0<1<y=x$, $0<1=y<x$, $0=y<1<x$, $0=y<1=x$, …

$$\underset{(5,1)}{\exists\overset{\Cup}{(x',y')}} \quad \underset{(5,0)}{\exists\overset{\Cup}{(x,y)}}: \quad \delta_a(x,\ y,\ x',\ y')$$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction

$0<y<1<x$, $0<x<1=y$, $0=1<y<x$, $0<1=y<x$, $0=y<1<x$, $0=y<1=x$, …

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

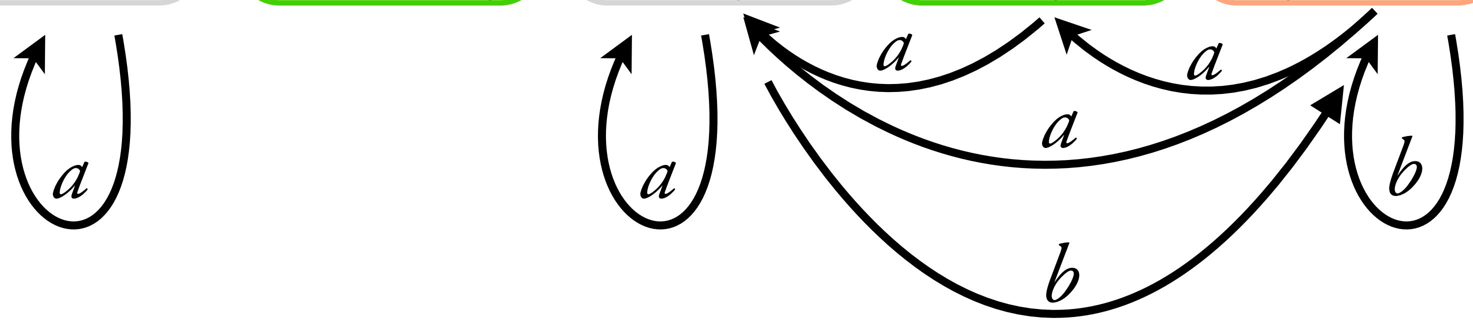
δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction

$0<y<1<x$, $0<x<1=y$, $0=1<y<x$, $0<1=y<x$, $0=y<1<x$, $0=y<1=x$, …



Dense case

The region construction

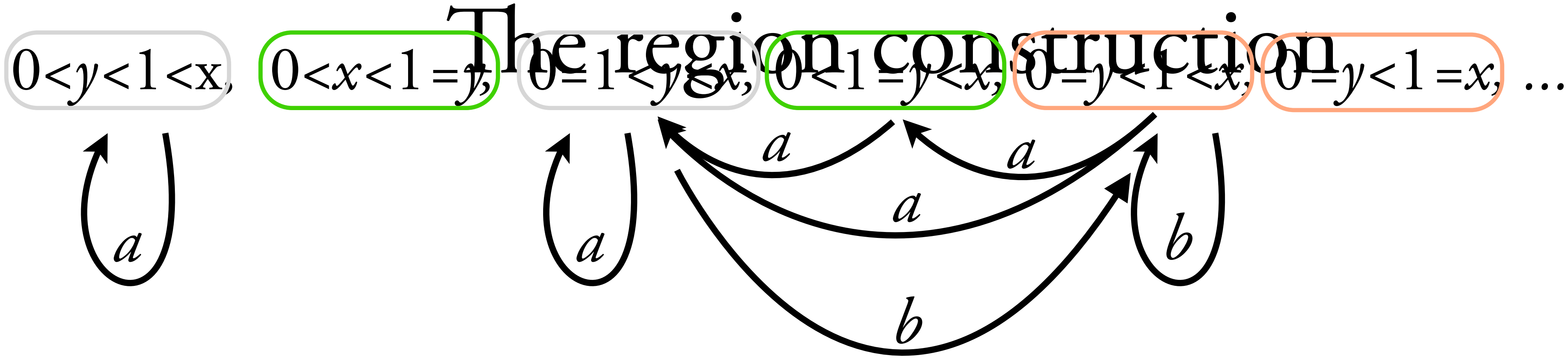
- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$



bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

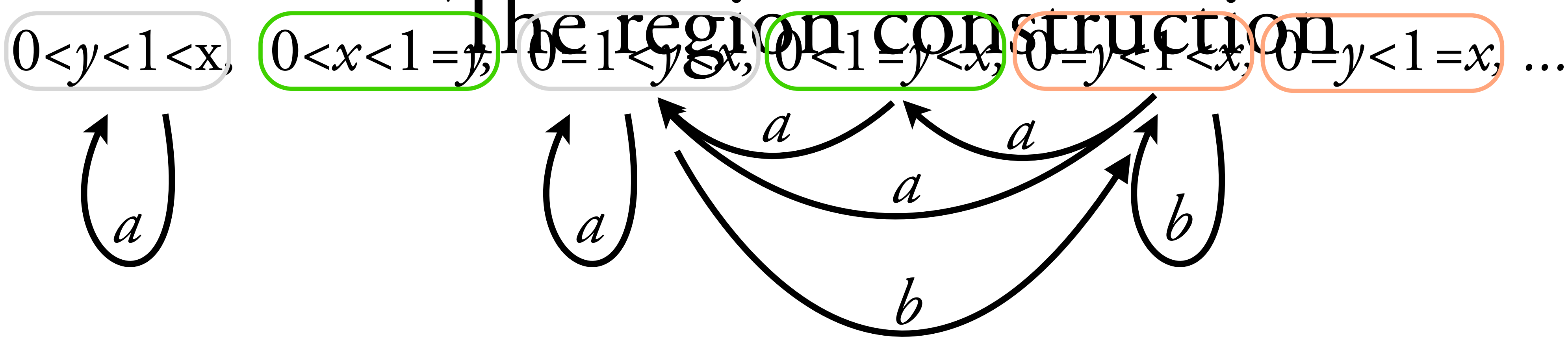
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



a a a b a b a a a a a

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

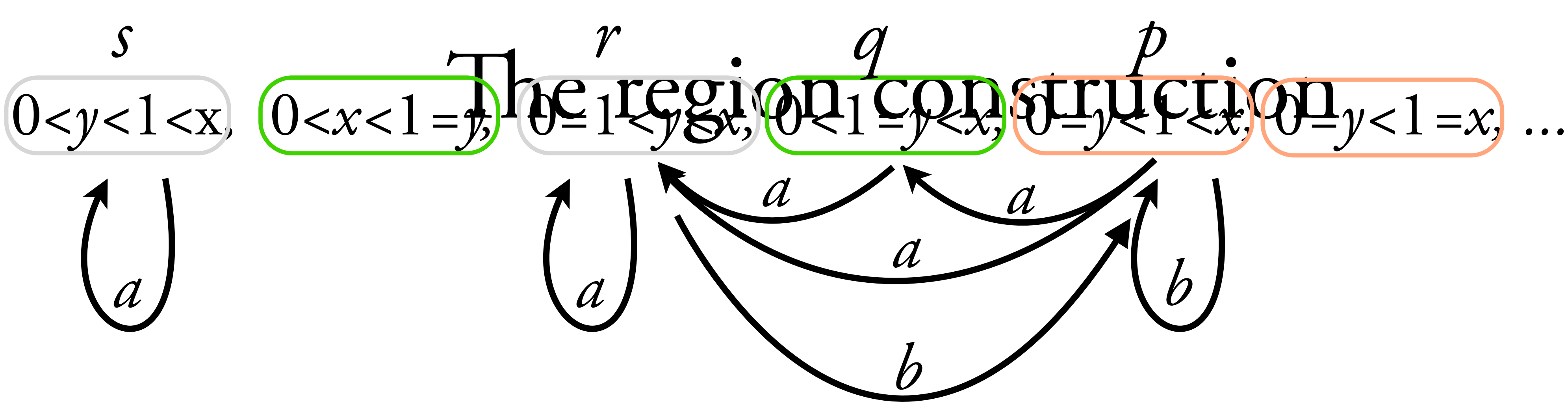
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



a a a b a b a a a a a

bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

Dense case

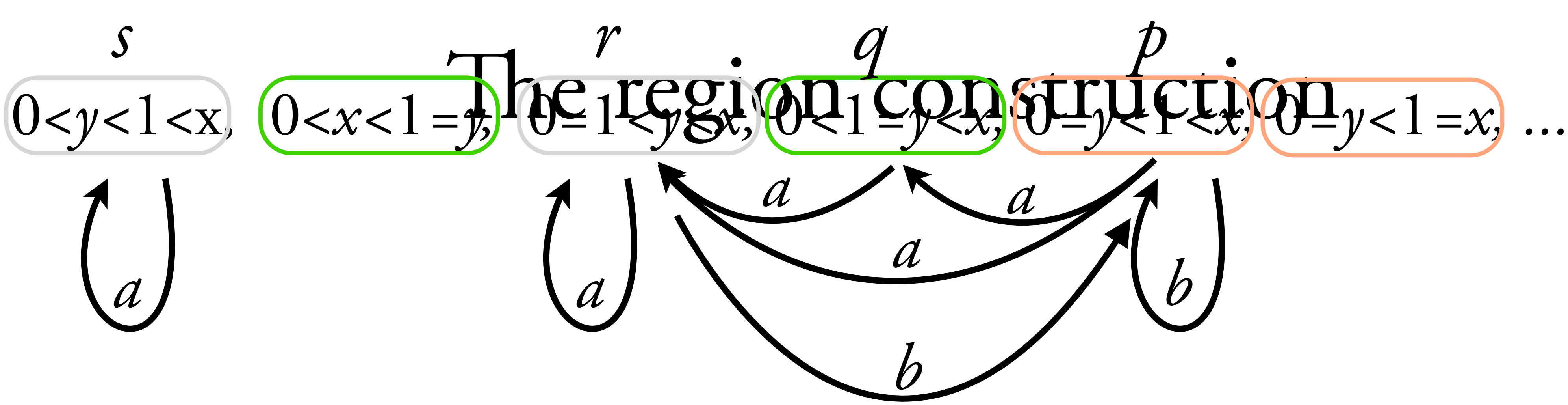
- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$



a a a b a b a a a a a

p q r r q r q s s s s q

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

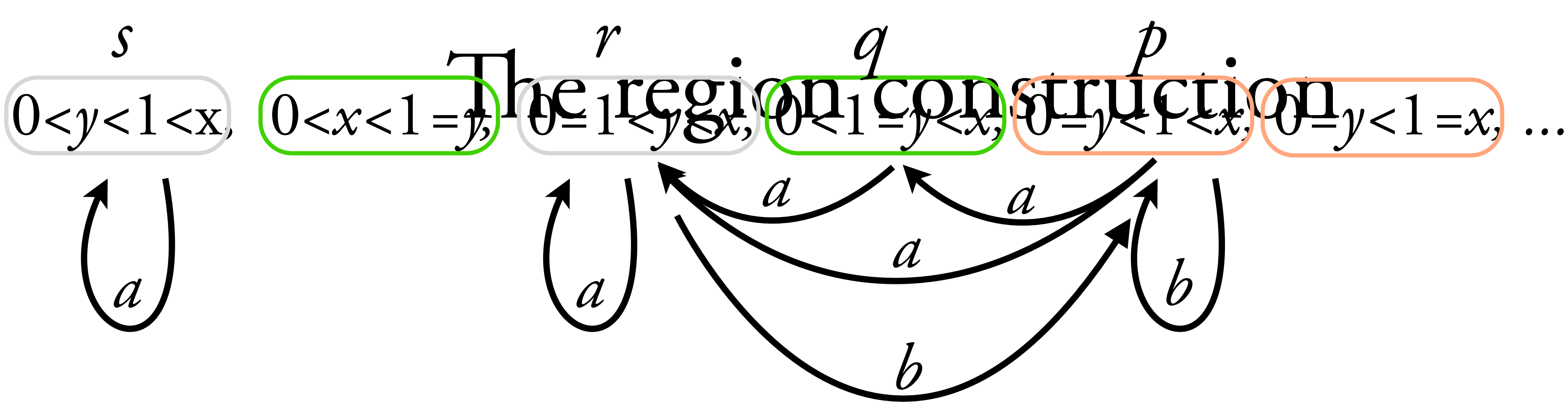
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

$p\ q\ r\ r\ q\ r\ q\ s\ s\ s\ s\ q$

x

y

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

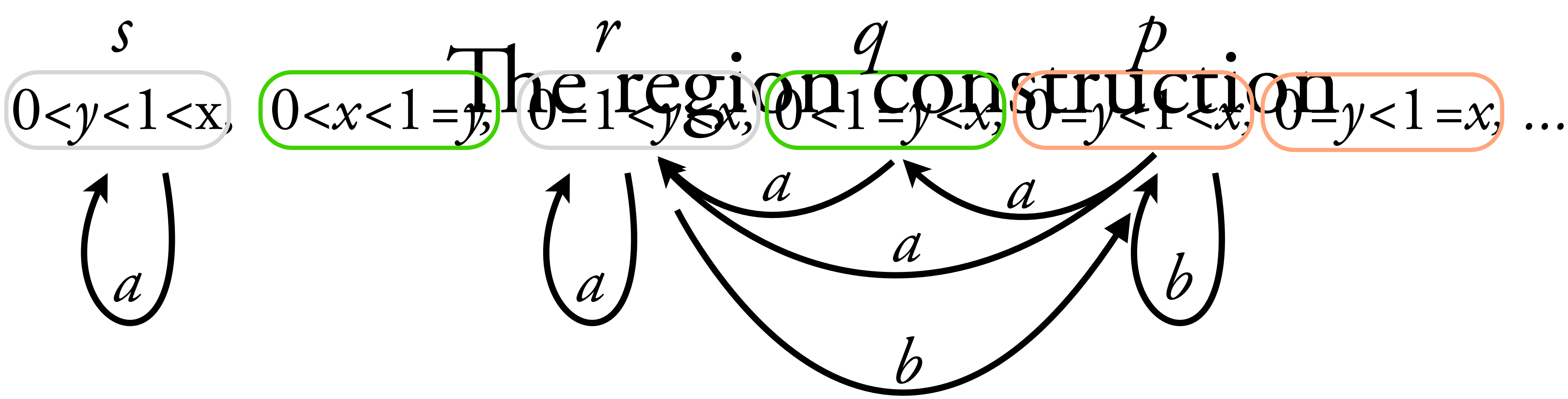
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

$p\ q\ r\ r\ q\ r\ q\ s\ s\ s\ s\ q$

x 5

y 0

bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

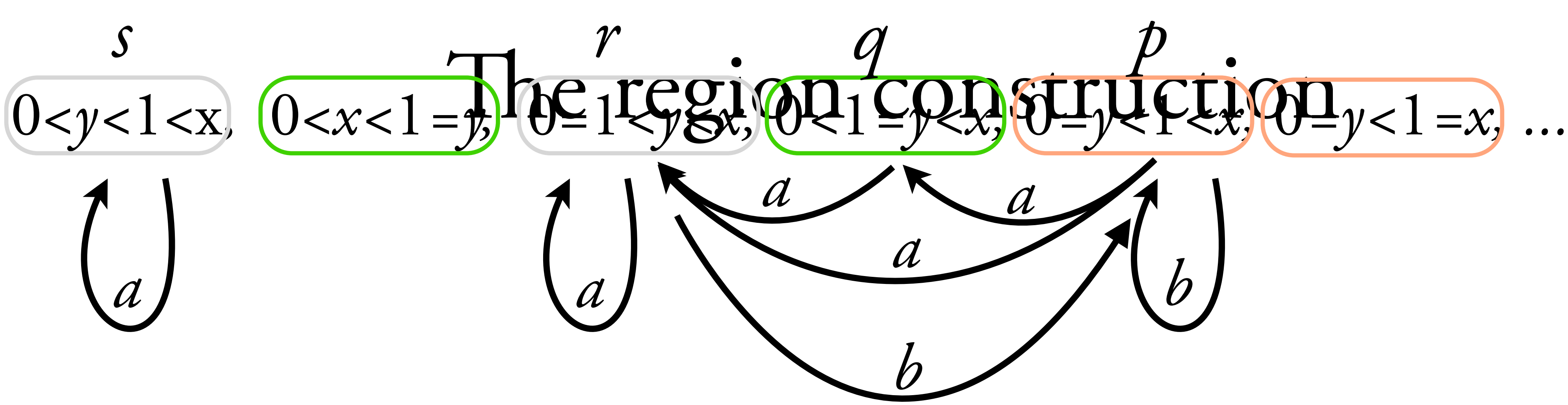
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



		a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*a*	*a*	*a*
	p	*q*	*r*	*r*	*q*	*r*	*q*	*s*	*s*	*s*	*s*	*q*
x	5	5										
y	0	1										

bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

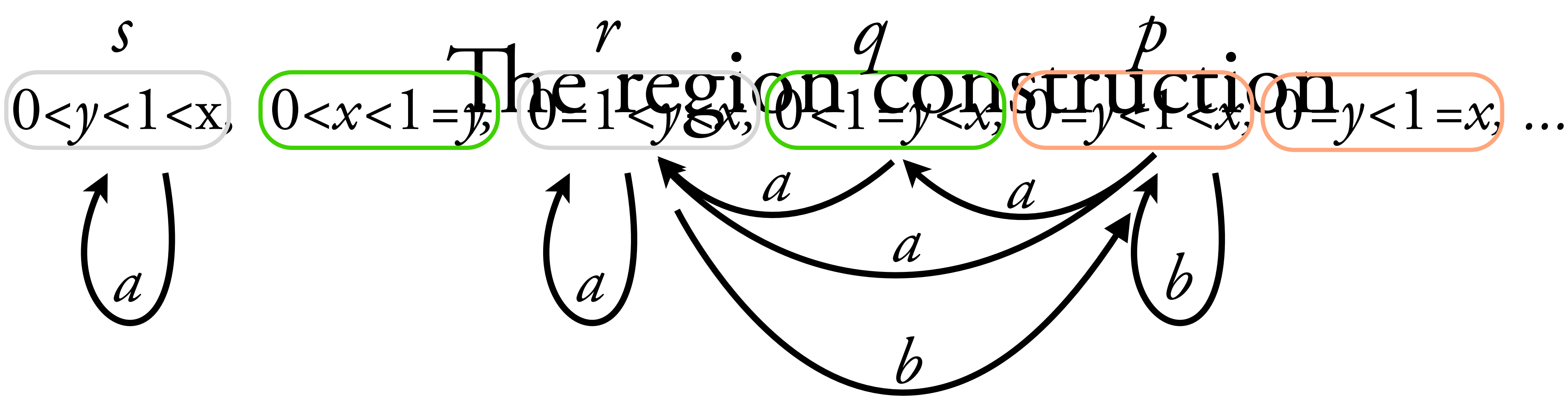
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



a a a b a b a a a a a

p q r r q r q s s s s q

x	5	5	5
y	0	1	2

bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

Dense case

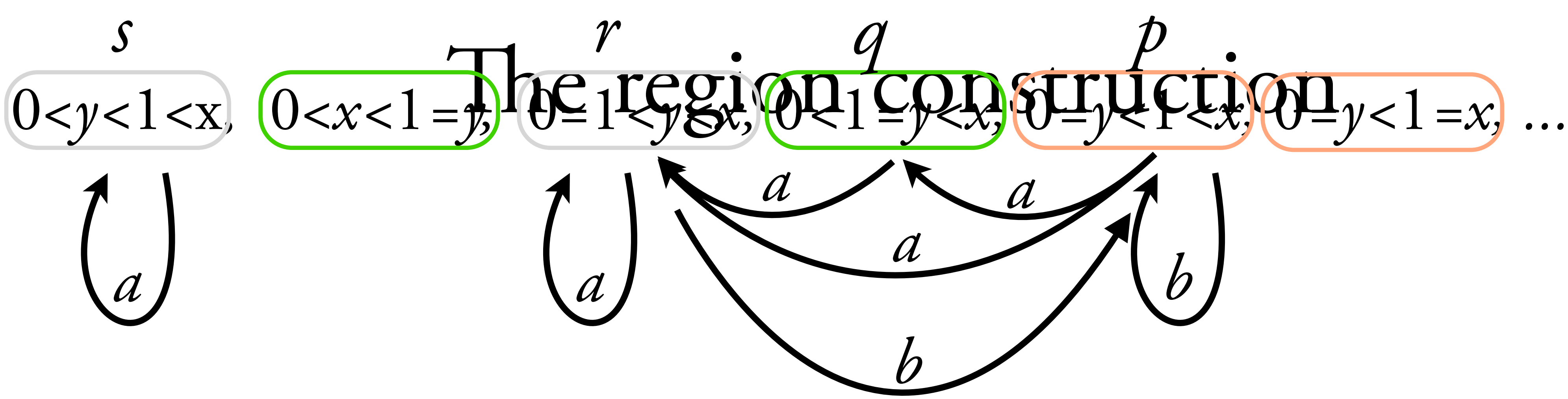
- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$



	a	a	a	b	a	b	a	a	a	a	a
	p	q	r	r	q	r	q	s	s	s	s	q
x	5	5	5	5							
y	0	1	2	4							

bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

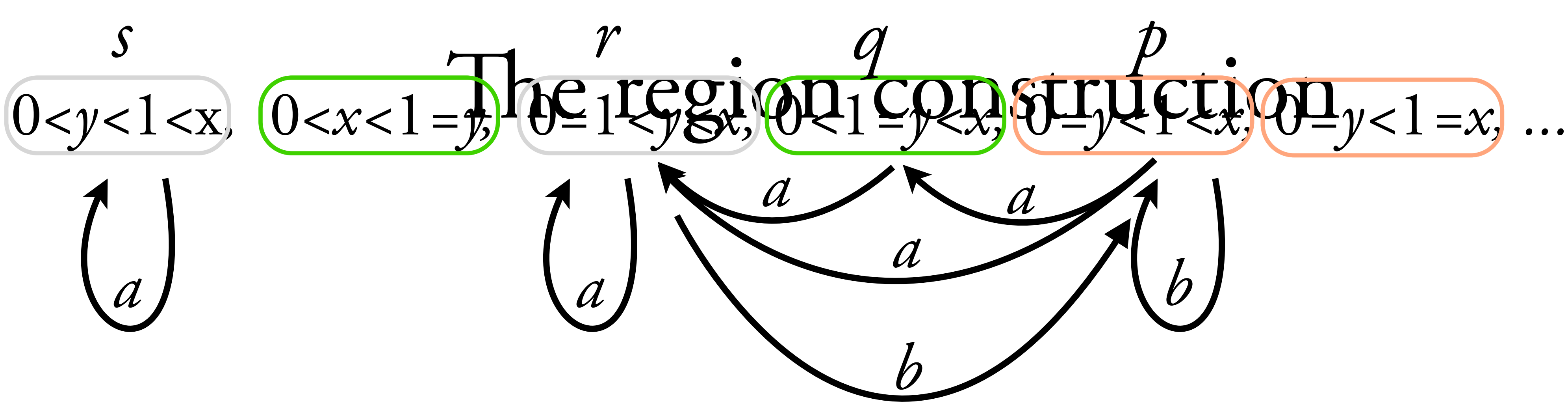
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

$p\ q\ r\ r\ q\ r\ q\ s\ s\ s\ s\ q$

x	5	5	5	5	5
y	0	1	2	4	0

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

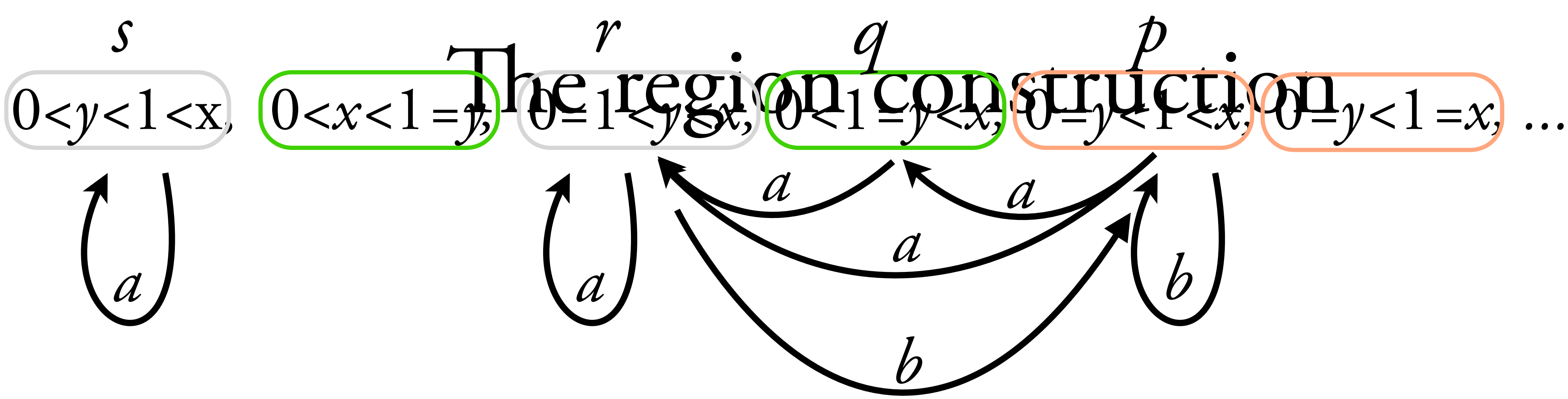
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



		a	a	a	b	a	b	a	a	a	a	a
	p	q	r	r	q	r	q	s	s	s	s	q
x	5	5	5	5	5	5						
y	0	1	2	4	0	3						

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

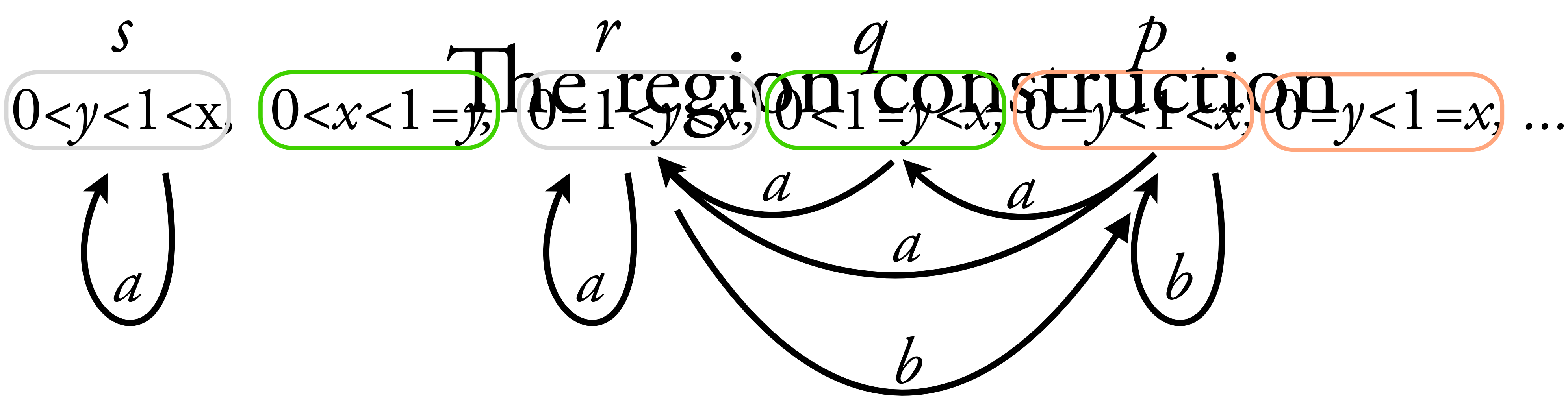
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



		a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*a*	*a* *a*
	p	*q*	*r*	*r*	*q*	*r*	*q*	*s*	*s*	*s*	*s* *q*
x	5	5	5	5	5	5	5				
y	0	1	2	4	0	3	0				

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

The region construction

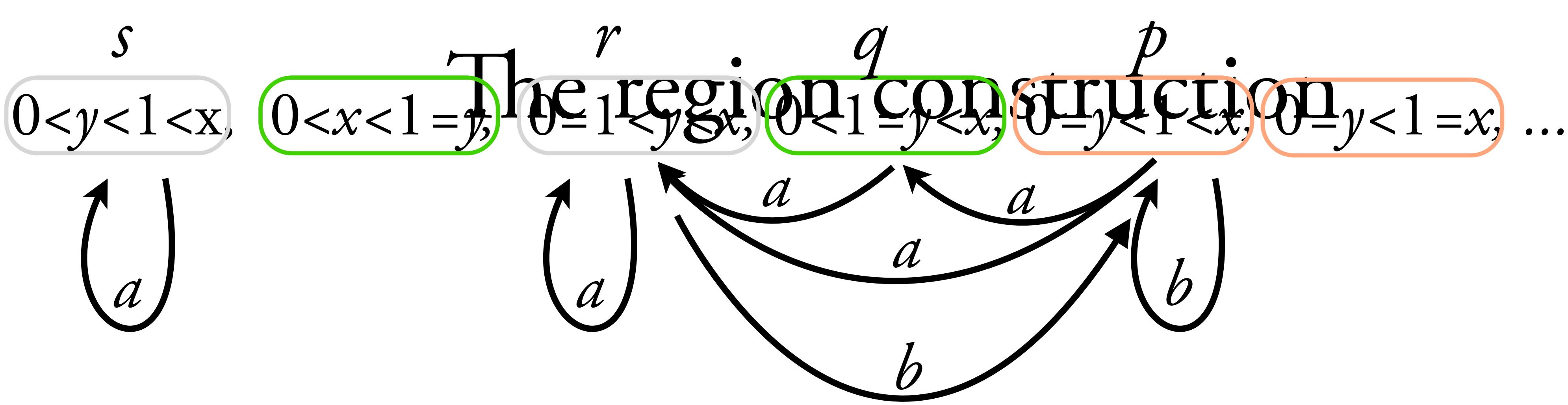
- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$



$$\begin{array}{cccccccccccc} & a & a & a & b & a & b & a & a & a & a & a \\ & p & q & r & r & q & r & q & s & s & s & s\ \ q \\ x & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & & & \\ y & 0 & 1 & 2 & 4 & 0 & 3 & 0 & .1 & & & \end{array}$$

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

- D = $\mathbb{Q}$
- finite words
- no database

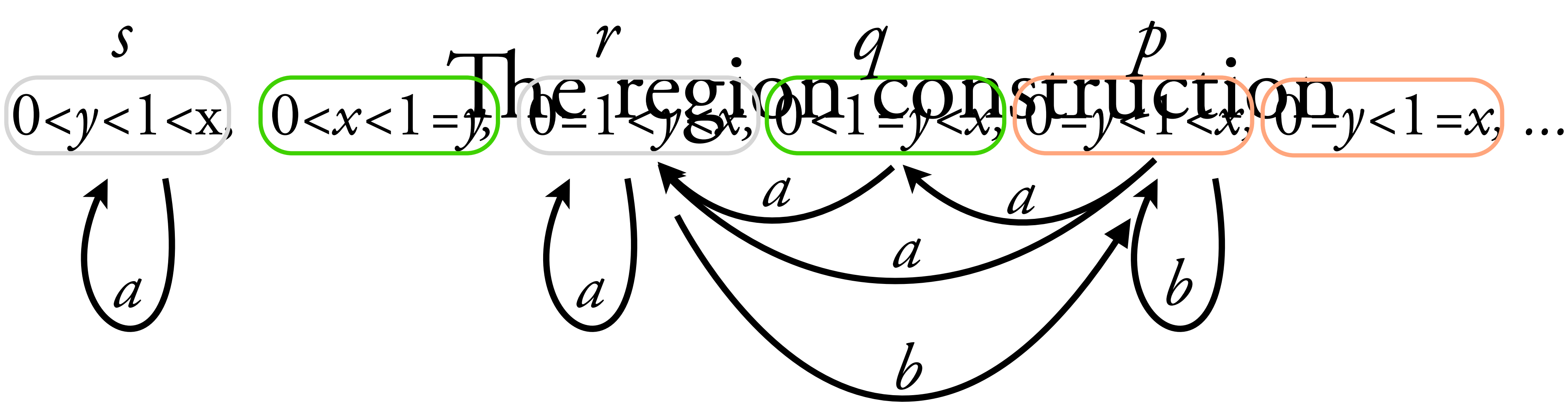
τ_I: $(x{>}0) \wedge (y{=}0)$

δ_a: $(x'{=}x) \wedge (y{<}y'{<}x)$

δ_b: $(x'{=}x) \wedge (y'{=}0)$

τ_F: $(y{=}1)$

The region construction



	a	a	a	b	a	b	a	a	a	a	a
	p	q	r	r	q	r	q	s	s	s	s	q
x	5	5	5	5	5	5	5	5	5		
y	0	1	2	4	0	3	0	.1	.2		

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

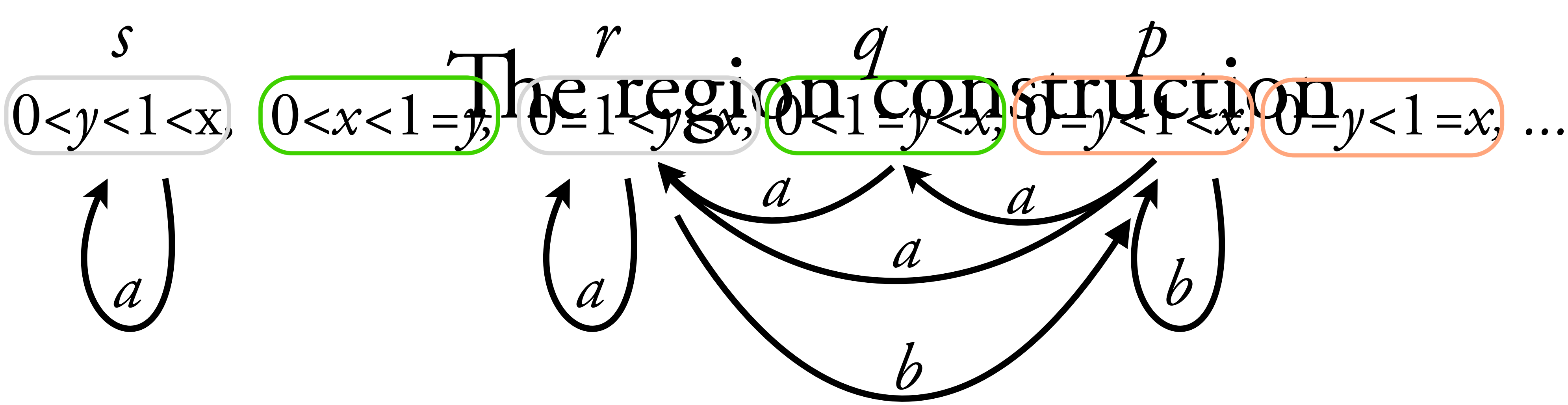
- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$



		a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*a*	*a*
	p	*q*	*r*	*r*	*q*	*r*	*q*	*s*	*s*	*s*	*s*

x	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	.1	.2	.4

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

- $D = \mathbb{Q}$
- finite words
- no database

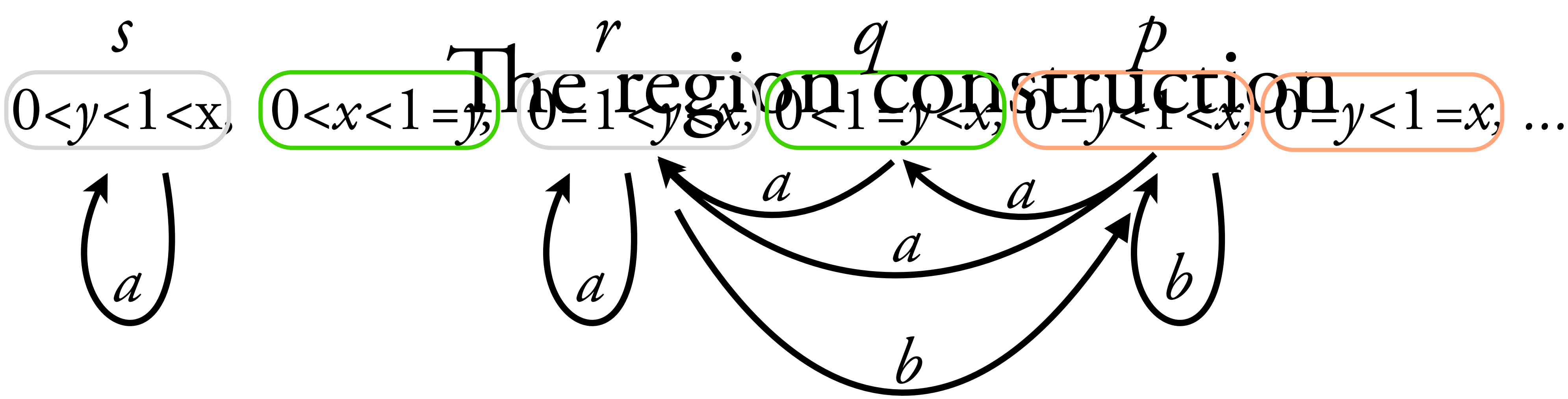
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



	a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*a*	*a*	*a*
p	*q*	*r*	*r*	*q*	*r*	*q*	*s*	*s*	*s*	*s*	*q*
x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	.1	.2	.4	.9

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

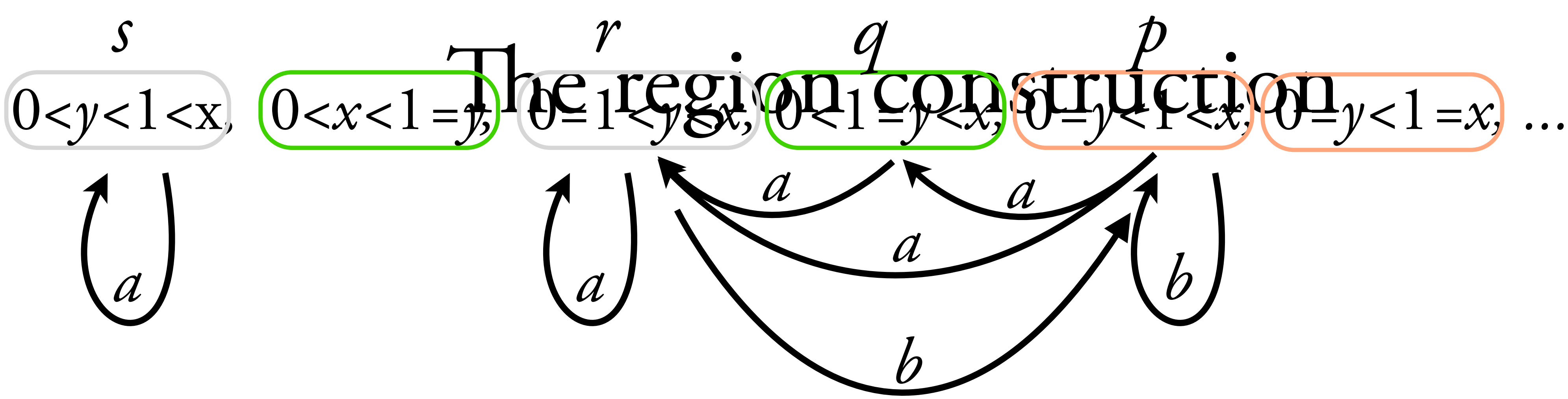
- $D = \mathbb{Q}$
- finite words
- no database

τ_{I}: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_{F}: $(y=1)$



The region construction

		a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*a*	*a*	*a*
	p	*q*	*r*	*r*	*q*	*r*	*q*	*s*	*s*	*s*	*s*	*q*
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	.1	.2	.4	.9	1

bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

Dense case

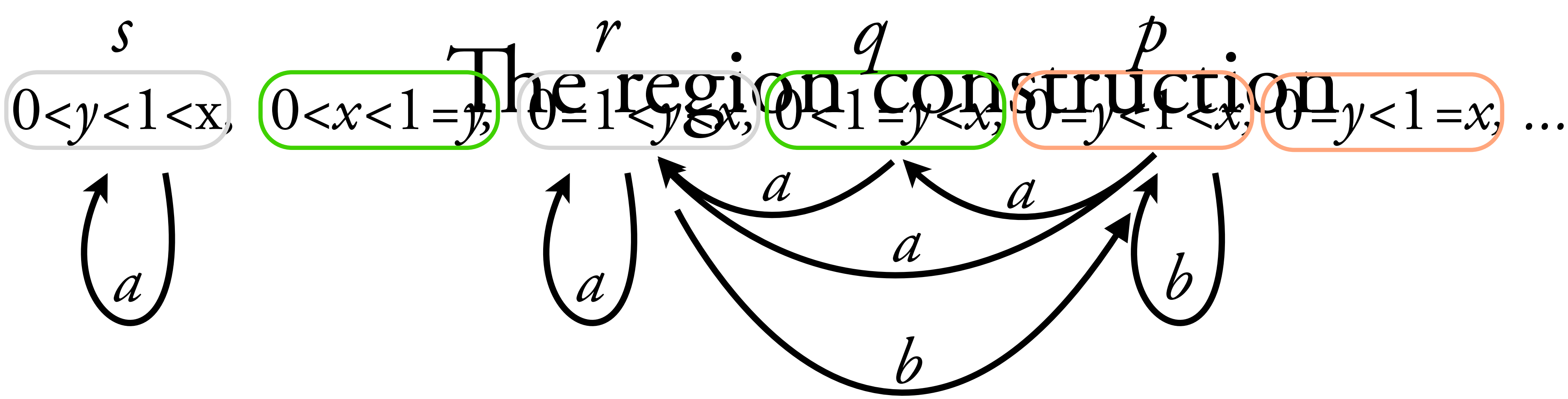
- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$



		a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*a*	*a*	*a*
	p	*q*	*r*	*r*	*q*	*r*	*q*	*s*	*s*	*s*	*s*	*q*
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	.1	.2	.4	.9	1

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

Dense case

- D = $\mathbb{Q}$
- finite words
- no database

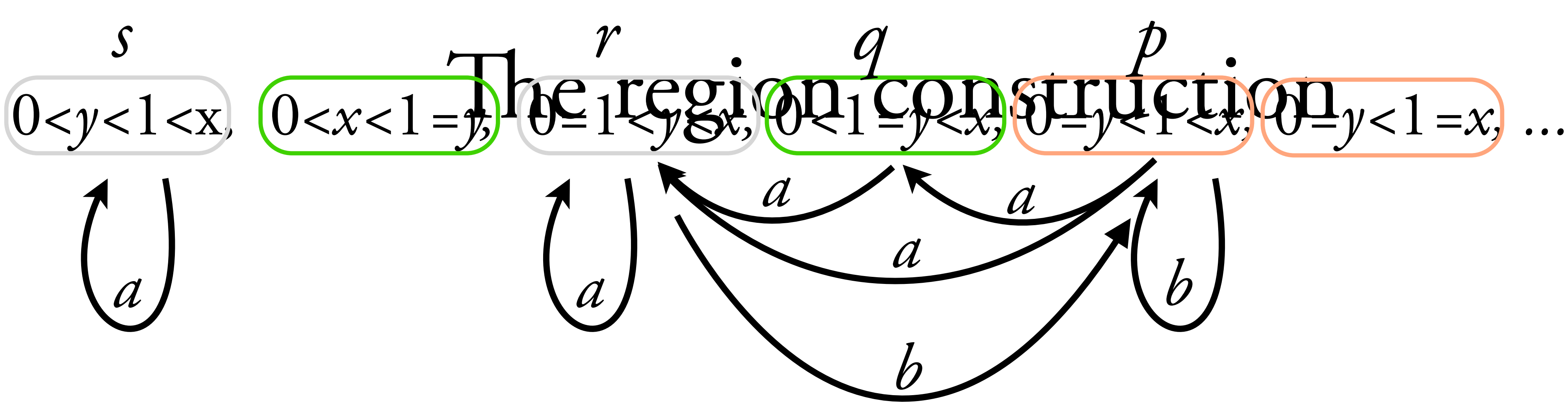
τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$

The region construction



		a	a	a	b	a	b	a	a	a	a	a
	p	q	r	r	q	r	q	s	s	s	s	q
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	.1	.2	.4	.9	1

bisimulation → runs of the region automaton correspond to runs of $\mathcal{A}$

accepted language: $(a+b)^*a$

The region construction

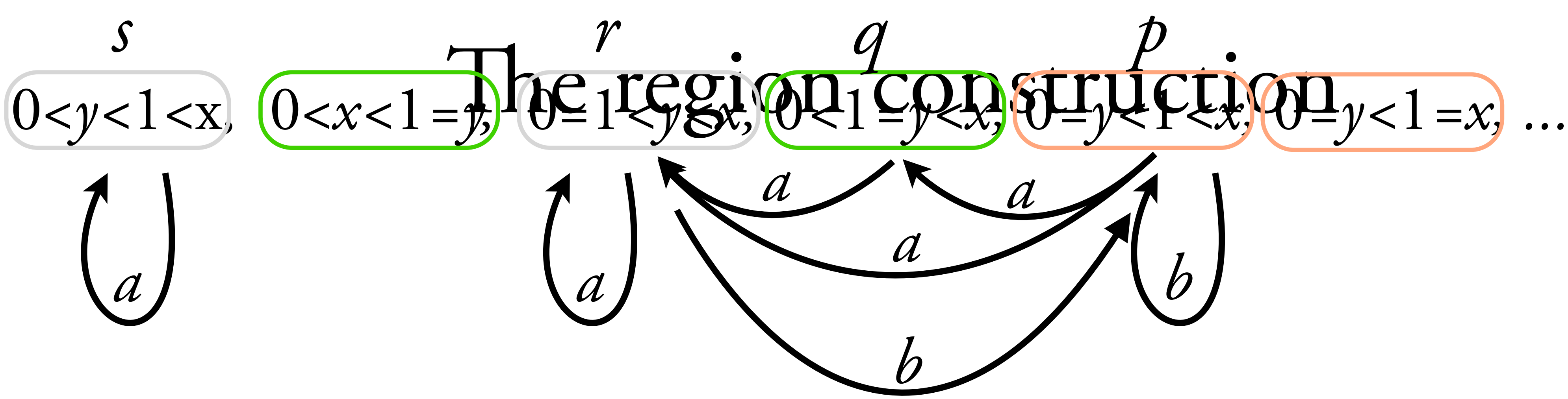
- $D = \mathbb{Q}$
- finite words
- no database

τ_I: $(x>0) \wedge (y=0)$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $(y=1)$



		a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*a*	*a*	*a*
	p	*q*	*r*	*r*	*q*	*r*	*q*	*s*	*s*	*s*	*s*	*q*
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	.1	.2	.4	.9	1

bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

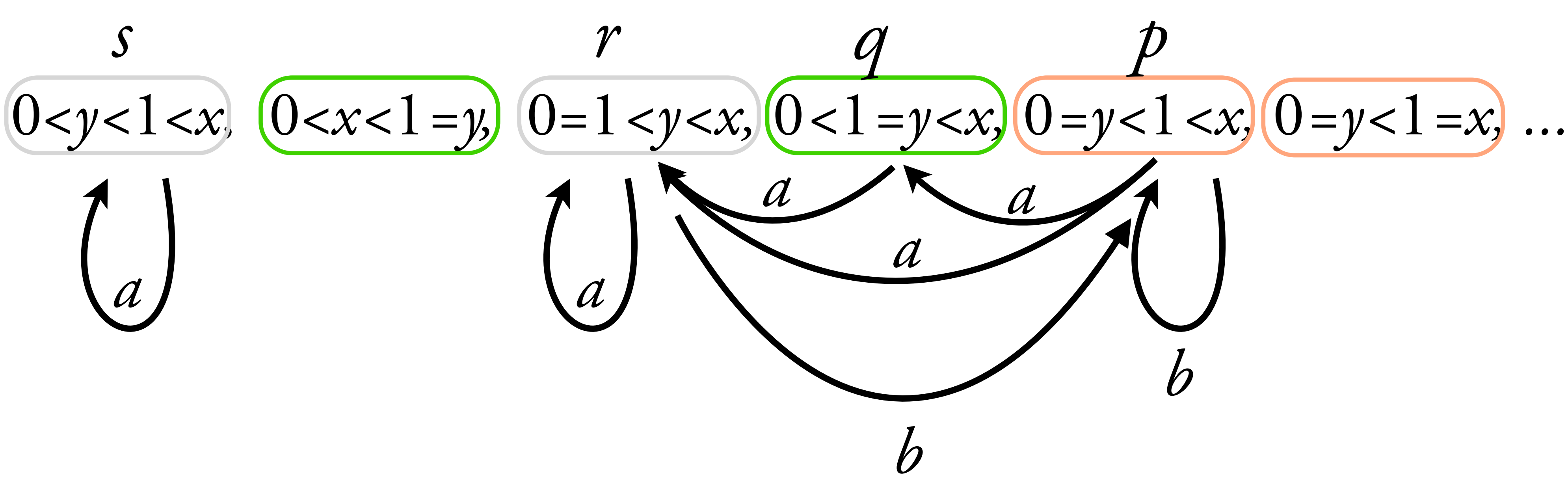
- $D = \mathbb{N}$
- finite words
- no database

τ_I: $\boxed{(x>0) \wedge (y=0)}$

δ_a: $(x'=x) \wedge (y<y'<x)$

δ_b: $(x'=x) \wedge (y'=0)$

τ_F: $\boxed{(y=1)}$



bisimulation → runs of the region automaton correspond to runs of $\mathscr{A}$

Discrete case

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- finite words
- no database

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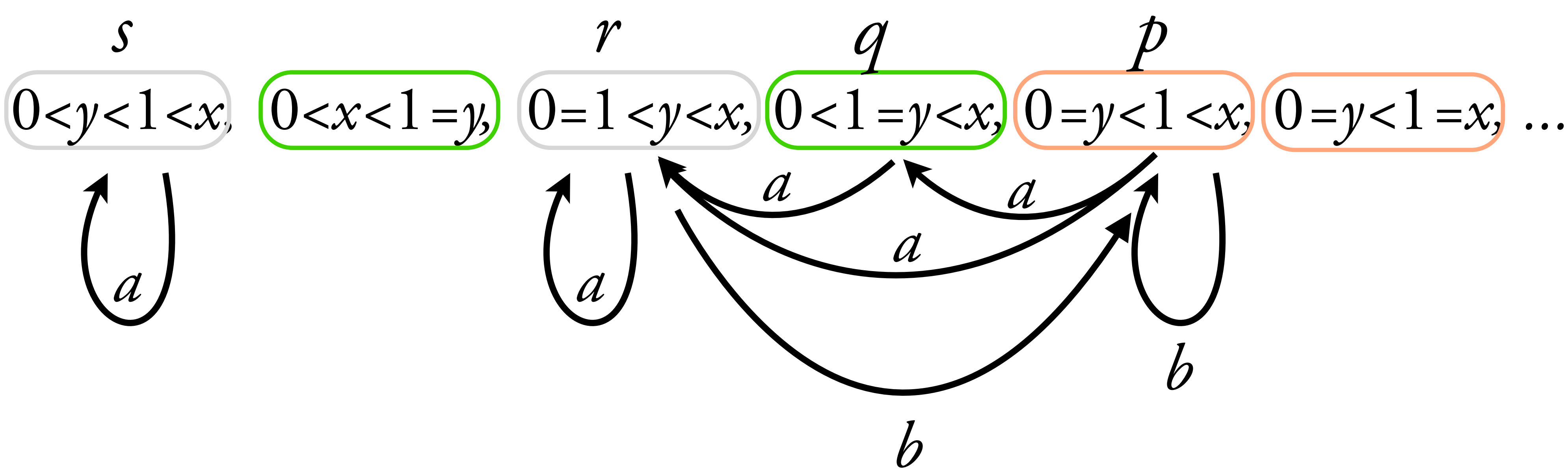

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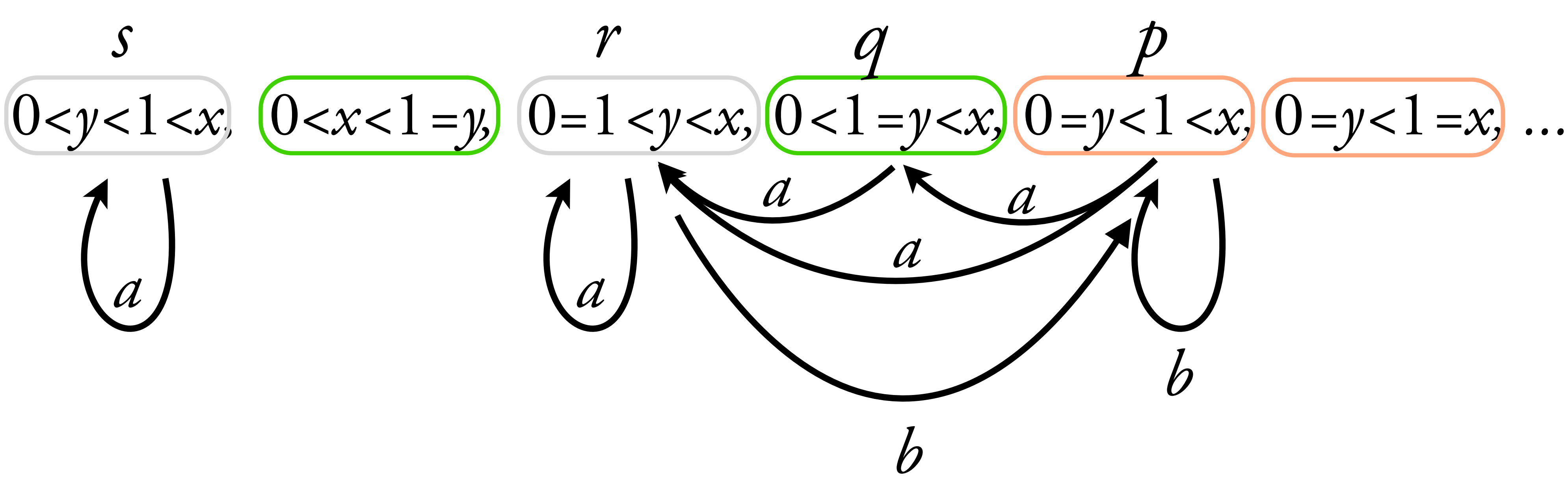

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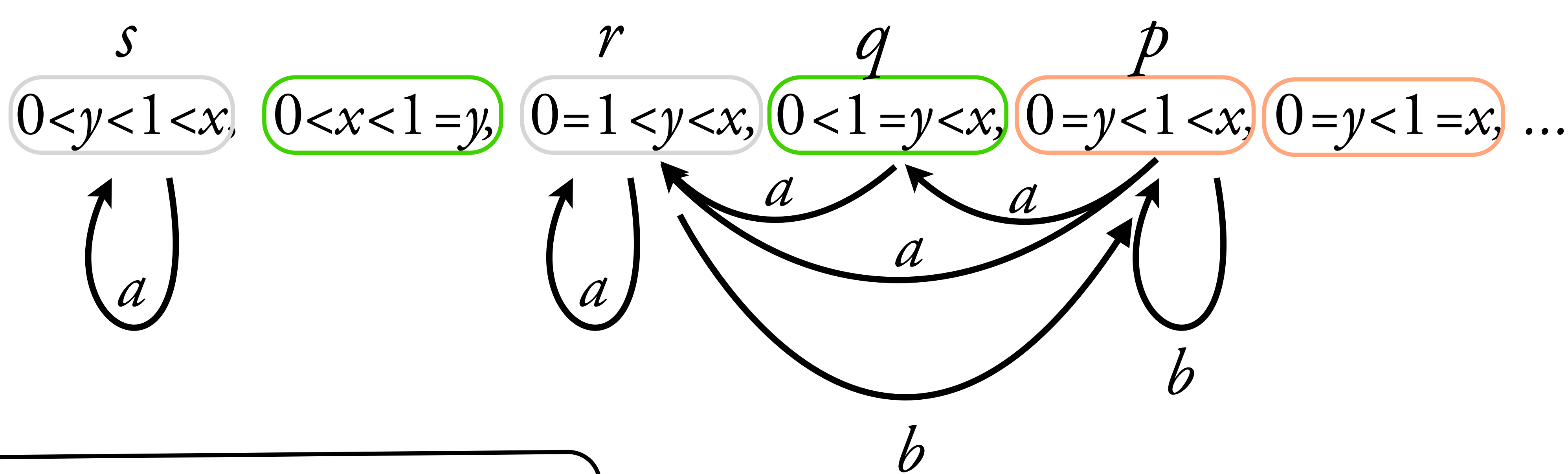

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$
$p\ s\ r\ r\ q\ r\ p\ s\ r\ r\ r\ q$

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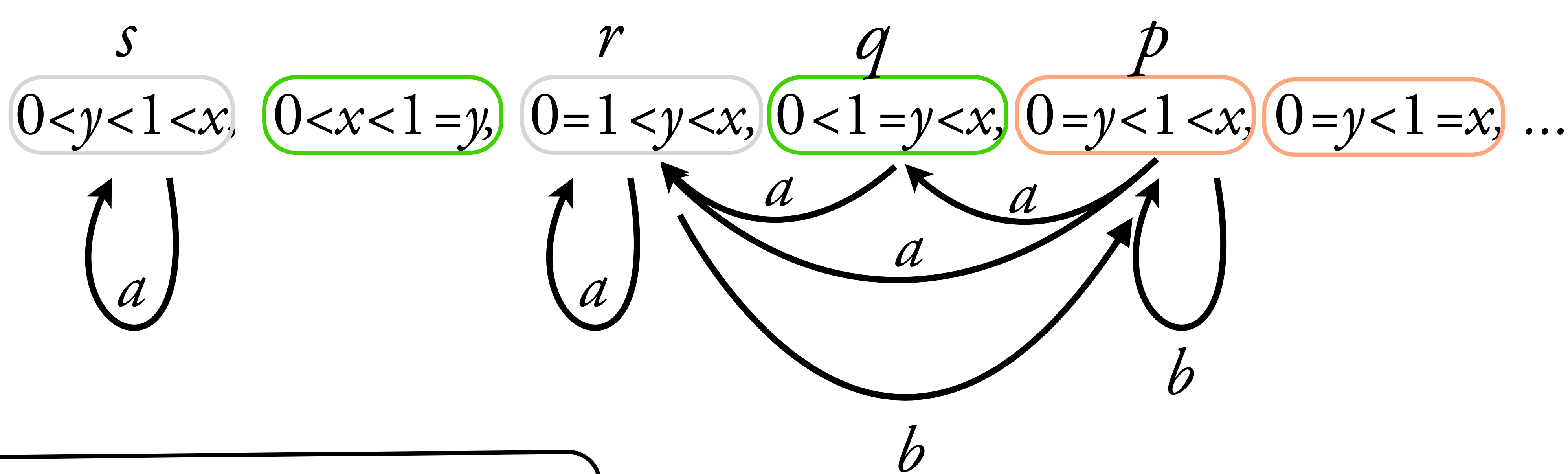

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y

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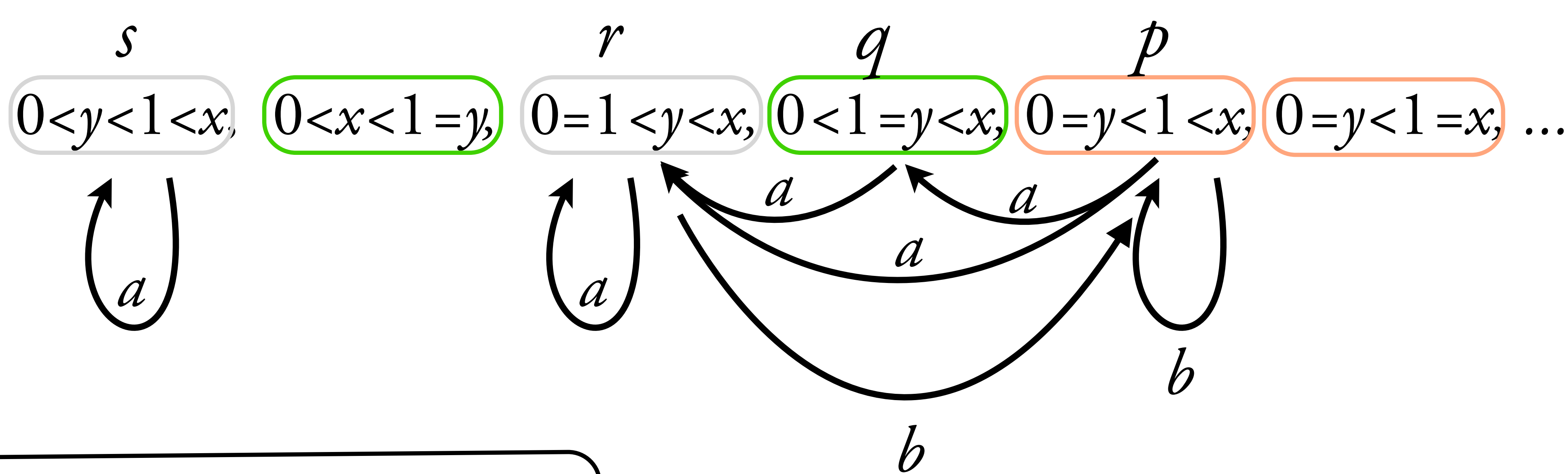

	a	*a*	*a*	*b*	*a*	*b*	*a*	*a*	*a*	*a*	*a*	
	p	*s*	*r*	*r*	*q*	*r*	*p*	*s*	*r*	*r*	*r*	*q*
x	5	5	5	5	5	5	5	5				
y	0	1	2	4	0	4	0	1				

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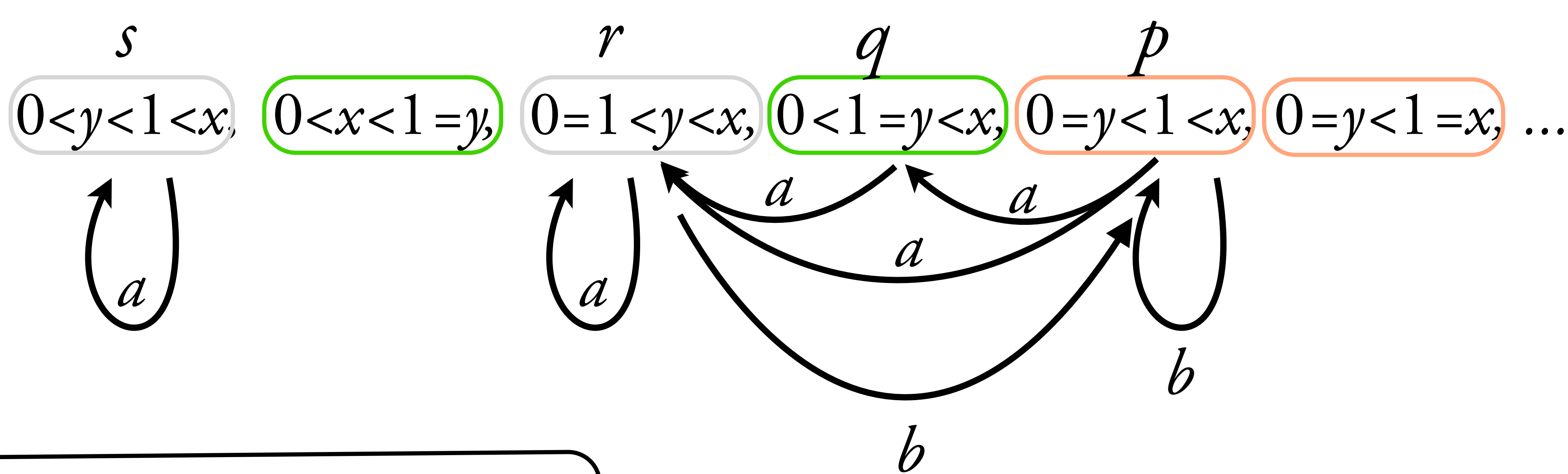

a a a b a b a a a a a

p s r r q r p s r r r q

x	5 5 5 5 5 5 5 5 5
y	0 1 2 4 0 4 0 1 4

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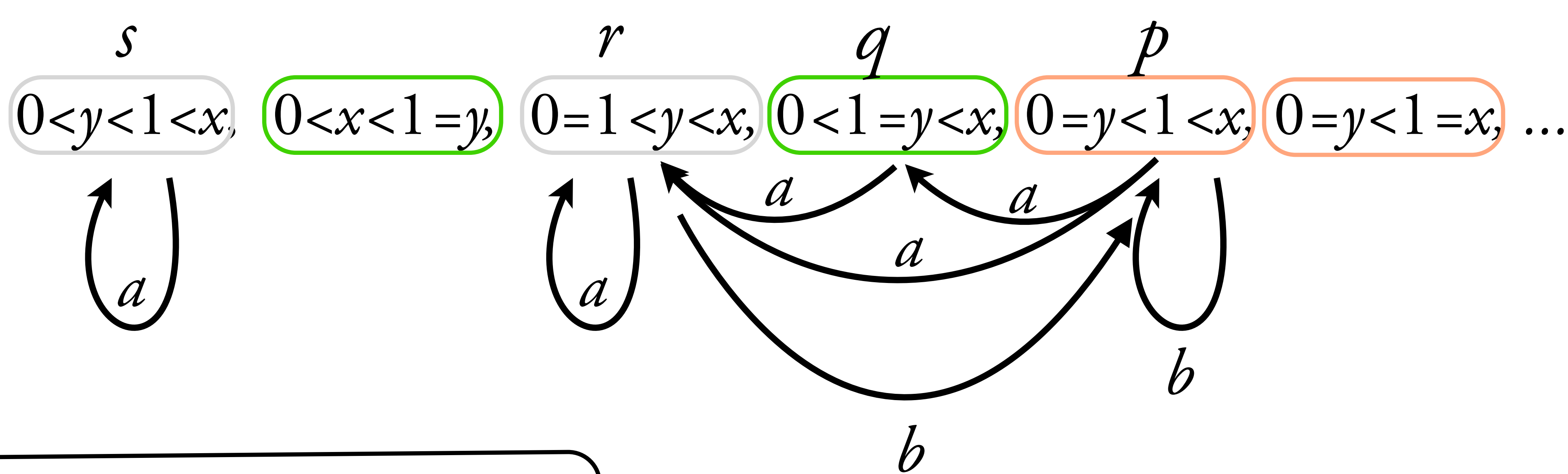

$a\ a\ a\ b\ a\ b\ a\ a\ a\ a\ a$

$p\ s\ r\ r\ q\ r\ p\ s\ r\ r\ r\ q$

x 5 5 5 5 5 5 5 5 5 ?

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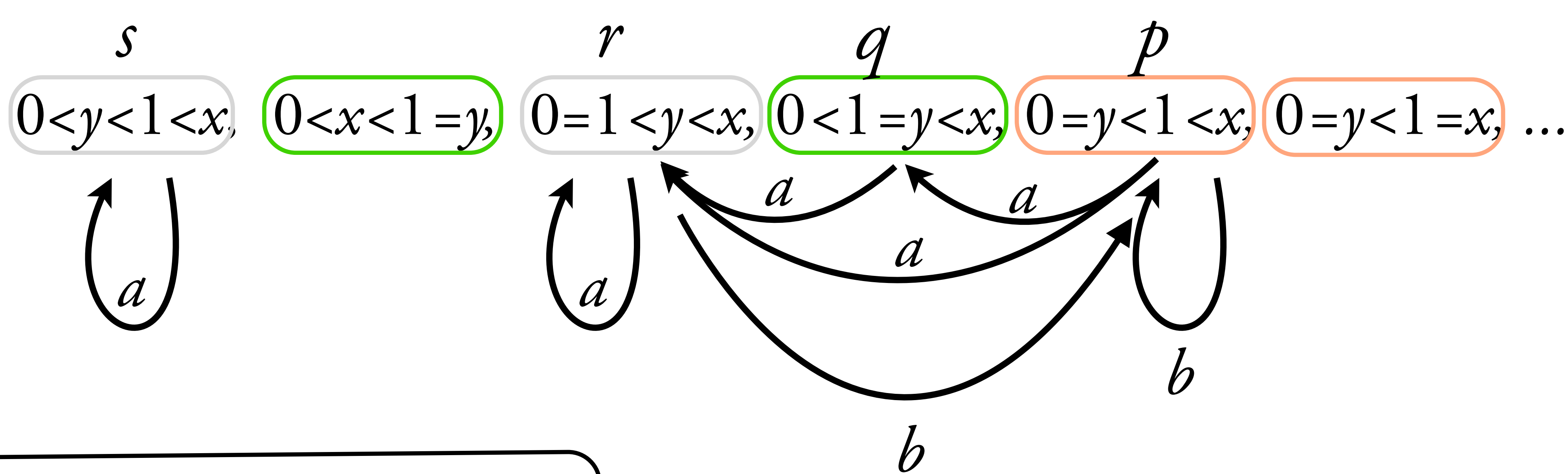

a a a b a b a a a a a
p s r r q r p s r r r q

x 5 5 5 5 5 5 5 5 5 ?
y 0 1 2 4 0 4 0 1 4 ?

but: in any cell and any n we can find configurations which are *n-bisimilar* → *finite* runs of the region automaton correspond to runs of $\mathcal{A}$

An elaboration of these ideas yields the results for infinite runs and database constraints and unary predicates in $\mathscr{D}$.

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Corollary. Deciding LTL+data tests properties of $\mathscr{D}$-automata is PSPACE-complete.

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