

Automata based verification over linearly ordered data domains

Luc Segoufin

INRIA and ENS Cachan

Szymon Toruńczyk

University of Warsaw

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Motivation

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Given a system which refers to a database and data values, verify properties of the system

Car Hire Home Page Return to Booking Help

Car Hire Offers Avis Locations Products & Services Join Avis Preferred Business Rentals Help and Contacts UK Fleet Home Delivery Newsletter

1 When and Where 2 Your Car Choice 3 Price and Extras 4 Checkout

Avis car hire quote and booking

Find pickup location by
Search for

City / town name

Go

Return location

Same as pickup location
[Change](#)

Rental Start Date/Time

0900

Rental End Date/Time

0900

Number of days

Avis Worldwide Discount (AWD) No.

[View, modify or cancel a booking](#)

Get a quote >

Sign in [Create an account](#)

[Sign in](#) to your Online account/Avis Preferred

*Car hire delivered
to your front door*

*Click here to book
Home Delivery! >*



UK from **£17** per day

Car hire from £17 per day

Enjoy great discounts this Autumn. Get on the road and explore the wonderful places the UK has to offer. Book Now!



Europe from **£15** per day

Car hire from £15 per day

Explore Europe this Autumn with great discounts which will take you further so you can see more. Book Now!



Car hire from £17 per day

Explore the world by taking advantage of our amazing Autumn Sale prices. Book Now!

Motivation

Given a system which refers to a database and data values, verify properties of the system

The screenshot shows the AVIS website's booking interface. At the top, the AVIS logo is on the left, and navigation links like 'Car Hire', 'Home Page', 'Return to Booking', and 'Help' are on the right. Below this is a secondary navigation bar with links for 'Car Hire Offers', 'Avis Locations', 'Products & Services', 'Join Avis Preferred', 'Business Rentals', 'Help and Contacts', 'UK Fleet', 'Home Delivery', and 'Newsletter'. A progress bar indicates four steps: '1 When and Where' (highlighted in red), '2 Your Car Choice', '3 Price and Extras', and '4 Checkout'. On the right side, there are links for 'Sign in' and 'Create an account', with a note to 'Sign in to your Online account/Avis Preferred'. The main form is titled 'Avis car hire quote and booking'. It includes fields for 'Find pickup location by Search for' (with 'edinburgh' entered), 'Return location' (set to 'Same as pickup location'), 'Rental Start Date/Time' (24 September 2010, 09:00), 'Rental End Date/Time' (28 September 2010, 09:00), 'Number of days' (4), and 'Avis Worldwide Discount (AWD) No.'. A 'Get a quote' button is at the bottom right. A red banner on the right side of the form reads 'Car hire delivered to your front door'. Annotations include a green box labeled 'input values' pointing to the search and date fields, and another green box labeled 'change state' pointing to the 'Get a quote' button.

input values

*Car hire delivered
to your front door*

change state

UK from **£17** per day



Car hire from £17 per day
Enjoy great discounts this Autumn. Get on the road and explore the wonderful places the UK has to offer. Book Now!

Europe from **£15** per day




Car hire from £15 per day
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[Car Hire](#) [Home Page](#) [Return to Booking](#) [Help](#)

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[1 When and Where](#) **[2 Your Car Choice](#)** [3 Price and Extras](#) [4 Checkout](#)

You are viewing vehicles available at Edinburgh Airport

[Small](#) [Medium](#) [Large](#) [Select Series](#)

Price	Size	Number of Luggage
	Small : Economy (Example of this range : Peugeot 207)	
	Best price £155.65 per rental	Hide Info
	Available to book now	
Vehicle Features Air Bag - Driver Short Wheel Base	Radio/Cassette	Driver age requirements You must be at least 23 years old to hire this vehicle If you are under 25 years old a Young Driver Surcharge will apply Young Driver surcharge is £11/day + VAT; max £110
		Credit card requirements The number of Credit Cards required when you pick up this vehicle is: 1.
	Medium : Economy (Example of this range : Nissan Note 1.4)	
	Best price £181.37 per rental	More Info
	Available to book now	

Sign in

[Create an account](#)
[Sign in](#) to your Online account/Avis Preferred


Your Booking

- 1 WHEN AND WHERE** [Change](#)
Pickup : Edinburgh Airport 23/09/2010 19:00
Return : Edinburgh Airport 28/09/2010 19:00
Rental Days : 5
- 2 YOUR CAR CHOICE**
- 3 PRICE AND EXTRAS**
- 4 CHECKOUT**

[Empty Basket](#)

Motivation

Given a system which refers to a database and data values, verify properties of the system



[Car Hire](#) [Home Page](#) [Return to Booking](#) [Help](#)

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Your Booking

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Return : Edinburgh Airport 28/09/2010 19:00
Rental Days : 5

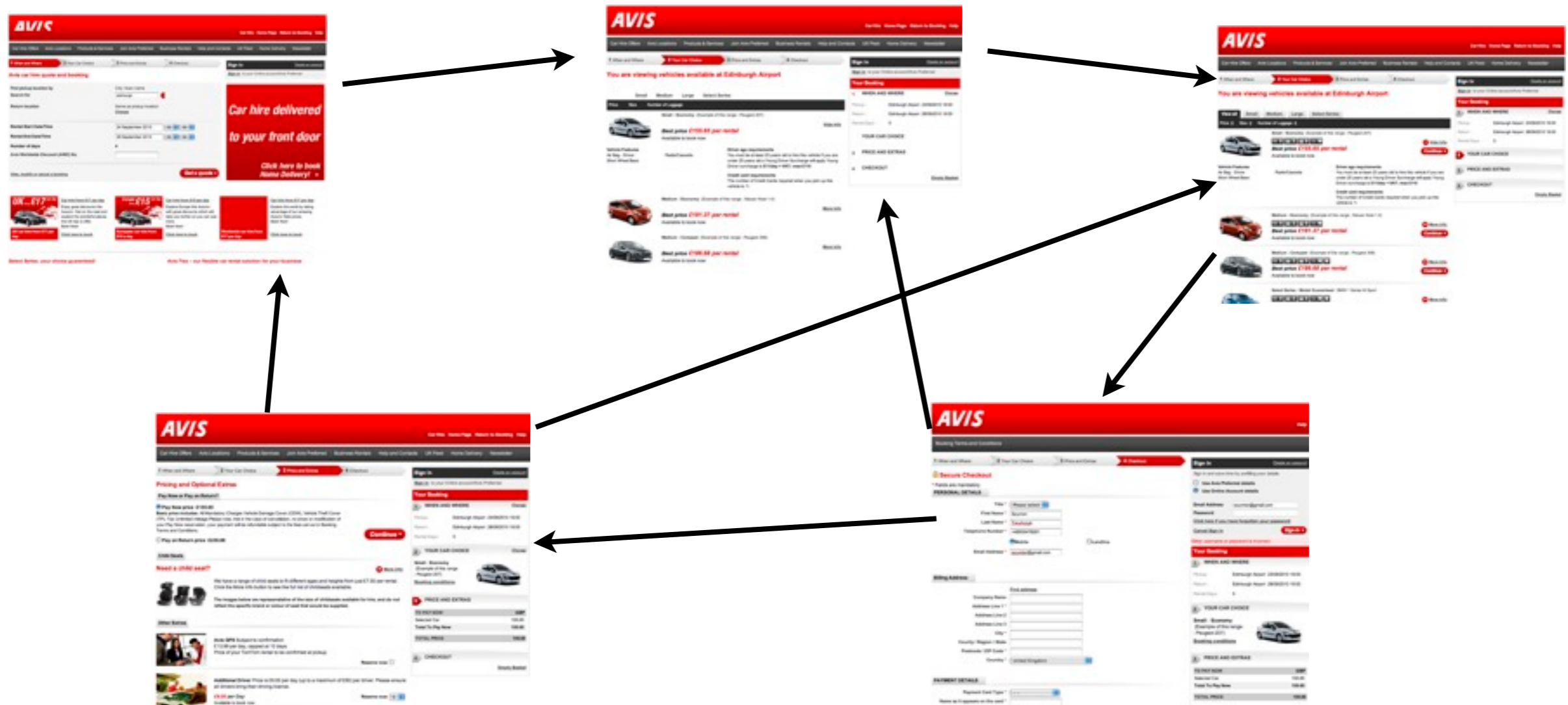
YOUR CAR CHOICE

3 PRICE AND EXTRAS

4 CHECKOUT [Empty Basket](#)

Motivation

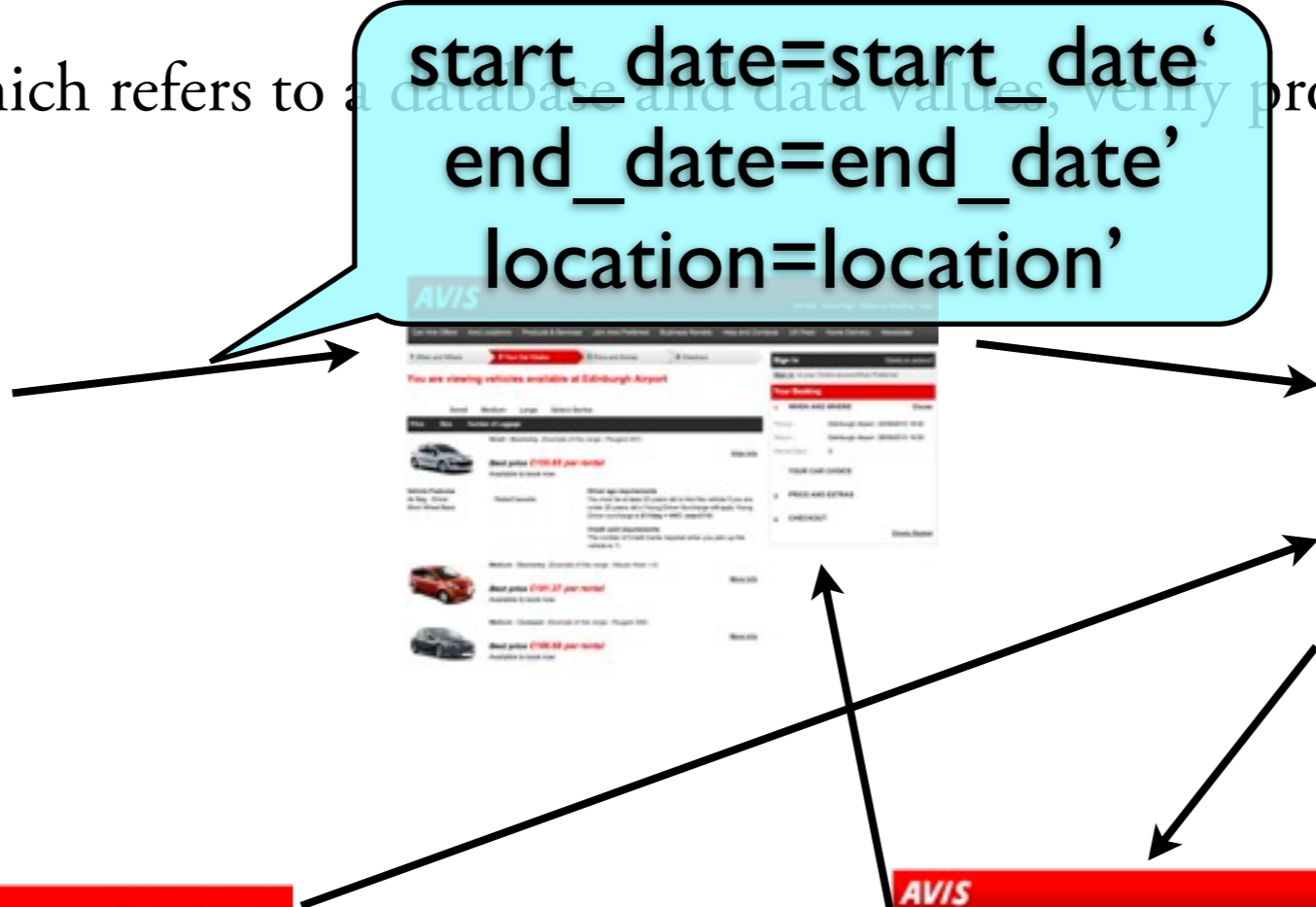
Given a system which refers to a database and data values, verify properties of the system



Motivation

Given a system which refers to a database and data values, verify properties of the system

`start_date=start_date'`
`end_date=end_date'`
`location=location'`

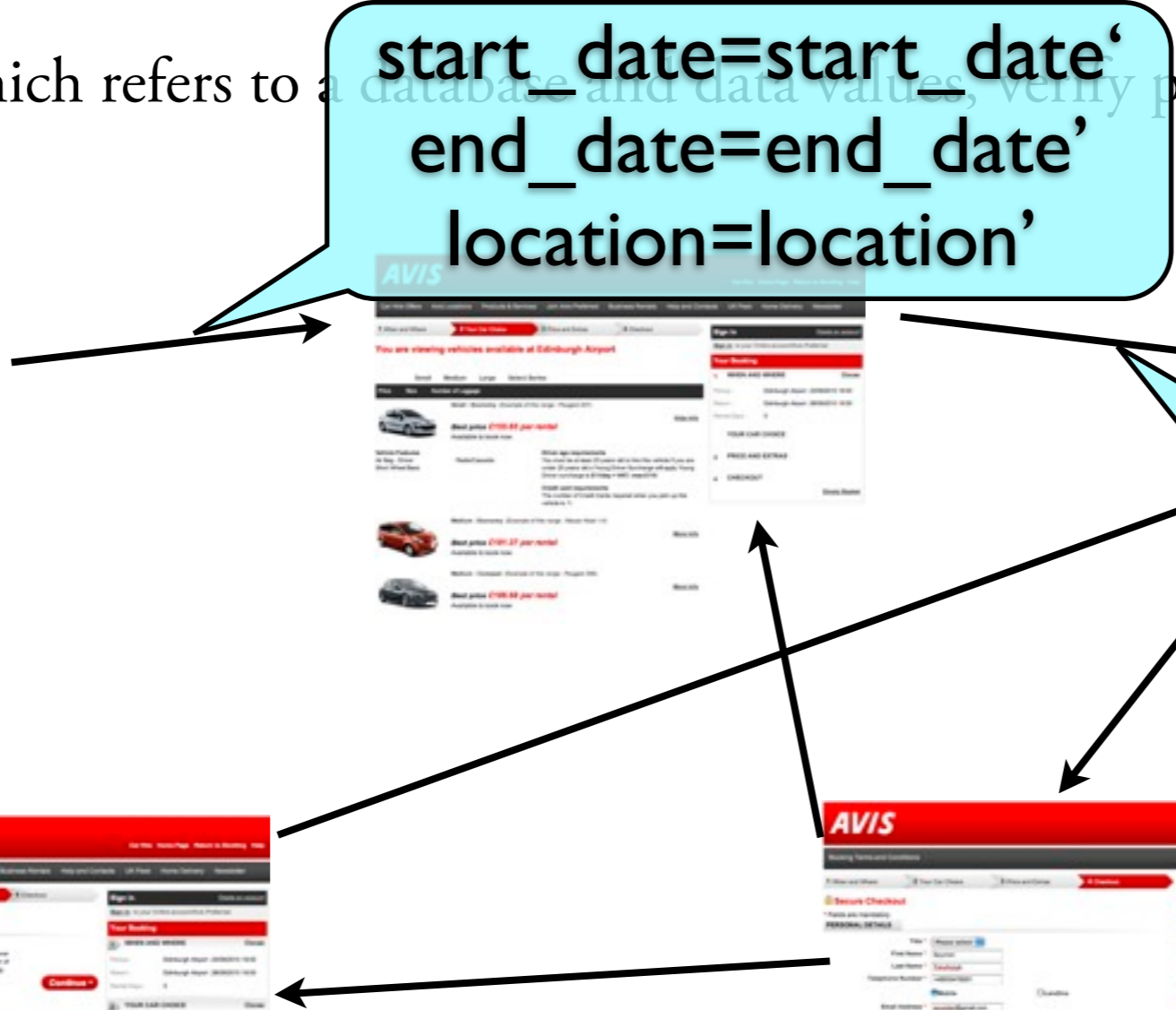


Motivation

Given a system which refers to a database and data values, verify properties of the system

$\text{start_date}=\text{start_date}'$
 $\text{end_date}=\text{end_date}'$
 $\text{location}=\text{location}'$

$\text{pricemin} \leq \text{price}$
 $\leq \text{pricemax}$



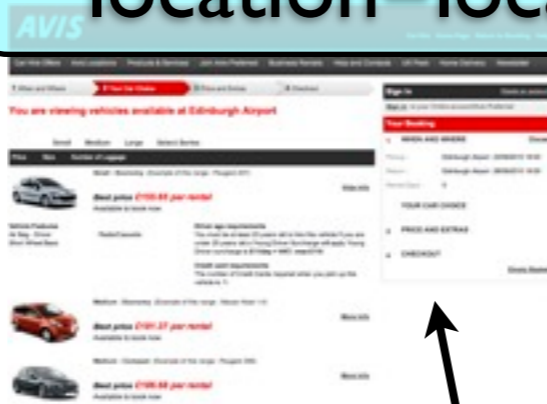
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$\text{CAR}(\text{car_id}, \text{price})$



Motivation

Given a system which refers to a database and data values, verify properties of the system

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$\text{CAR}(\text{car_id}, \text{price})$

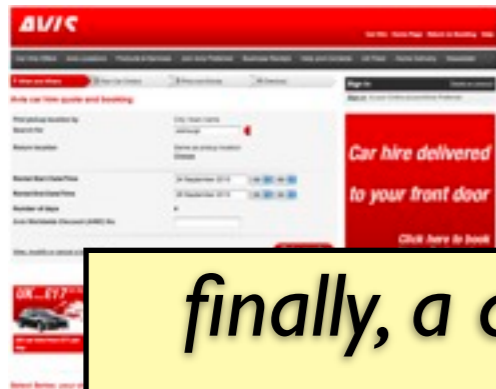
$\text{PAYMENT}(\text{user_id}, \text{car_id},$
 $\text{price}, \text{order_id})$



Motivation

Given a system which refers to a database and data values, verify properties of the system

`start_date=start_date'`
`end_date=end_date'`
`location=location'`



`pricemin < price`

finally, a car is rented and

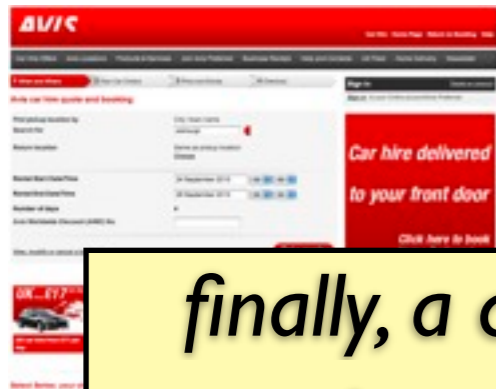
`PAYMENT(user_id, car_id,
price, order_id)`



Motivation

Given a system which refers to a database and data values, verify properties of the system

`start_date=start_date'`
`end_date=end_date'`
`location=location'`



`pricemin < price`

finally, a car is rented and

- no payment was recieved*

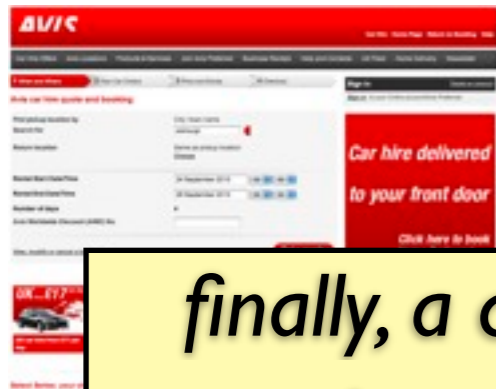


`PAYMENT(user_id, car_id, price, order_id)`

Motivation

Given a system which refers to a database and data values, verify properties of the system

`start_date=start_date'`
`end_date=end_date'`
`location=location'`



`pricemin < price`

finally, a car is rented and

- *no payment was recieved*
- *a payment was recieved but payed=0*

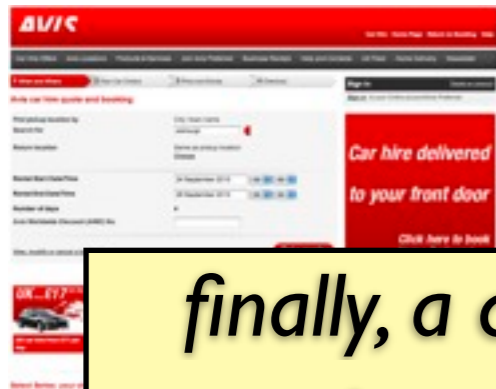


`PAYMENT(user_id,car_id,
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Motivation

Given a system which refers to a database and data values, verify properties of the system

`start_date=start_date'`
`end_date=end_date'`
`location=location'`



`pricemin < price`

finally, a car is rented and

- *no payment was recieved*
- *a payment was recieved but payed=0*
- *there was a payment, but payed < 100 & end_date > 2050*

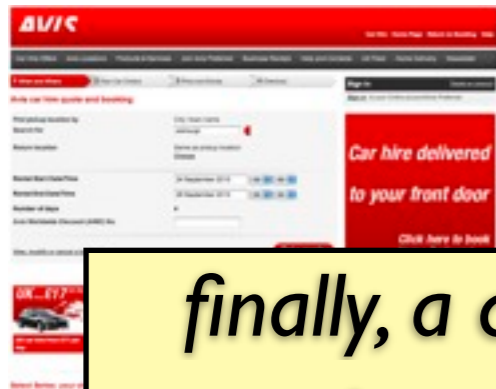


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`start_date=start_date'`
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`pricemin < price`

finally, a car is rented and

- *no payment was recieved*
- *a payment was recieved but payed=0*
- *there was a payment, but payed<100 & end_date>2050*
- *there was a payment, but payed<car_price*



`PAYMENT(user_id,car_id,
price, order_id)`

Extended automaton

a definition that captures timed automata, or vector addition systems

D – fixed domain

Q – finite set of states

X – finite set of variables

$D^X = \{ v: X \rightarrow D \}$ – space of variable valuations

$Q \times D^X$ – space of configurations

restricted
form

$\delta \subseteq (Q \times D^X) \times (Q \times D^X)$ – set of allowed transitions

$I \subseteq (Q \times D^X)$ – set of initial configurations

$F \subseteq (Q \times D^X)$ – set of final configurations

Examples

- *Vector Addition System:* $D = \mathbb{N}$

$$(q, v) \xrightarrow[v' = v + w]{} (q', v')$$

- *Timed Automata:* $D = \mathbb{R}$

$$(q, v) \xrightarrow[c_2' = 0]{c_1 < 2} (q', v')$$

- *(Lossy) channel system:* $D = \{a+b\}^*$

$$(q, v) \xrightarrow[c_1' = c_1 \cdot b]{first_a(c_1)} (q', v')$$

Our setting

(without the database)

The domain:

$$\mathcal{D} = \langle D, <, P_1, P_2, P_3, \dots, P_l \rangle$$

linearly ordered set

unary predicates
(subsets of D)

Examples

$$\langle \mathbb{N}, <, 0, 100, P_{\text{even}}, P_{\text{prime}} \rangle$$

$$\langle \mathbb{Q}, <, 0, 100, P_{\text{integer}}, P_{<\pi} \rangle$$

$$\langle \{a+b\}^*, <_{\text{lex}}, P_{(ab)^*} \rangle$$

Transitions:

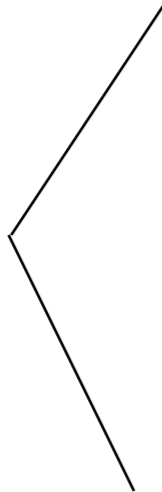
$$I, F \subseteq (Q \times D^X), \quad \delta \subseteq (Q \times D^X) \times (Q \times D^X)$$

are specified by quantifier free formulas over \mathcal{D}

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

no states

\mathcal{A} 

$$\tau_I: (x > 0) \wedge (y = 0)$$

$$\delta_a: (x' = x) \wedge (y < y' < x)$$

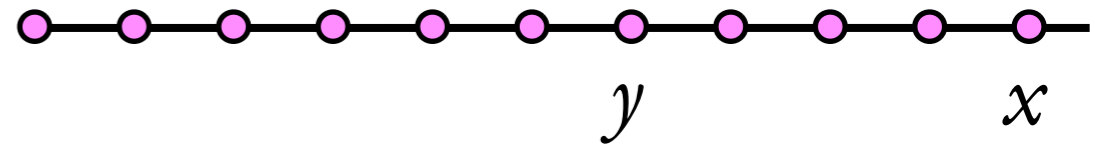
$$\delta_b: (x' = x) \wedge (y' = 0)$$

$$\tau_F: (y = 1)$$

Example D -automaton \mathcal{A}

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Example D -automaton \mathcal{A}

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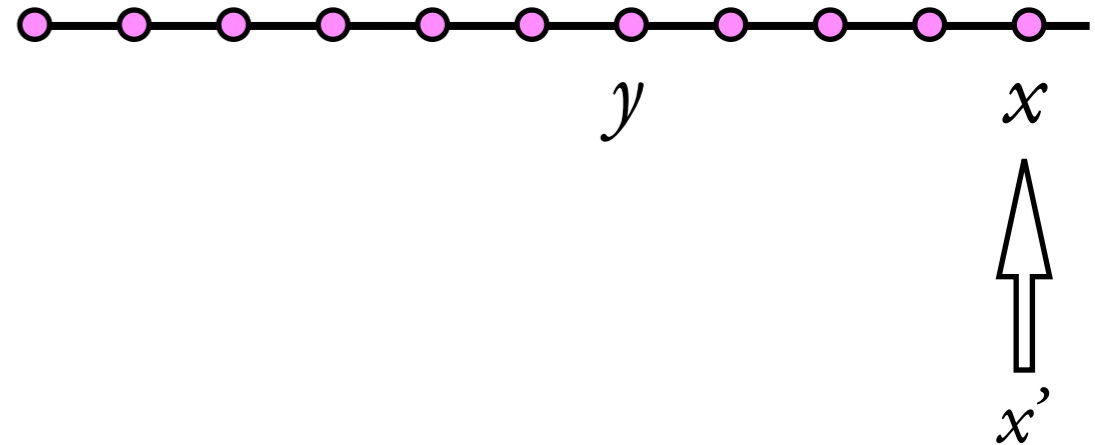
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Example D -automaton \mathcal{A}

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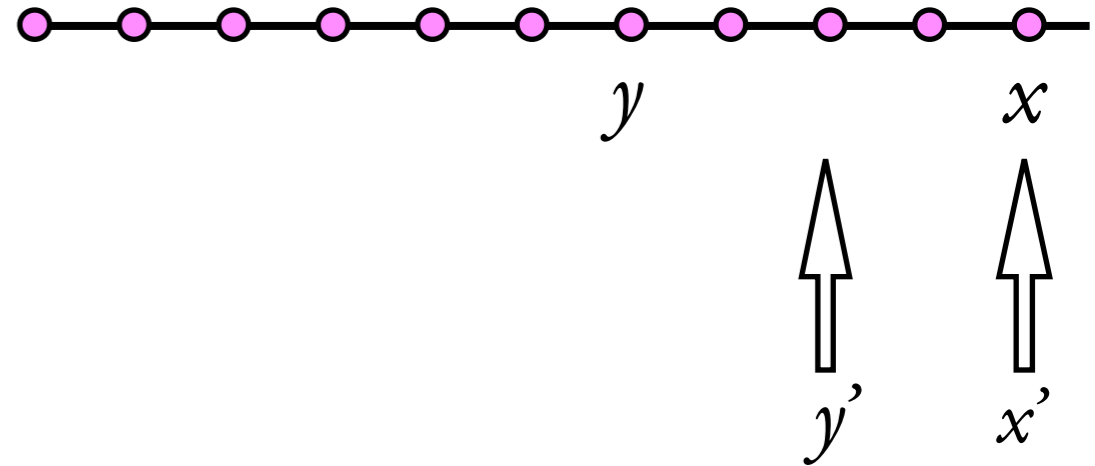
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x, y – variables of \mathcal{A}

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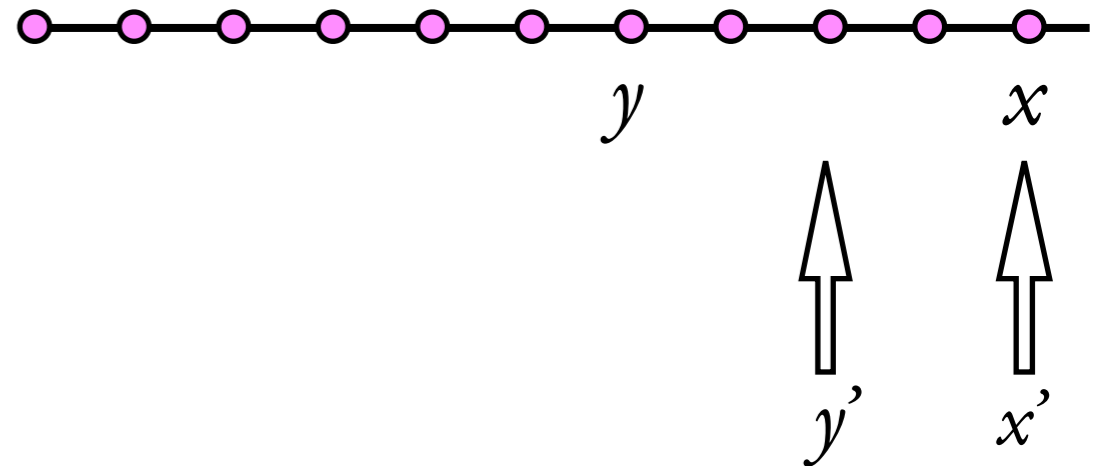
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a finite run over \mathbb{Q} :

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

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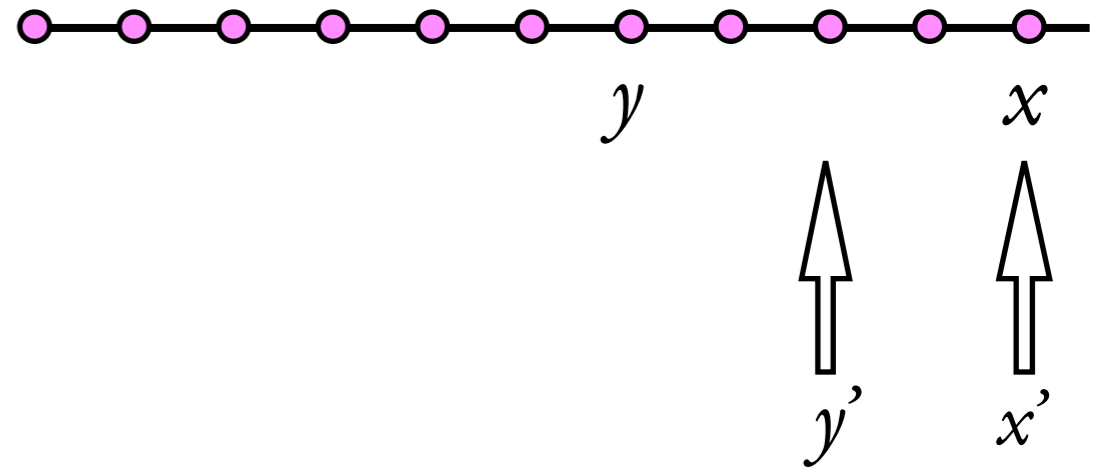
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a finite run over \mathbb{Q} :

$a a a b a b a a a a$

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

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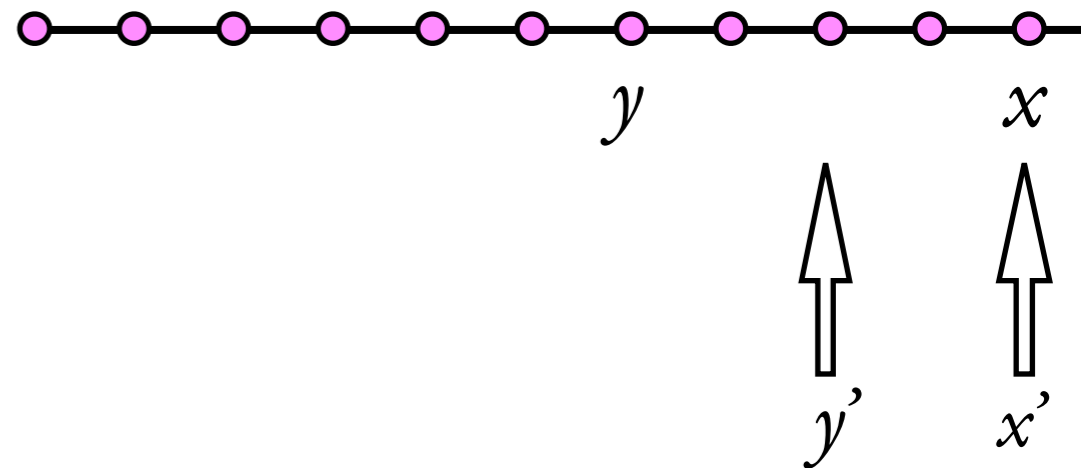
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a finite run over \mathbb{Q} :

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x

y

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

no states

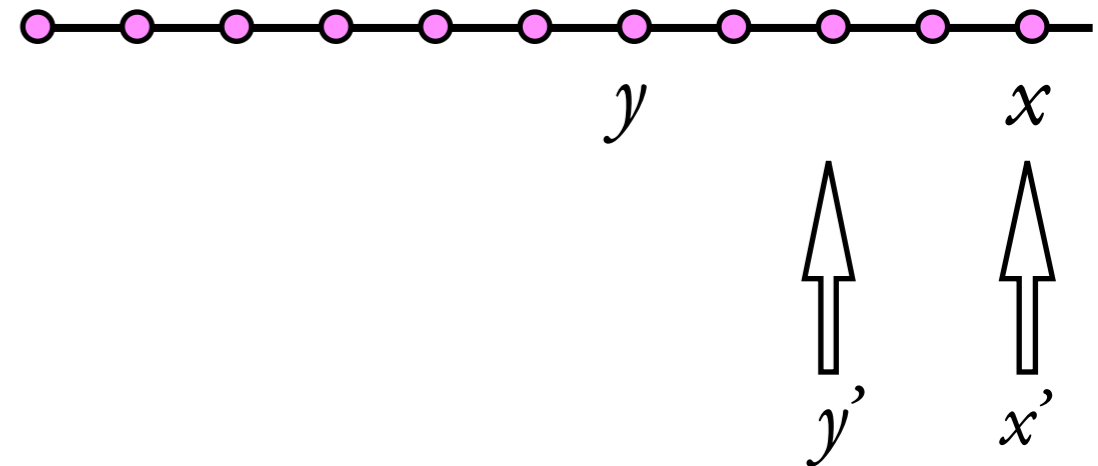
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a finite run over \mathbb{Q} :

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x 5

y 0

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

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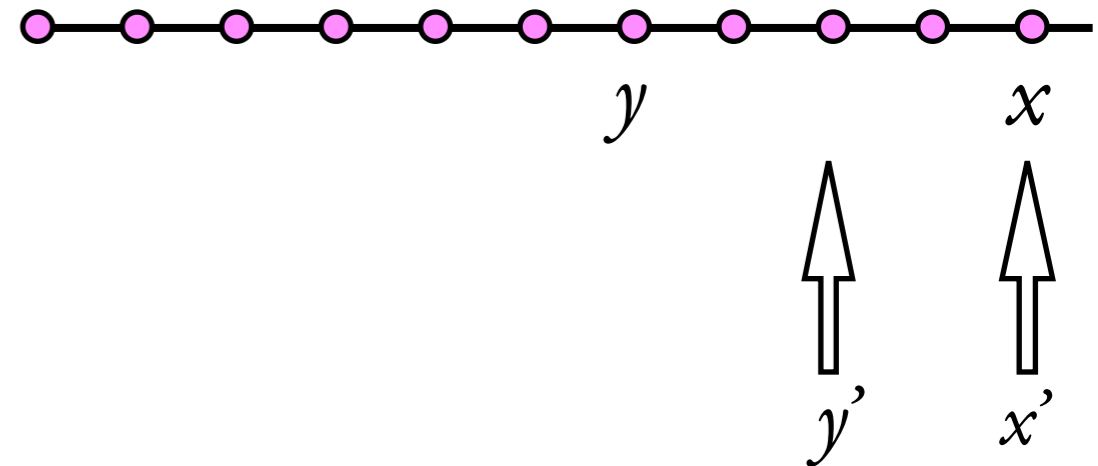
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y 0 1

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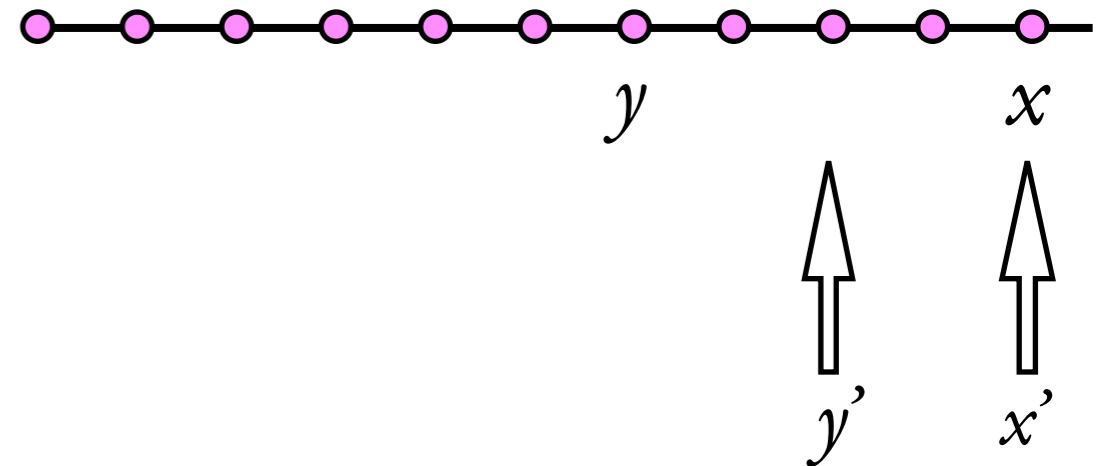
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x 5 5 5

y 0 1 2

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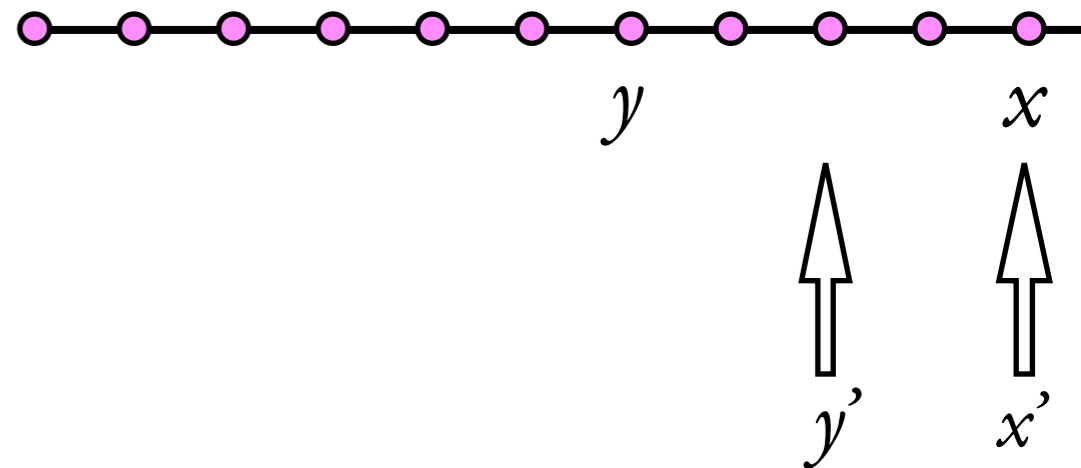
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a finite run over \mathbb{Q} :

$a a a b a b a a a a$

x 5 5 5 5

y 0 1 2 4

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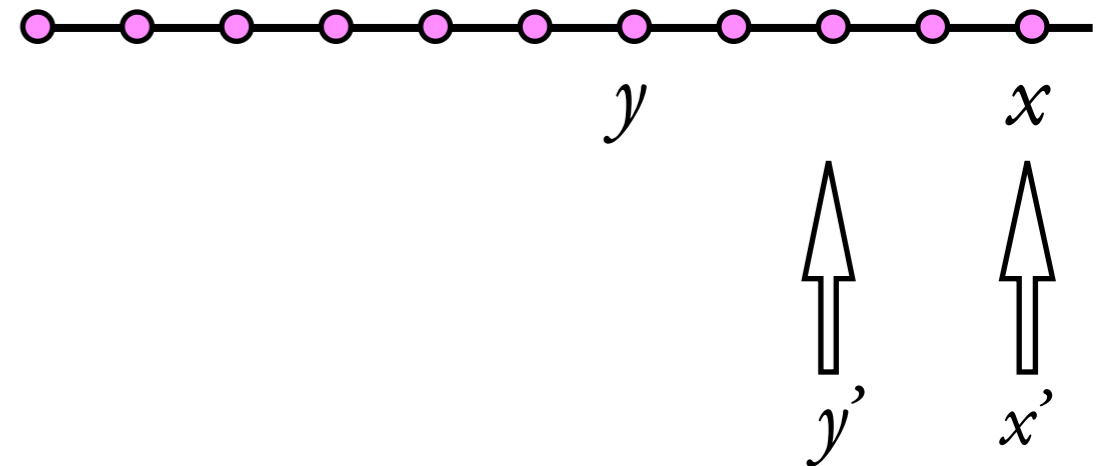
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a finite run over \mathbb{Q} :

$a a a b a b a a a a$

x 5 5 5 5 5

y 0 1 2 4 0

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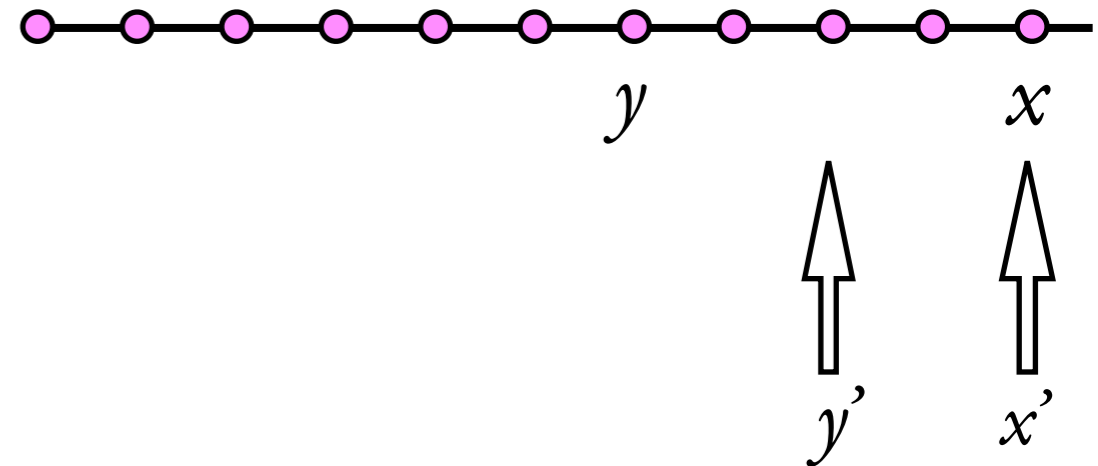
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a finite run over \mathbb{Q} :

$a a a b a b a a a a$

x 5 5 5 5 5 5

y 0 1 2 4 0 3

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

no states

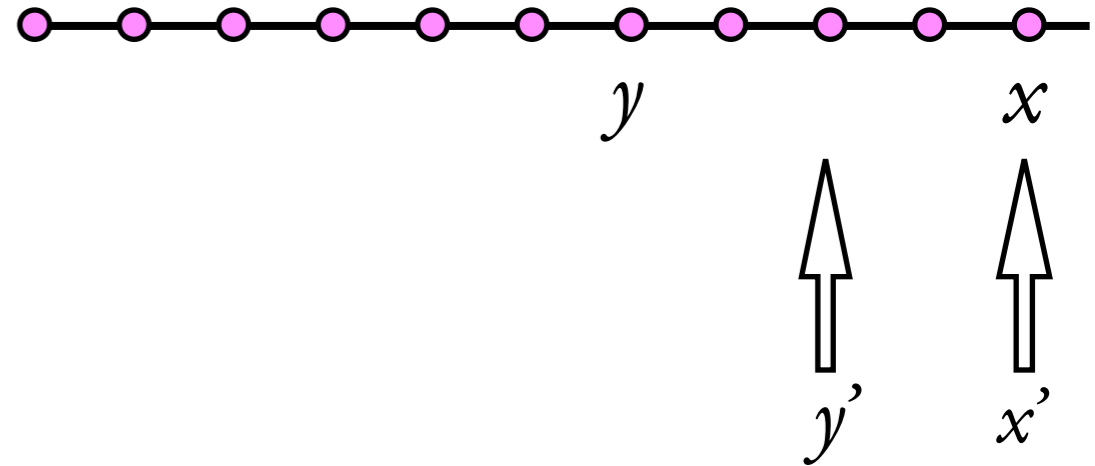
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a finite run over \mathbb{Q} :

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x 5 5 5 5 5 5 5

y 0 1 2 4 0 3 0

Example D -automaton \mathcal{A}

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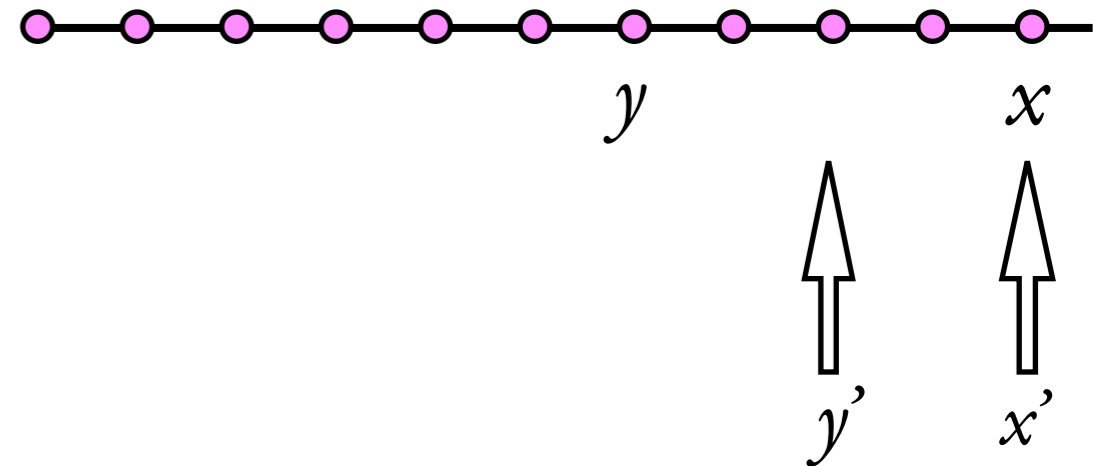
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a finite run over \mathbb{Q} :

$a a a b a b a a a a$

x	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

no states

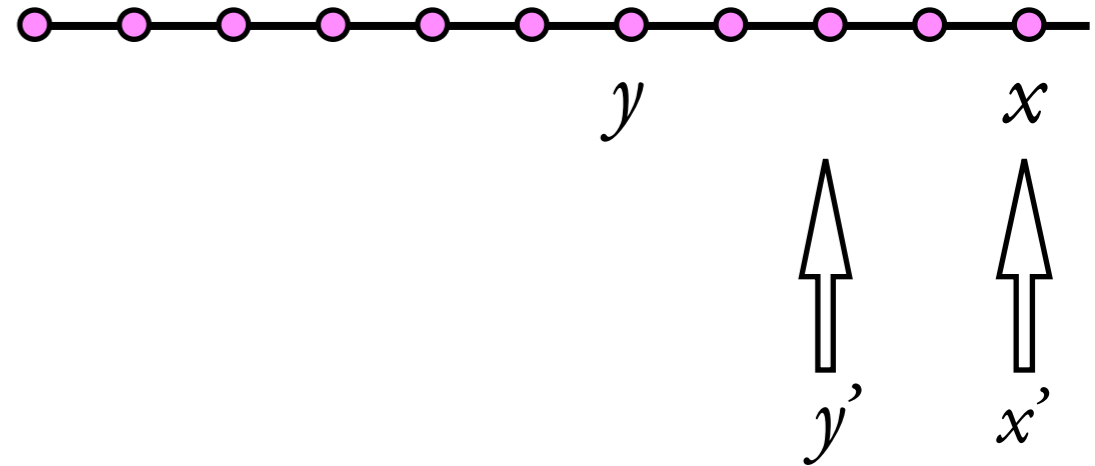
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a finite run over \mathbb{Q} :

$a a a b a b a a a a$

x	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

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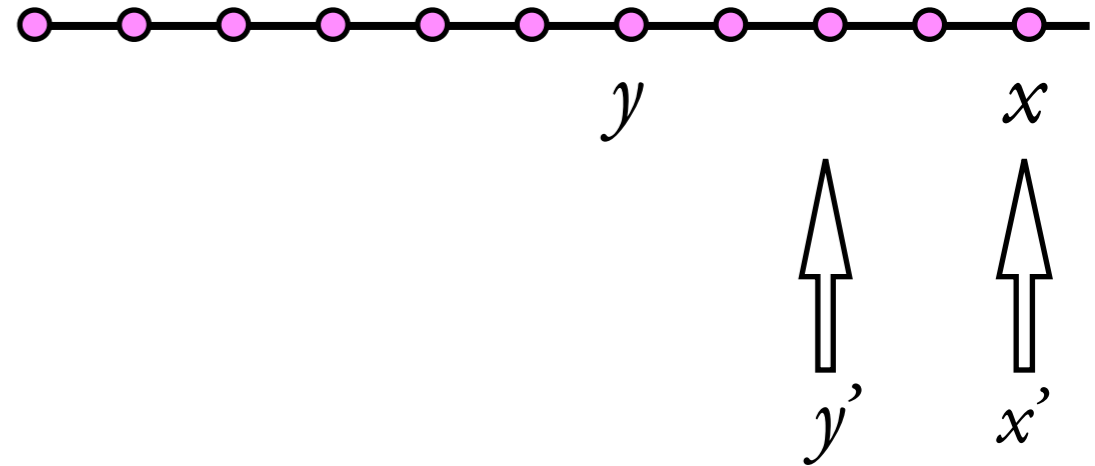
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a finite run over \mathbb{Q} :

$a a a b a b a a a a a$

x	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

no states

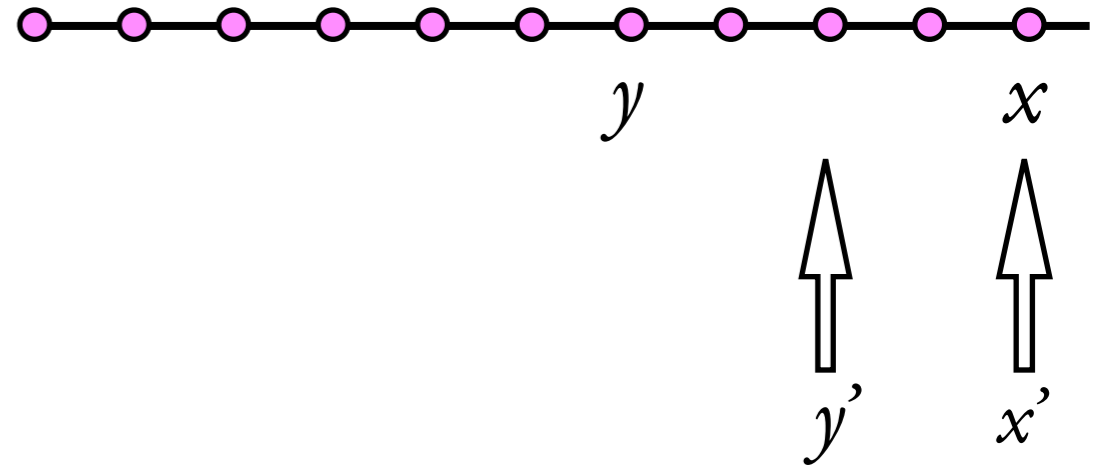
\mathcal{A}

$$\tau_I: (x > 0) \wedge (y = 0)$$

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$$\delta_b: (x' = x) \wedge (y' = 0)$$

$$\tau_F: (y = 1)$$



a finite run over \mathbb{Q} :

$a a a b a b a a a a$

x	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$

Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

no states

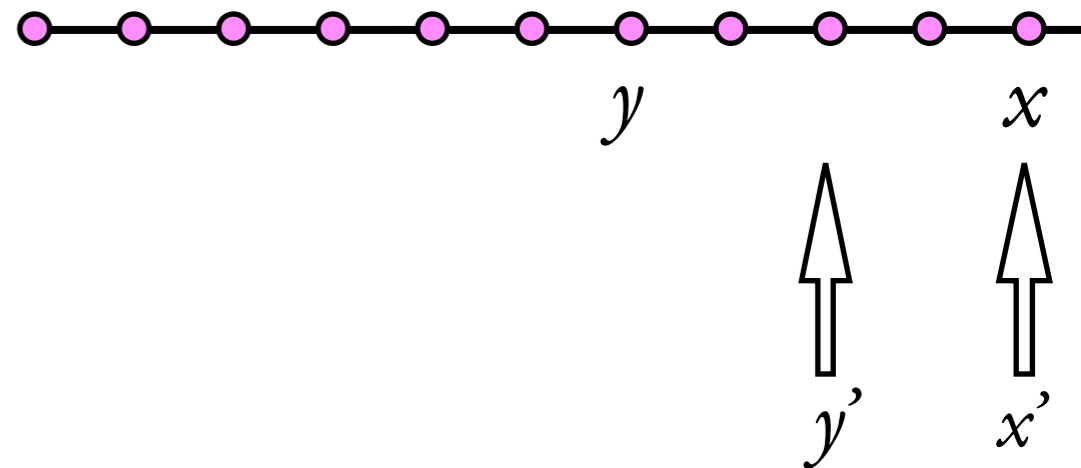
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Example D -automaton \mathcal{A}

x, y – variables of \mathcal{A}

no states

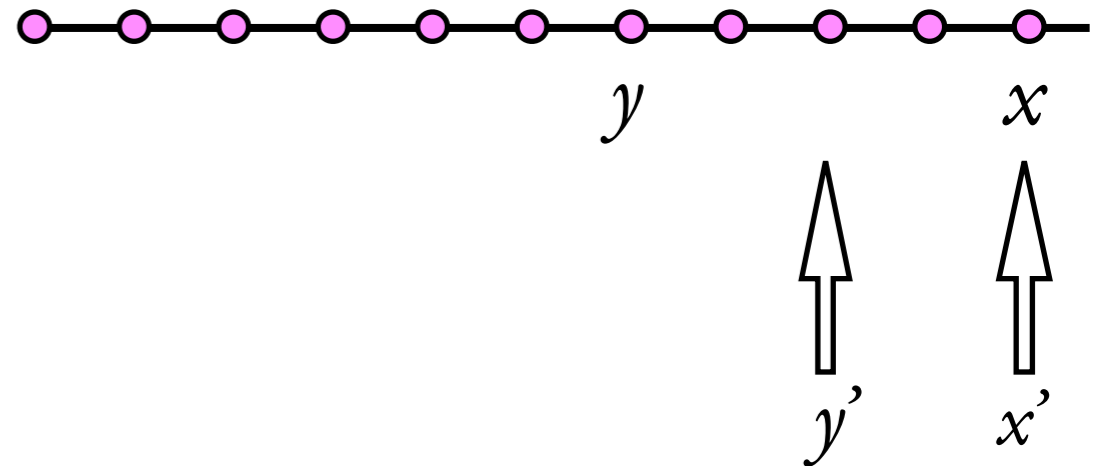
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x, y – variables of \mathcal{A}

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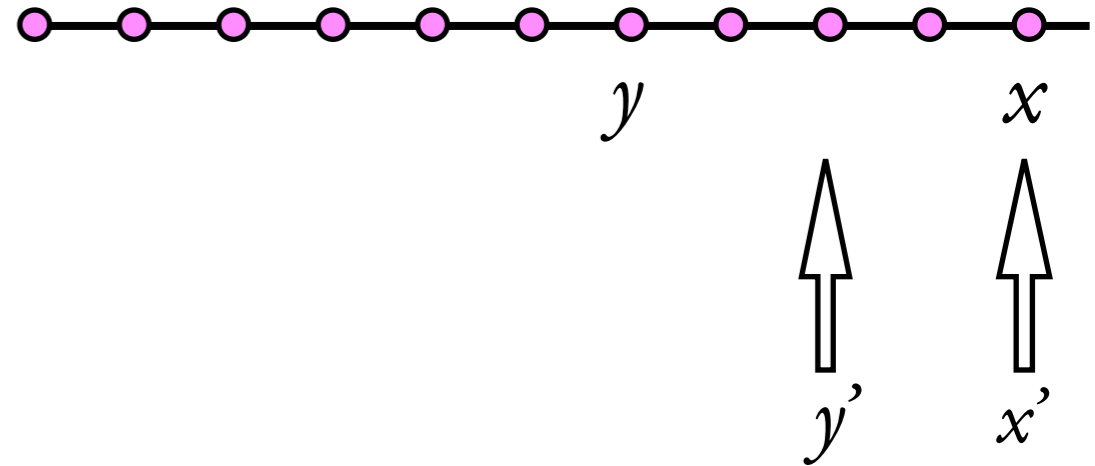
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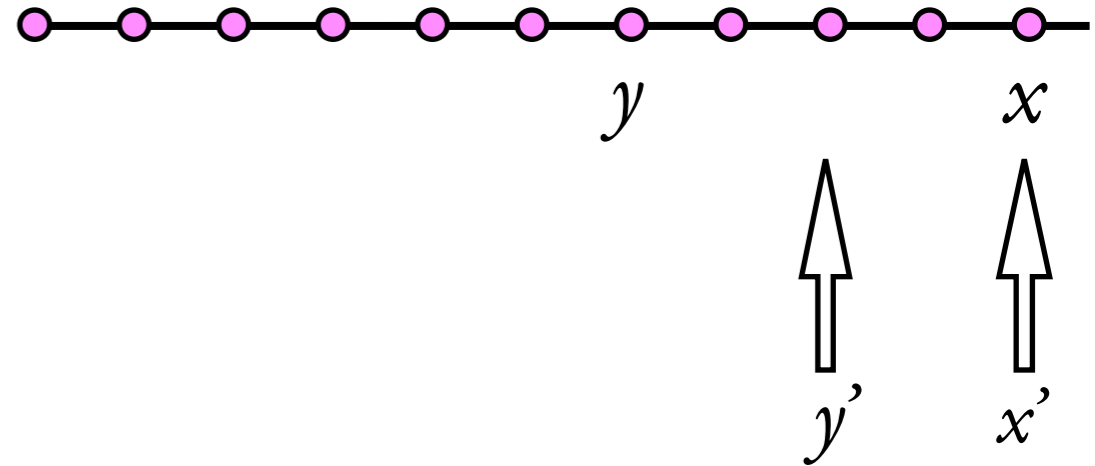
x	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1

accepted language: $(a+b)^*a$

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x, y – variables of \mathcal{A}

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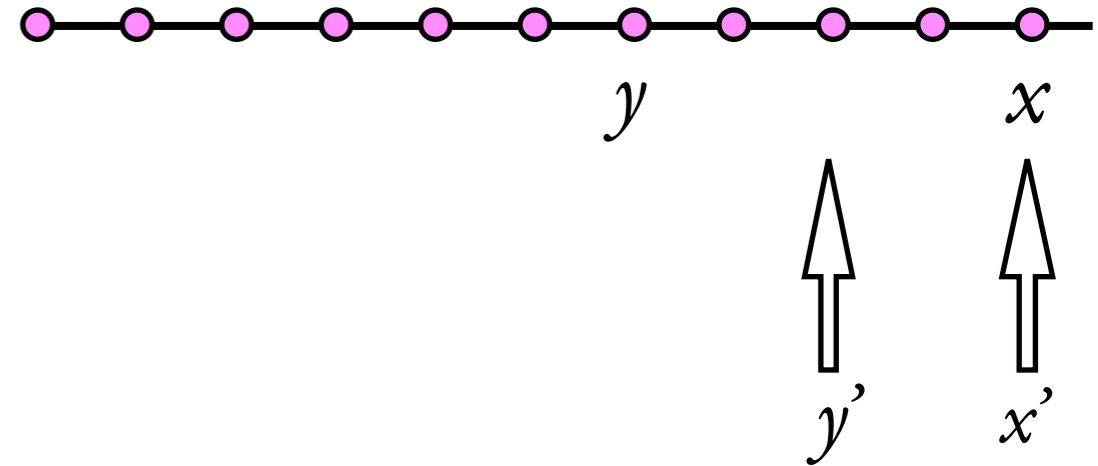
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x, y – variables of \mathcal{A}

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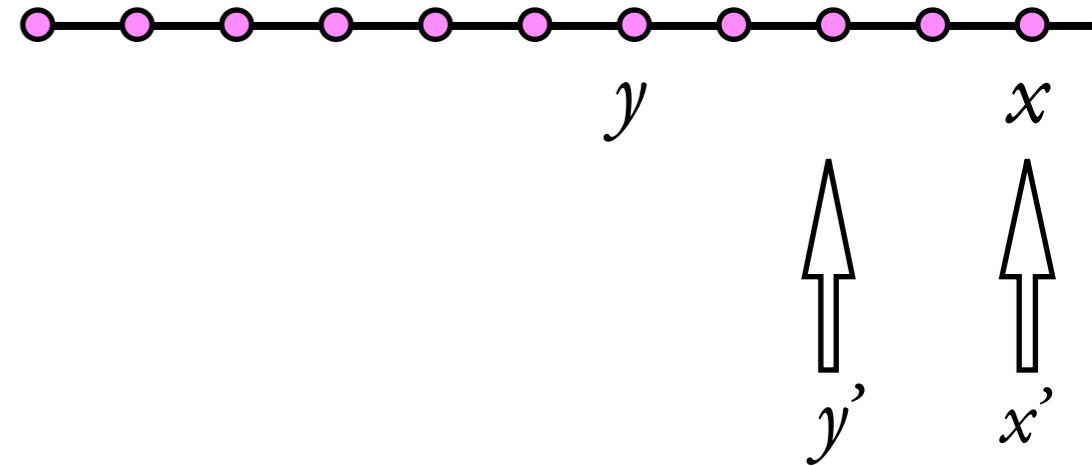
a finite run over \mathbb{N} :

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x, y – variables of \mathcal{A}

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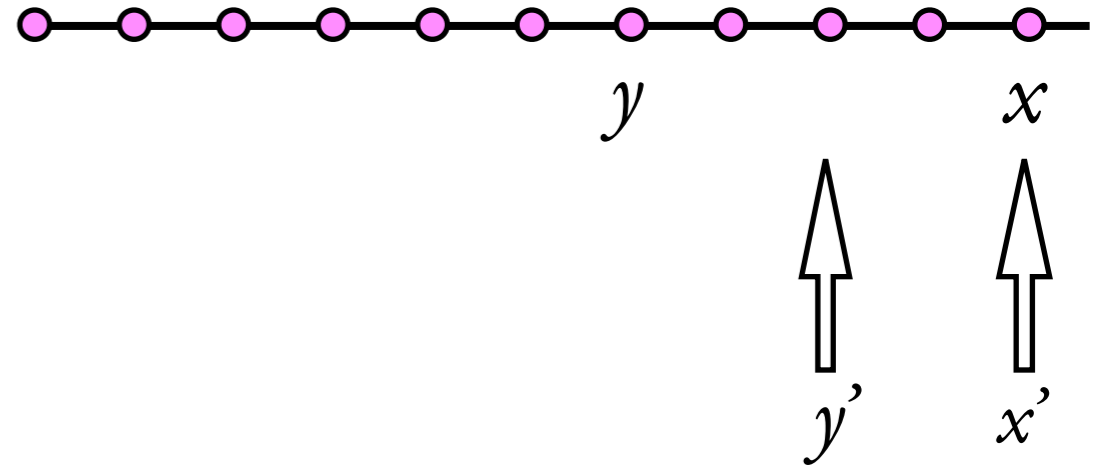
$a a a b a b a a a a$

x
 y

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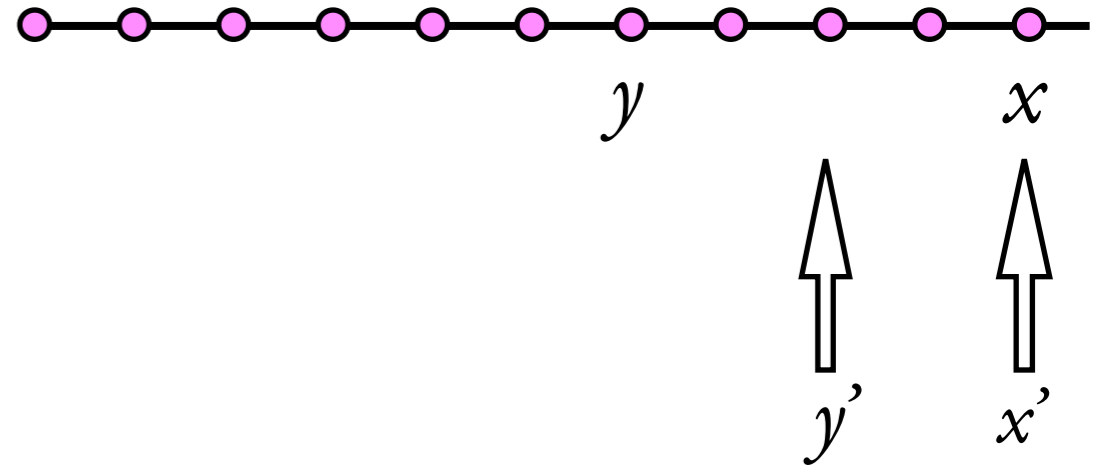
$a a a b a b a a a a$

x	5	5	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4			

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a finite run over \mathbb{N} :

$a a a b a b a a a a$

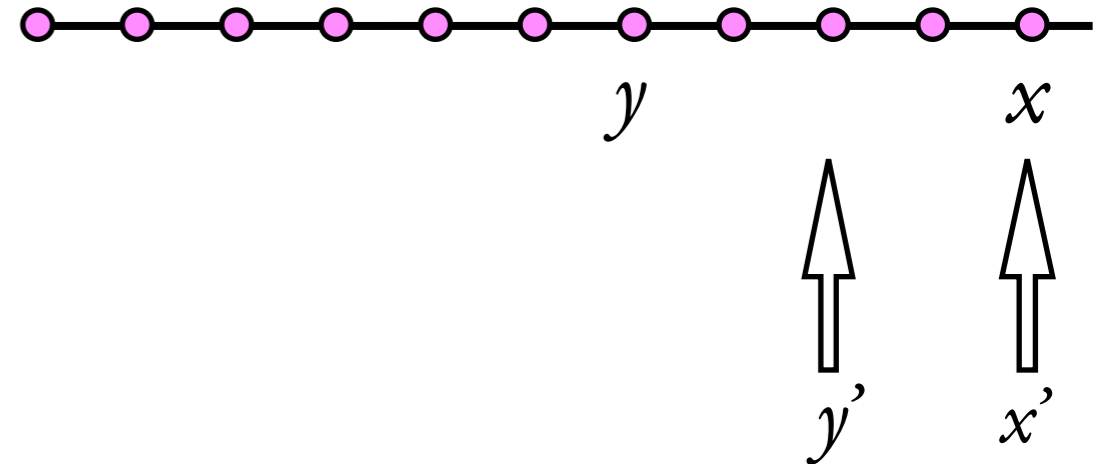
x	5	5	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4			

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a finite run over \mathbb{Q} :

$aaababaaaa$

x	5	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{4}{5}$	1	1

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a finite run over \mathbb{N} :

$aaababaaaa$

x	5	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4	4	4

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an infinite run over \mathbb{N} :

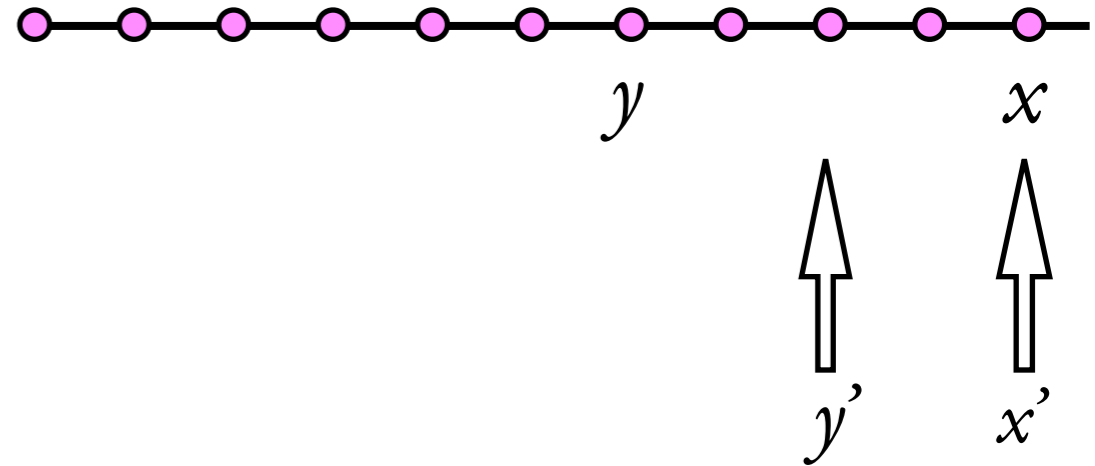
$aaababaaaa$

x	5	5	5	5	5	5	5	5	5	5	5	5	5
y	0	1	2	4	0	3	0	1	2	3	4	4	4

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accepted language: $(a^B b)^\omega$

Adding database constraints

x, y, z, \dots – a finite set of variables

R, S, \dots – a finite set of relational symbols

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EMPTINESS: is there a *finite* database M , a word w , and an accepting run over w consistent with M ?

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x 0 2 3 4 0 3 0 1

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Decide emptiness
of D -automata

$\tau_F: (x=)$

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Known results deciding emptiness

	no database	with database
$\mathbb{N}, =$	Kaminski, Francez (94)	Deutsch, Sui, Vianu, Zhou (06) PSPACE
$\mathbb{Q}, <$	Čerans (94) PSPACE	Deutsch, Hull, Patrizi, Vianu (09) PSPACE
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$\mathcal{D}, <$	PSPACE	PSPACE

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$\mathcal{D}, <$	PSPACE	PSPACE

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$\mathcal{D}, <$	PSPACE	PSPACE

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- $D = \mathbb{Q}$
- finite words
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The region construction

$0 < y < 1 < x$, $0 < x < 1 = y$, $0 = 1 < y < x$, $0 < 1 = y < x$, $0 = y < 1 < x$, $0 = y < 1 = x$, ...

Dense case

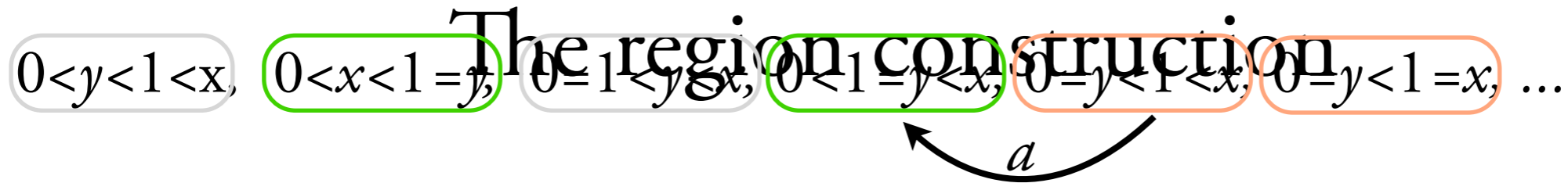
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Dense case

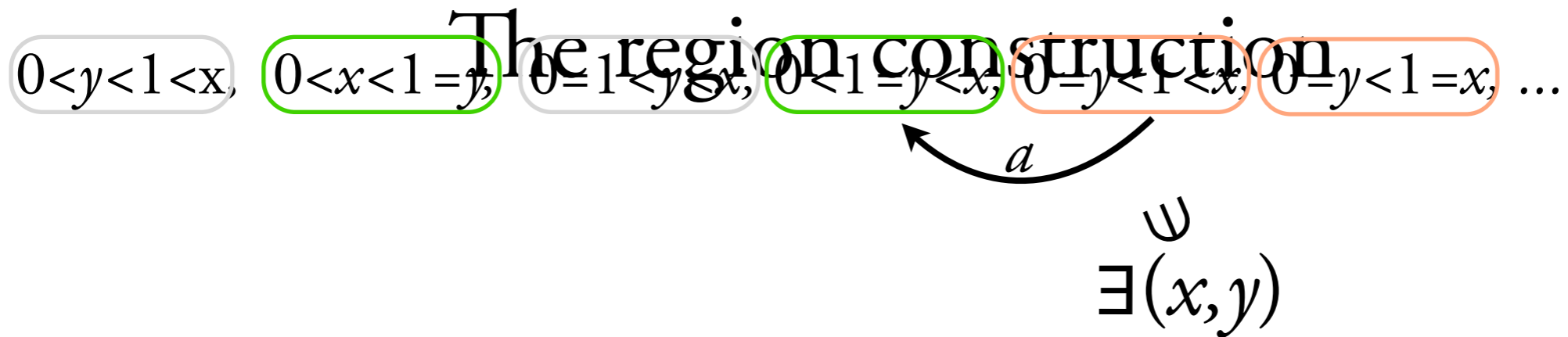
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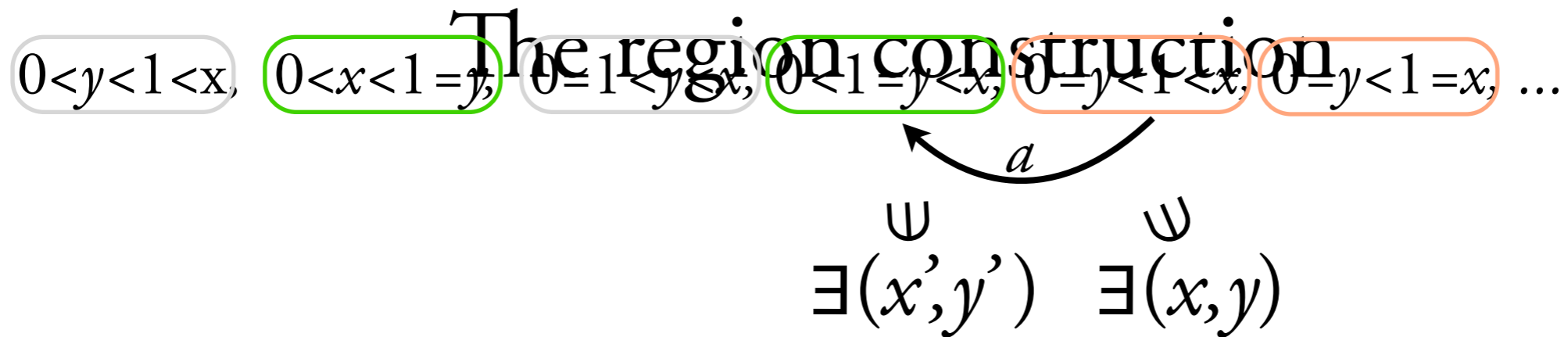
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$$\tau_I: (x > 0) \wedge (y = 0)$$

$$\delta_a: (x' = x) \wedge (y < y' < x)$$

$$\delta_b: (x' = x) \wedge (y' = 0)$$

$$\tau_F: (y = 1)$$



Dense case

- $D = \mathbb{Q}$
- finite words
- no database

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The region construction

$0 < y < 1 < x$, $0 < x < 1 = y$, $0 = 1 < y < x$, $0 < 1 = y < x$, $0 = y < 1 < x$, $0 = y < 1 = x$, ...

$$\begin{array}{ccc} & \curvearrowright a & \\ \Downarrow & & \Downarrow \\ \exists(x', y') & \exists(x, y): & \delta_a(x, y, x', y') \end{array}$$

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The region construction

$0 < y < 1 < x$, $0 < x < 1 = y$, $0 = 1 < y < x$, $0 < 1 = y < x$, $0 = y < 1 < x$, $0 = y < 1 = x$, ...

$$\begin{array}{ccc}
 & \Downarrow & \Downarrow \\
 \exists(x', y') & \xrightarrow{a} & \exists(x, y): \delta_a(x, y, x', y') \\
 & & (5, 0)
 \end{array}$$

Dense case

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The region construction

$0 < y < 1 < x$, $0 < x < 1 = y$, $0 = 1 < y < x$, $0 < 1 = y < x$, $0 = y < 1 < x$, $0 = y < 1 = x$, ...

$$\begin{array}{ccc}
 \Downarrow & & \Downarrow \\
 \exists(x', y') & \exists(x, y): & \delta_a(x, y, x', y') \\
 (5, 1) & & (5, 0)
 \end{array}$$

Dense case

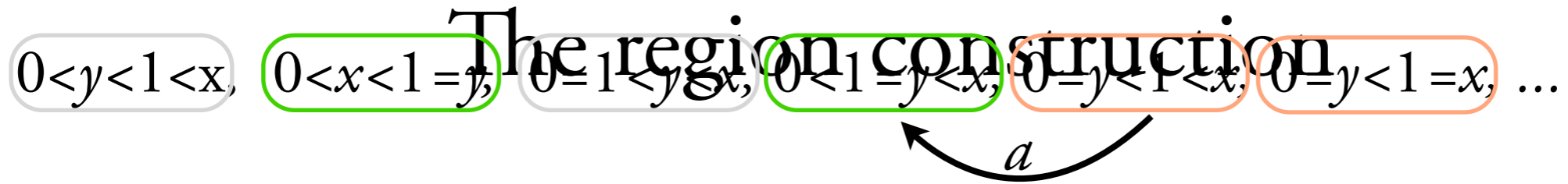
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Dense case

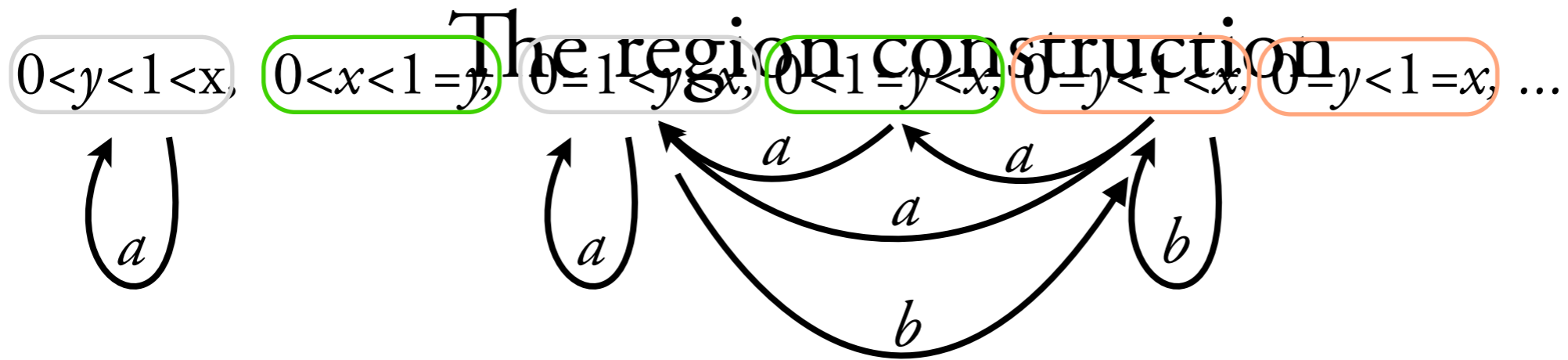
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Dense case

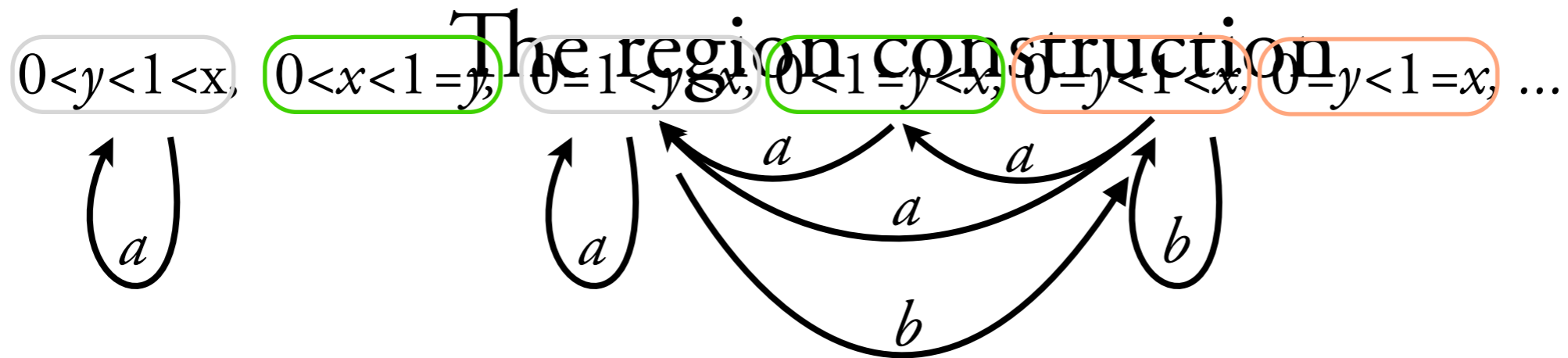
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bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}

Dense case

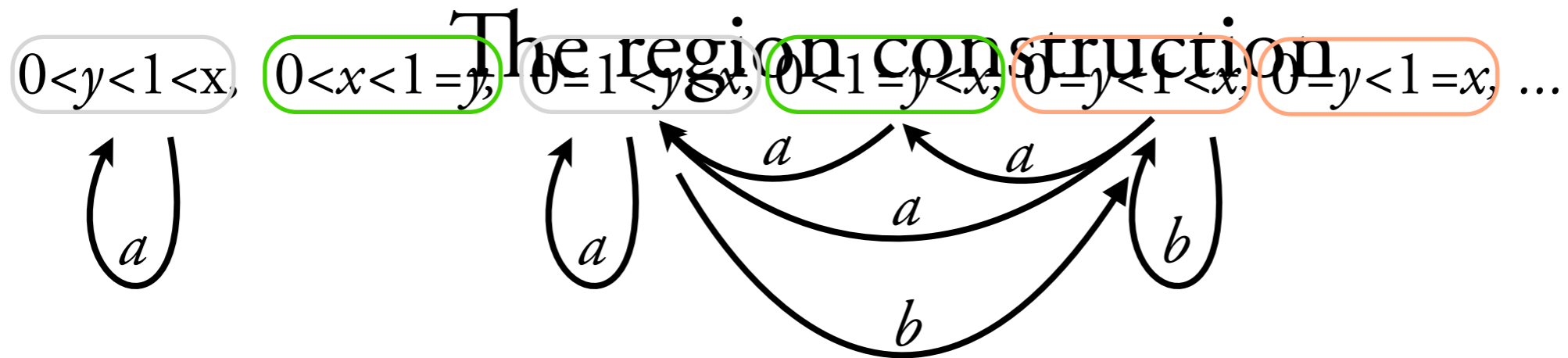
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a a a b a b a a a a

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Dense case

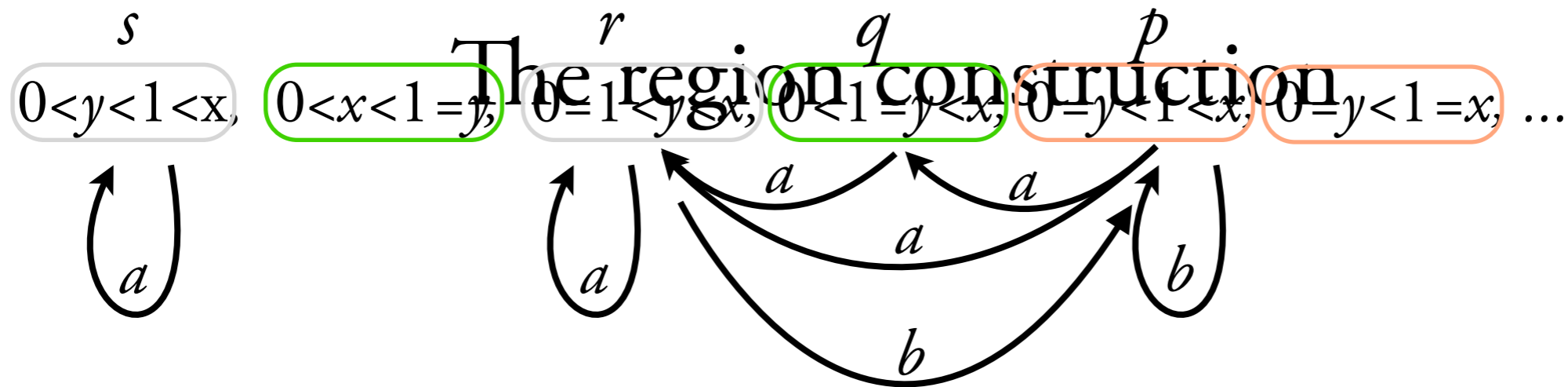
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Dense case

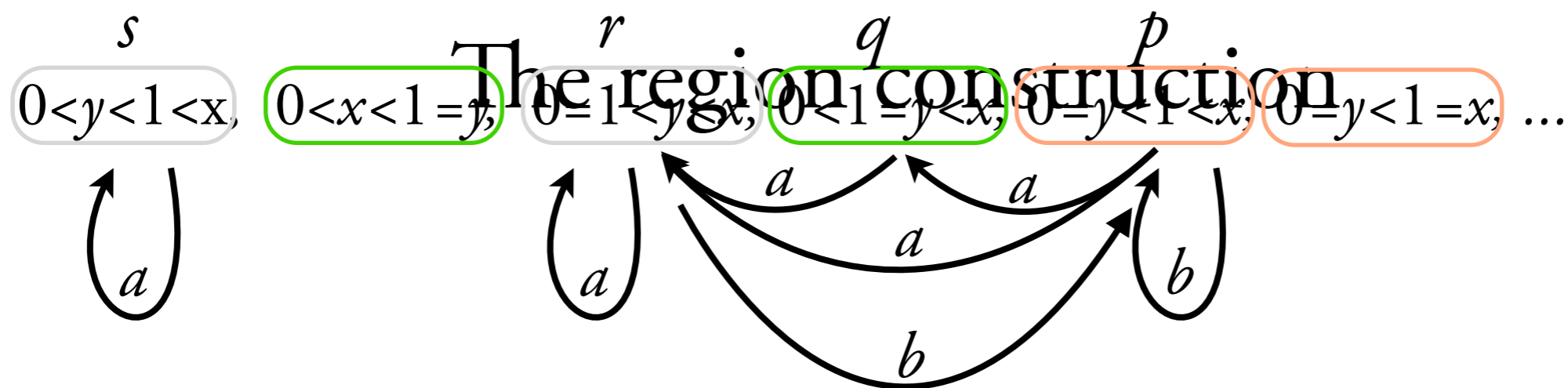
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$a a a b a b a a a a$
 $p q r r q r q s s s s q$

bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}

Dense case

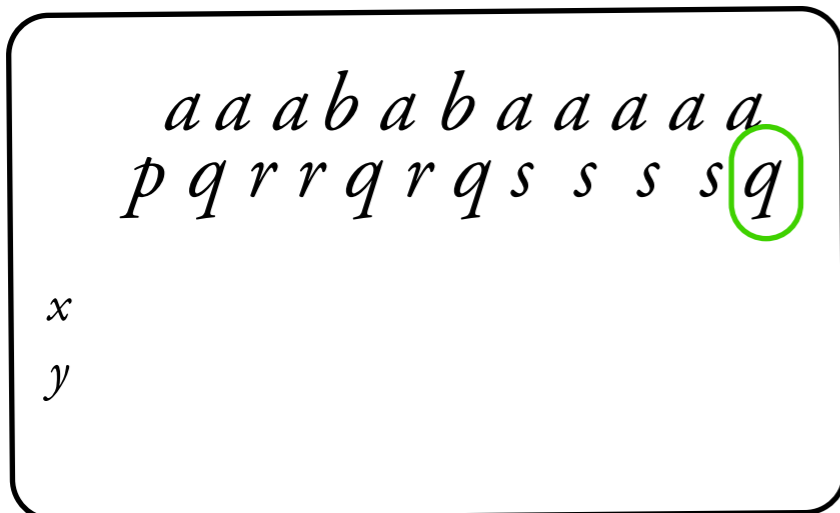
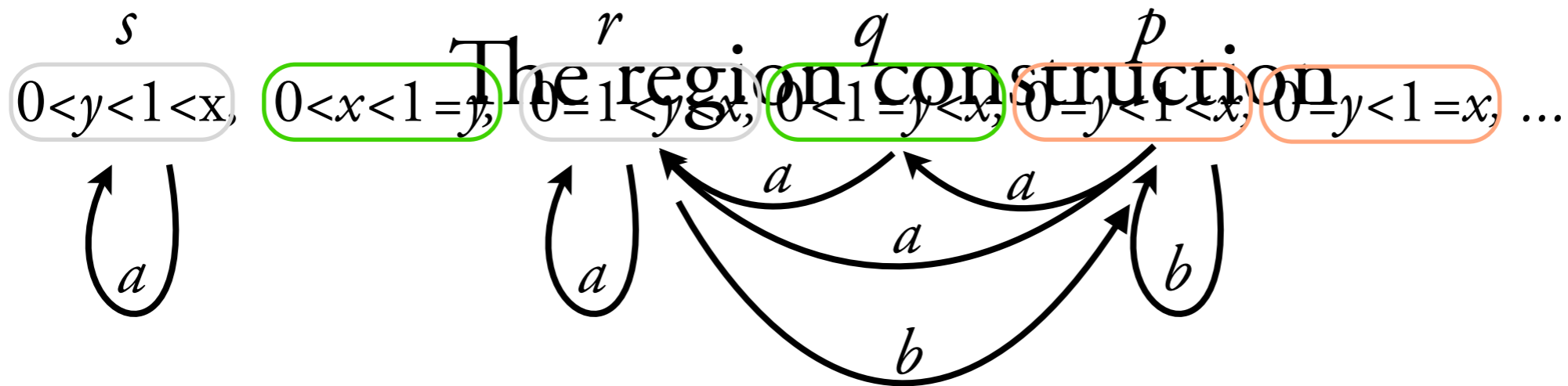
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Dense case

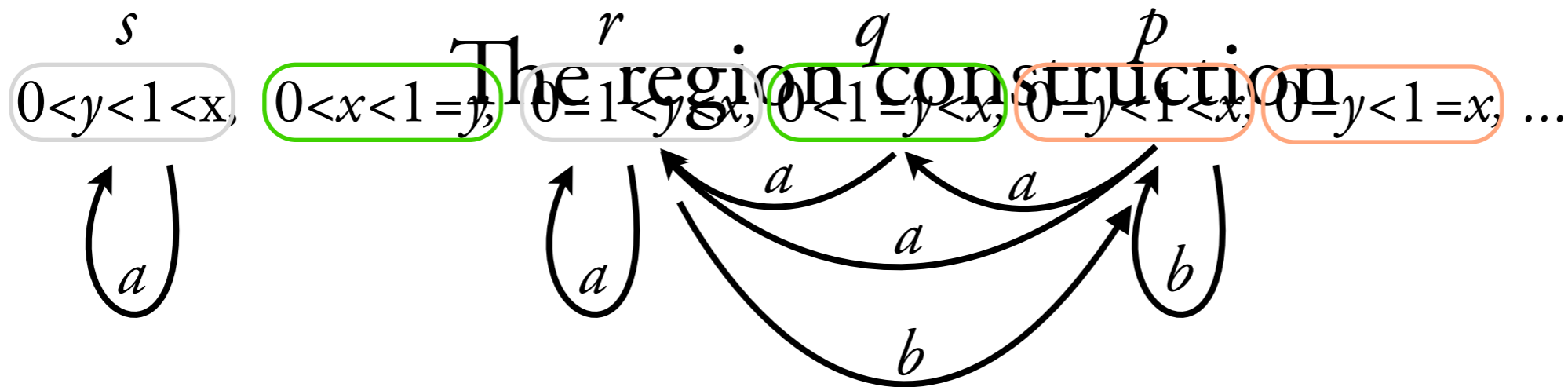
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	<i>a a a b a b a a a a</i>
	<i>p q r r q r q s s s s q</i>
<i>x</i>	5
<i>y</i>	0

bisimulation → runs of the region automaton correspond to runs of \mathcal{A}

Dense case

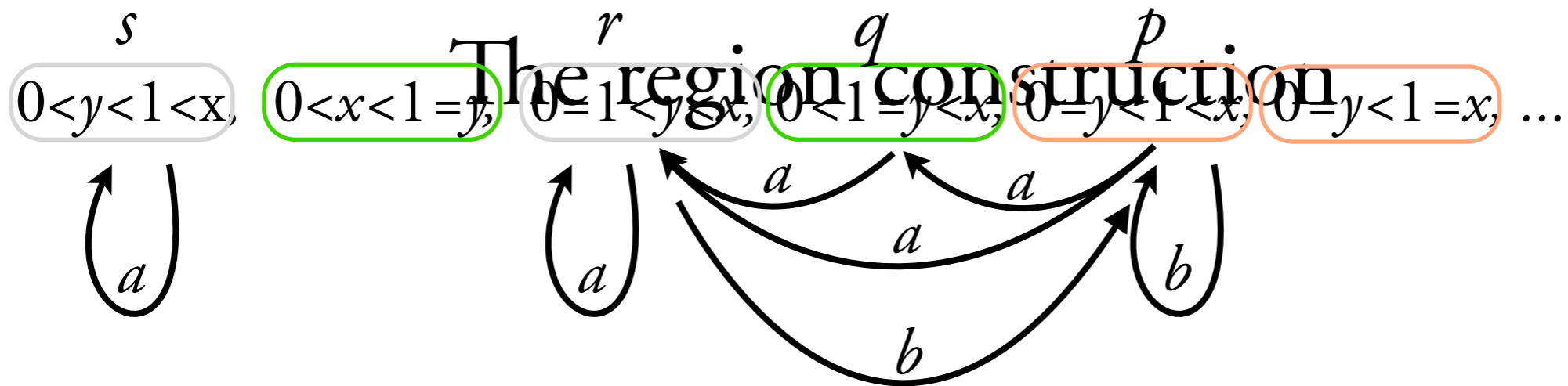
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	<i>a a a b a b a a a a</i>
	<i>p q r r q r q s s s s q</i>
<i>x</i>	5 5
<i>y</i>	0 1

bisimulation → runs of the region automaton correspond to runs of \mathcal{A}

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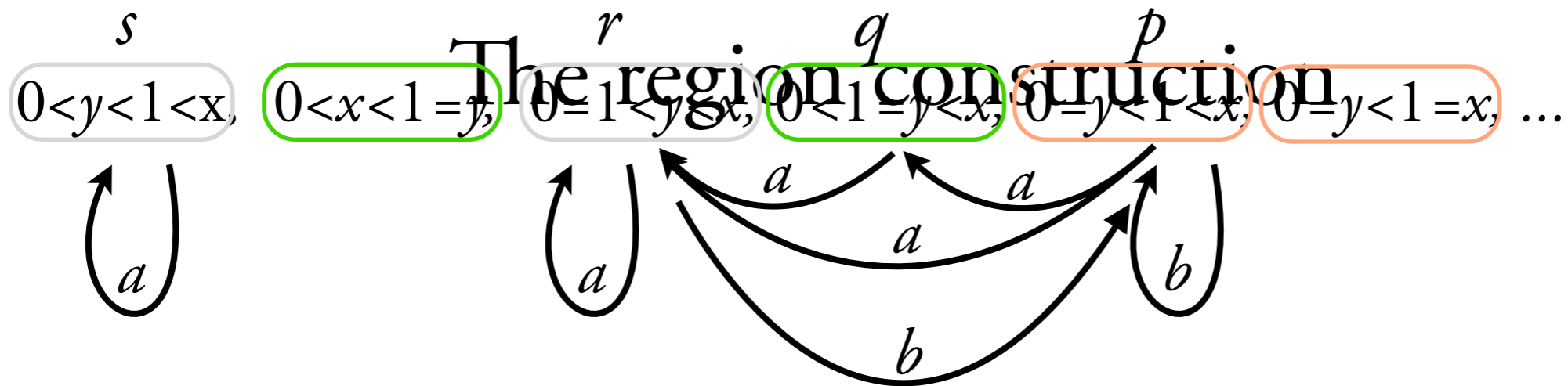
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	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>
<i>x</i>	5	5	5						
<i>y</i>	0	1	2						

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Dense case

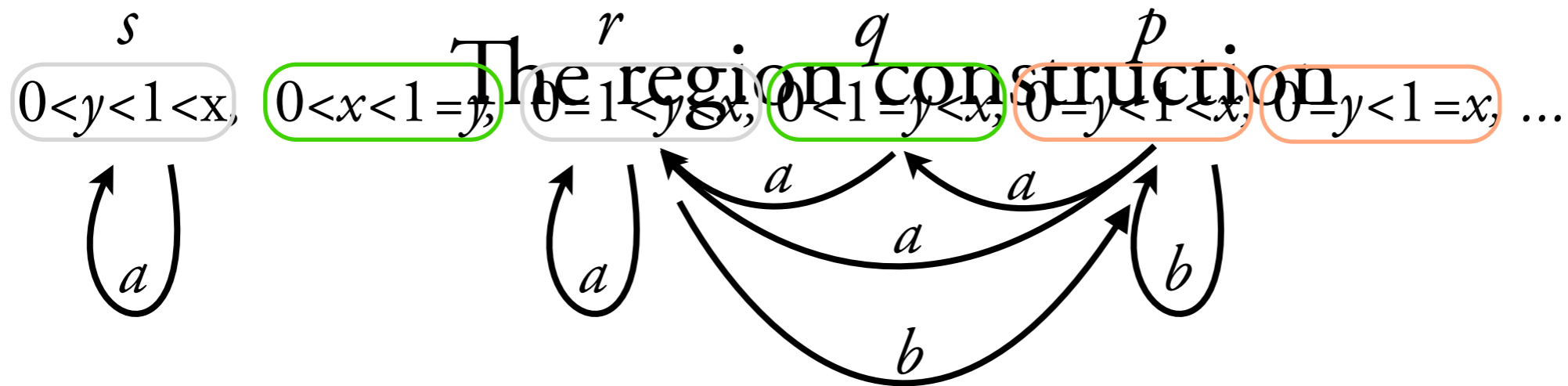
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5							
<i>y</i>	0	1	2	4							

bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}

Dense case

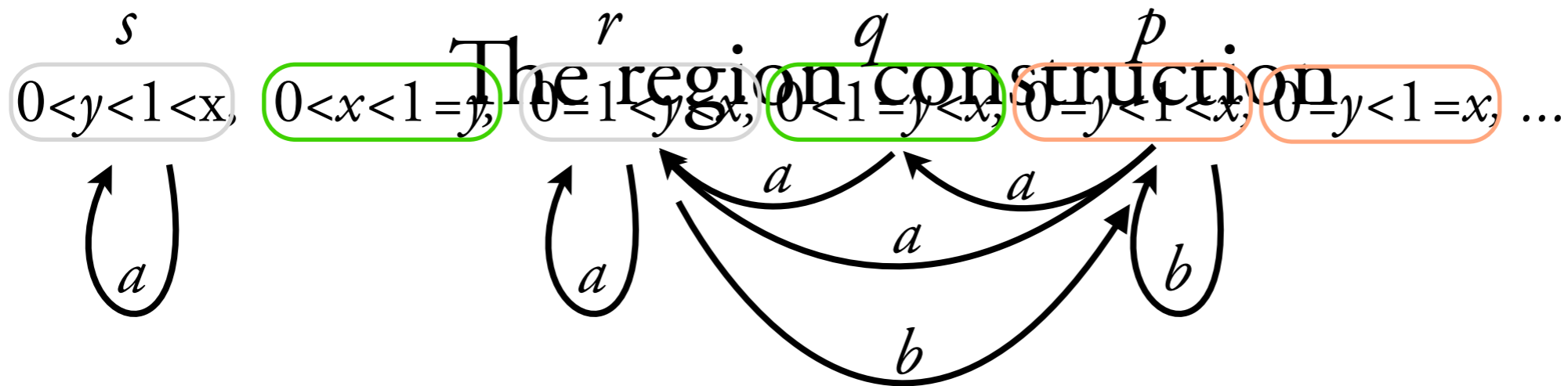
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	1	2	4	0	1	2

bisimulation → runs of the region automaton correspond to runs of \mathcal{A}

Dense case

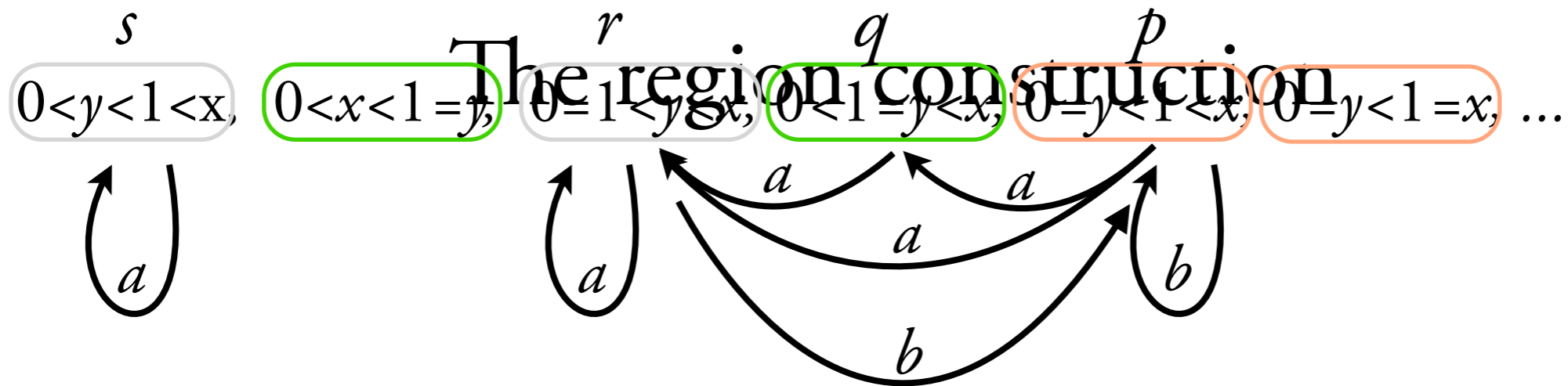
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	3	0	3	3	3	3

bisimulation → runs of the region automaton correspond to runs of \mathcal{A}

Dense case

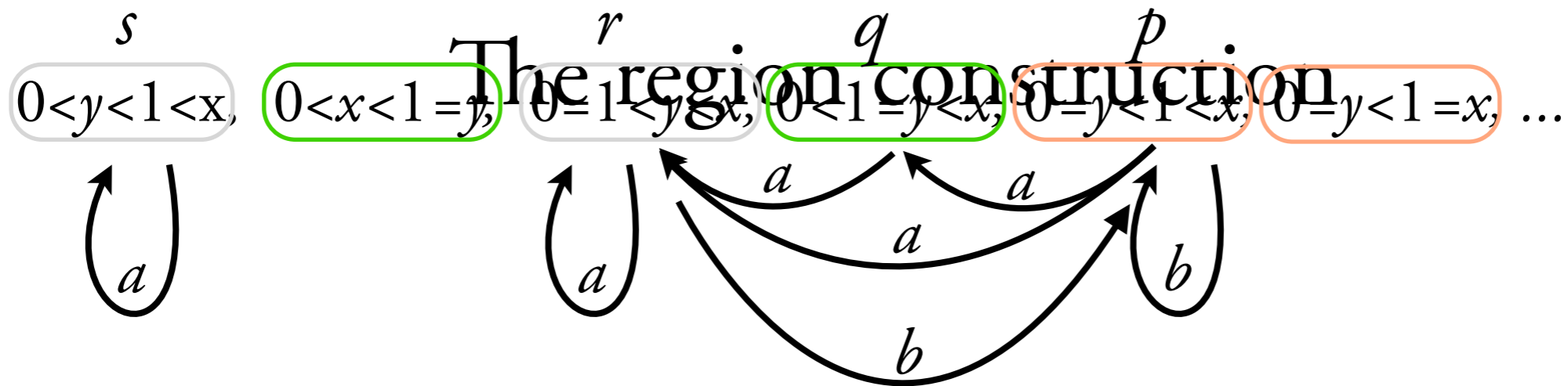
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	3	0				

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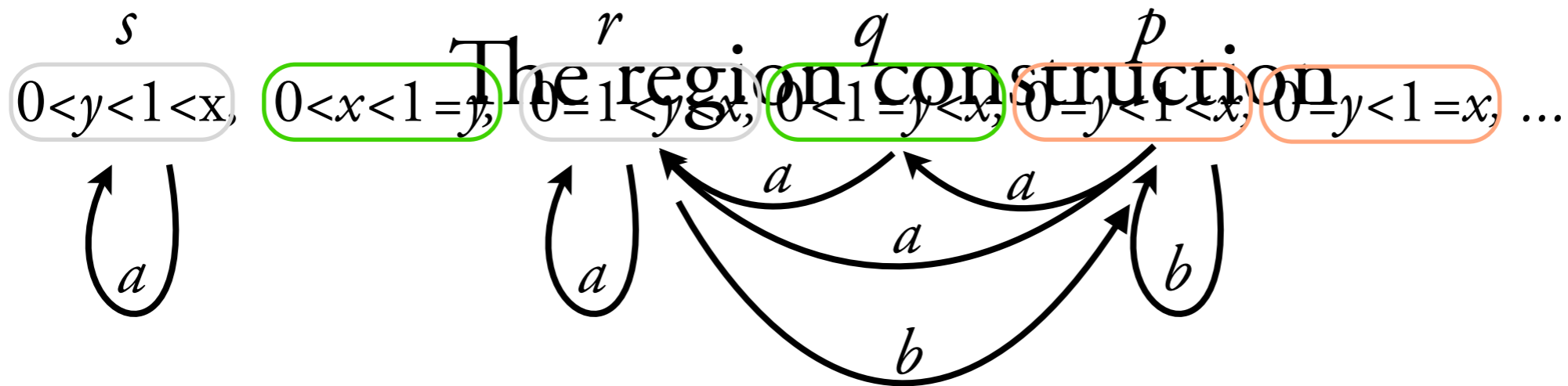
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	
<i>y</i>	0	1	2	4	0	3	0	.	.	.	

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Dense case

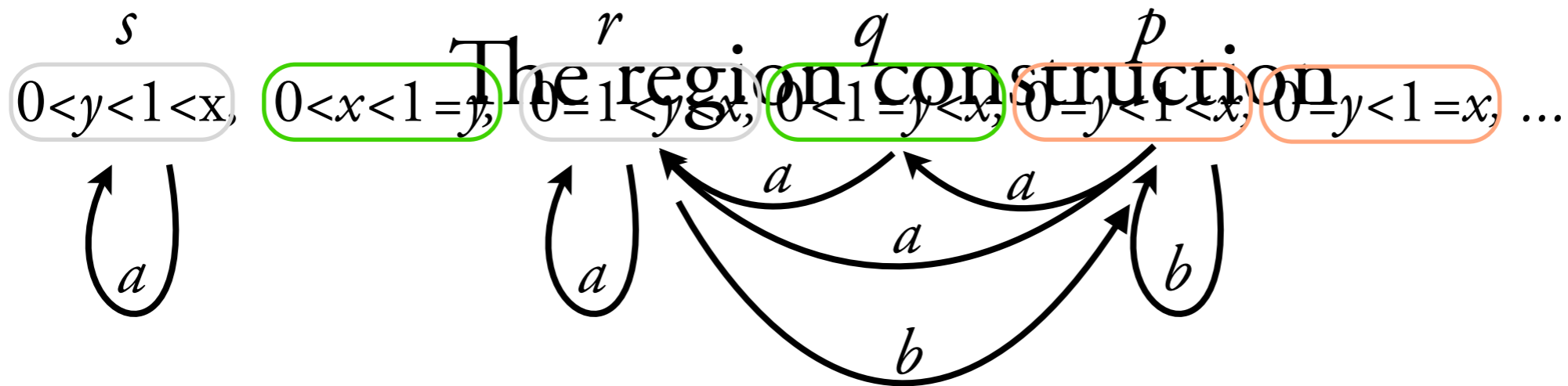
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>		
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	3	0	.1	.2			

bisimulation → runs of the region automaton correspond to runs of \mathcal{A}

Dense case

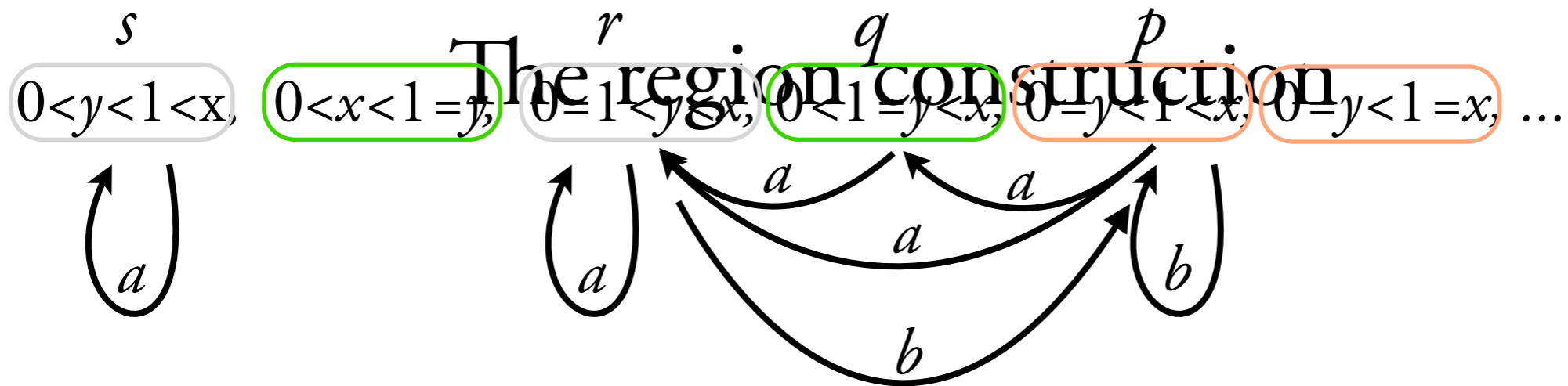
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	3	0	.1	.2	.4	

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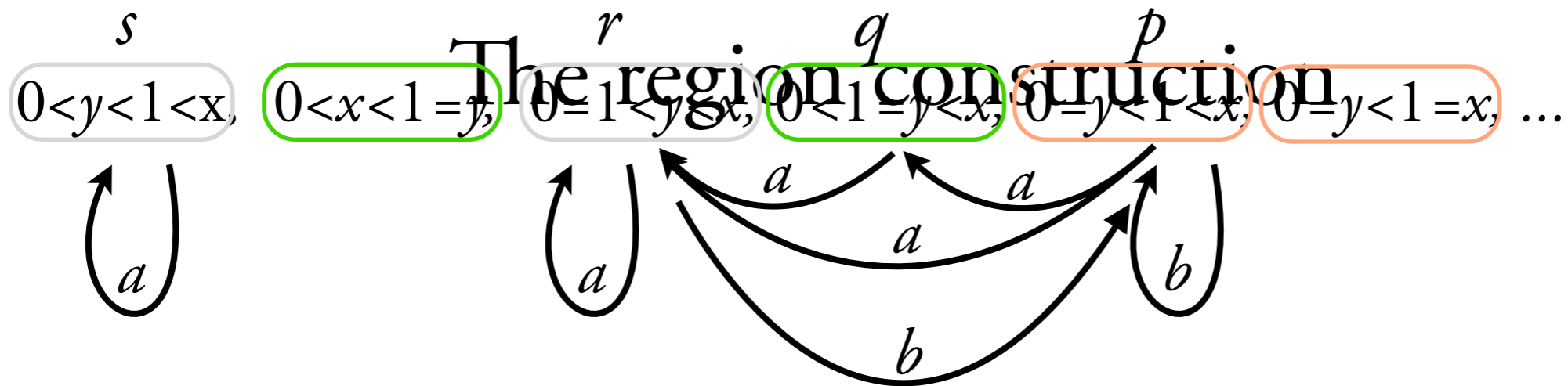
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	3	0	.1	.2	.4	.9

bisimulation → runs of the region automaton correspond to runs of \mathcal{A}

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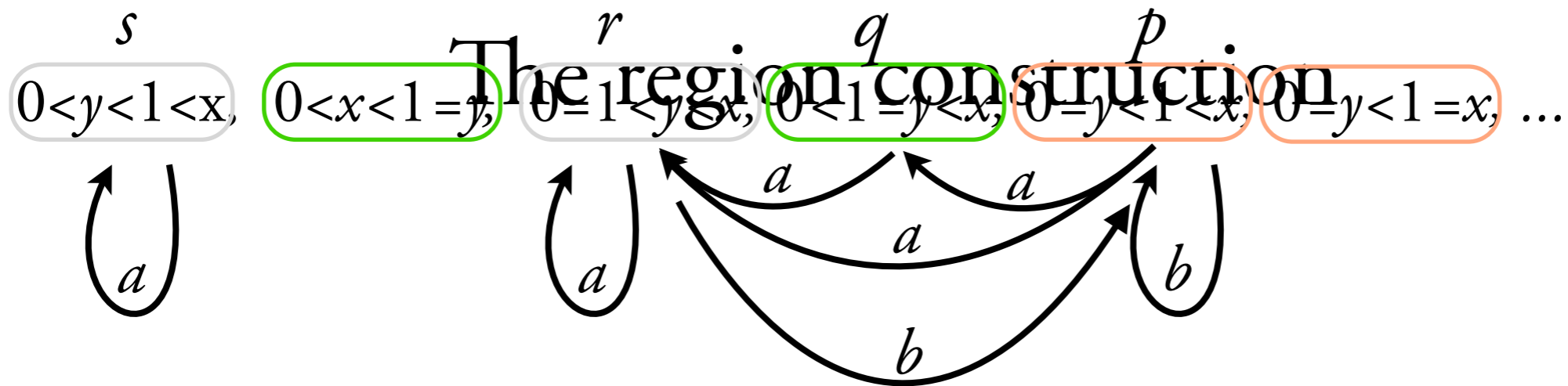
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>		
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>	
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5	
<i>y</i>	0	1	2	4	0	3	0	.1	.2	.4	.9	1

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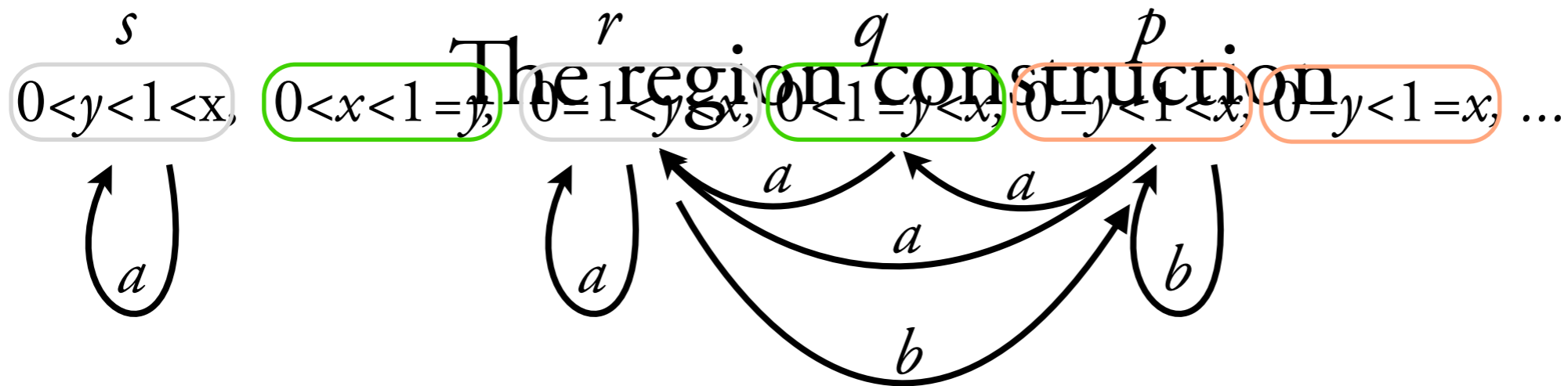
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	3	0	.1	.2	.4	.9	1

bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}

Dense case

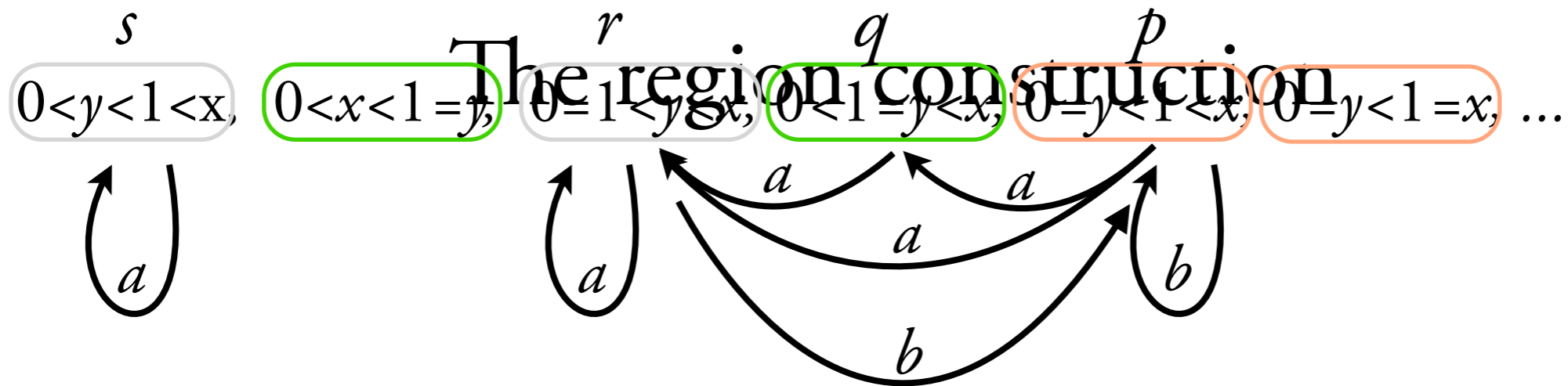
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	3	0	.1	.2	.4	.9	1

bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}

accepted language: $(a+b)^*a$

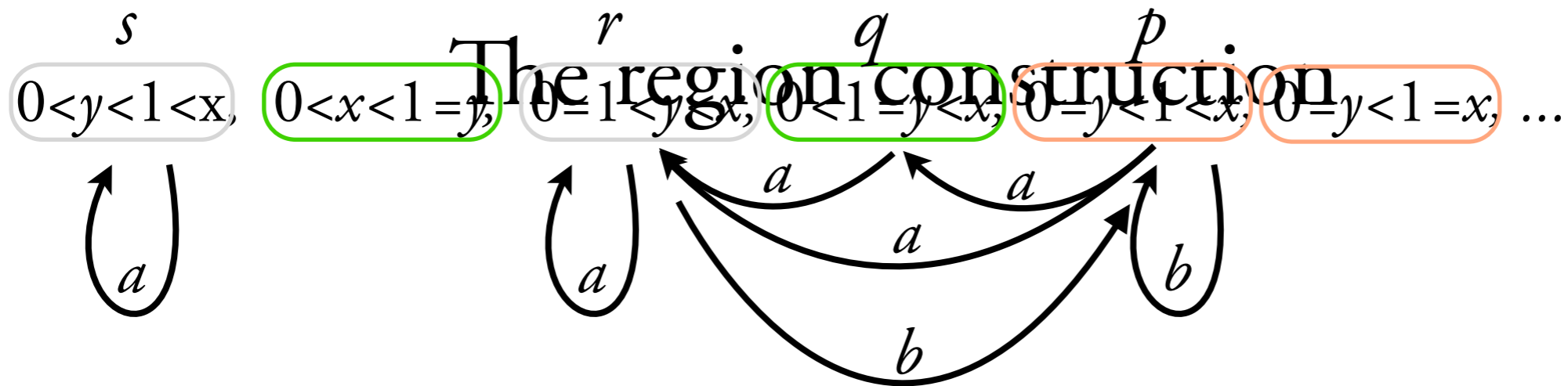
- $D = \mathbb{Q}$
- finite words
- no database

$$\tau_I: (x > 0) \wedge (y = 0)$$

$$\delta_a: (x' = x) \wedge (y < y' < x)$$

$$\delta_b: (x' = x) \wedge (y' = 0)$$

$$\tau_F: (y = 1)$$



	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>
	<i>p</i>	<i>q</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>q</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>s</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	3	0	.1	.2	.4	.9	1

bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}

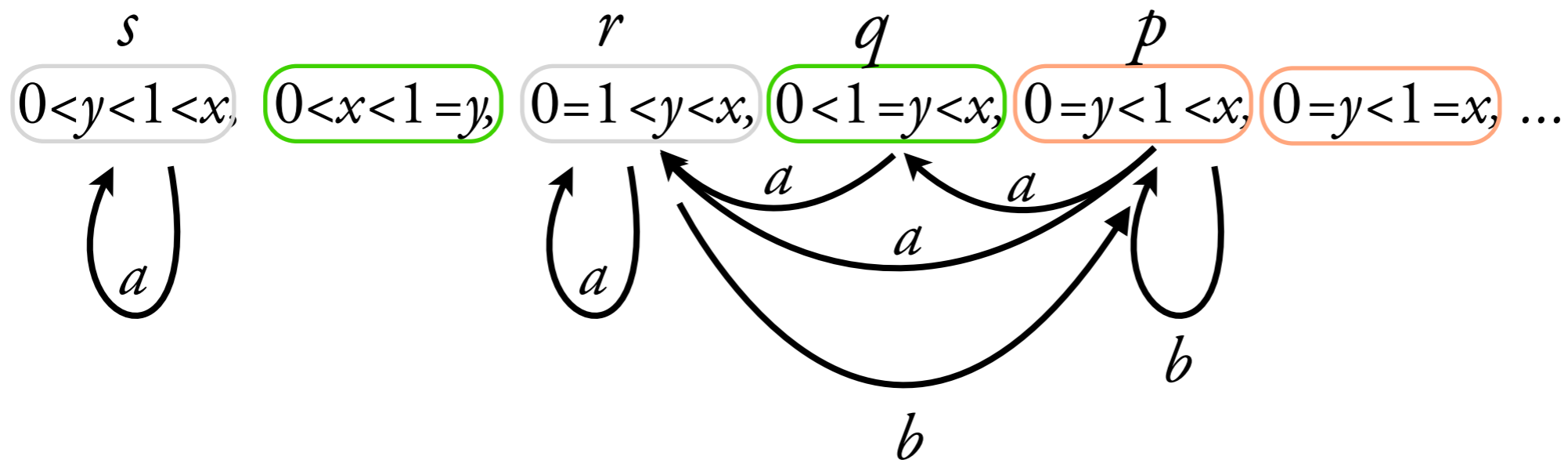
- $D = \mathbb{N}$
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- no database

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$$\delta_a: (x' = x) \wedge (y < y' < x)$$

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bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}

Discrete case

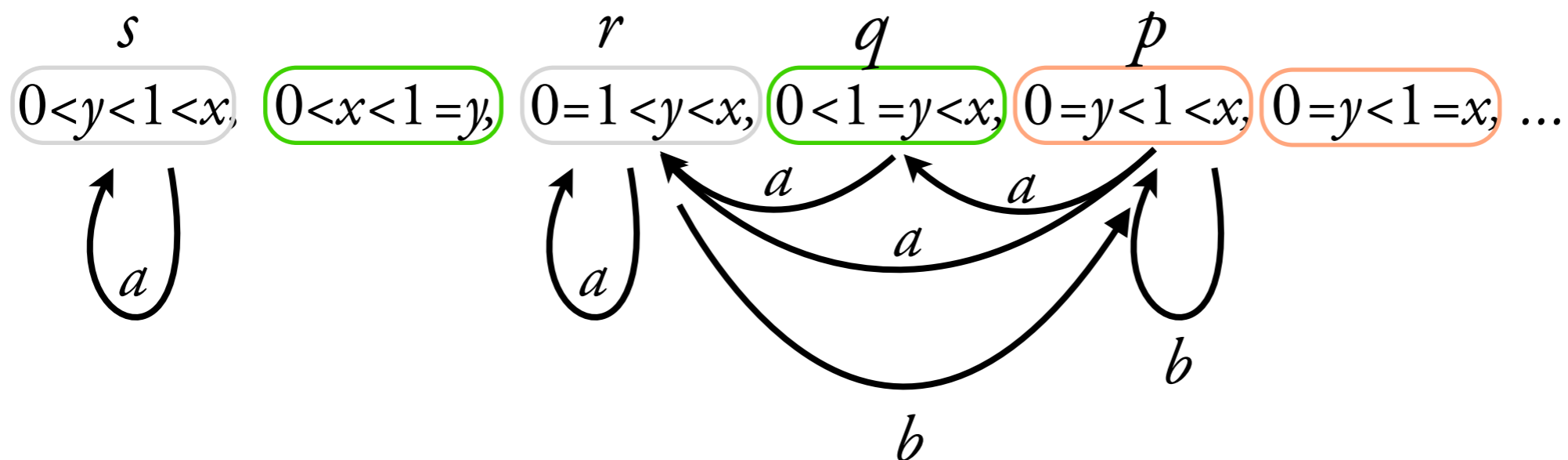
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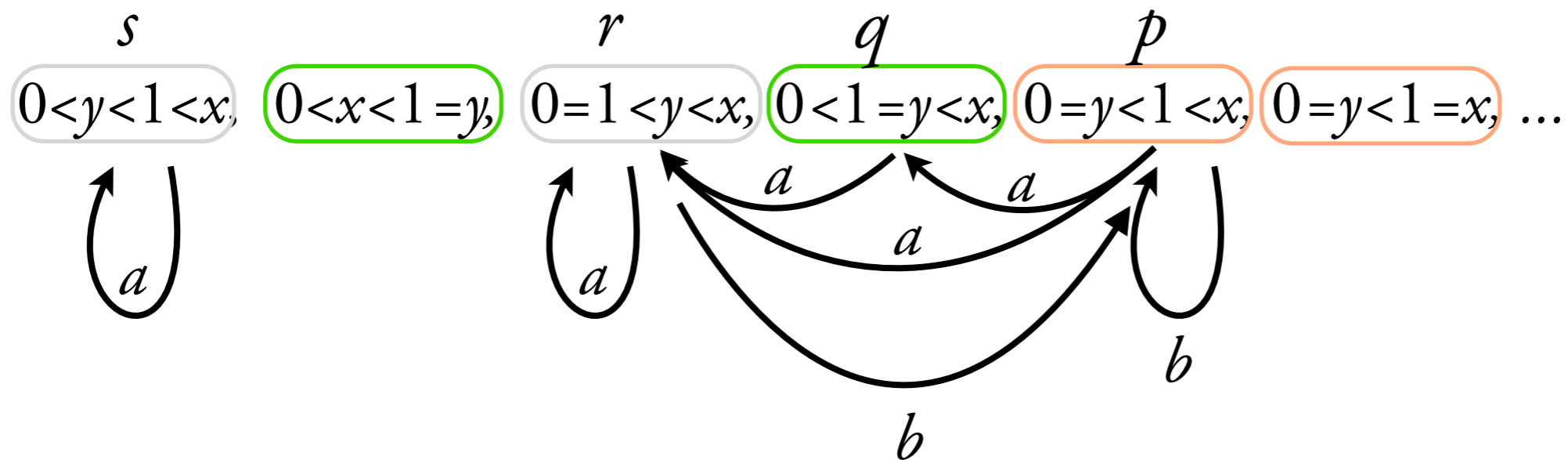
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~~bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}~~

Discrete case

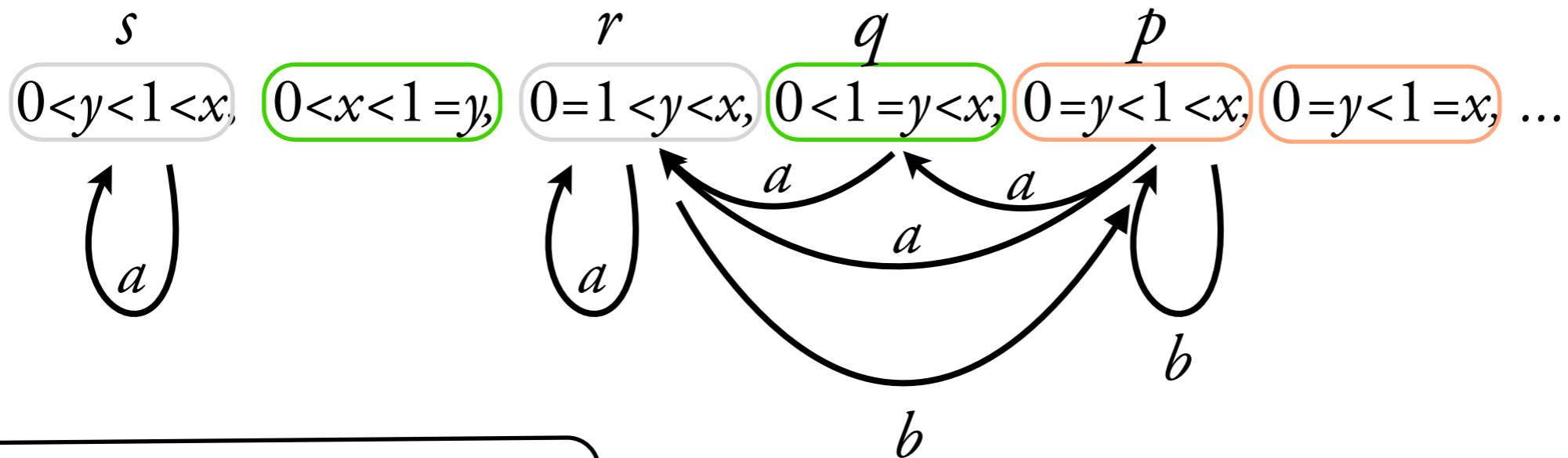
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$aaababaaaa$
 $psrrqrpsrrrq$

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Discrete case

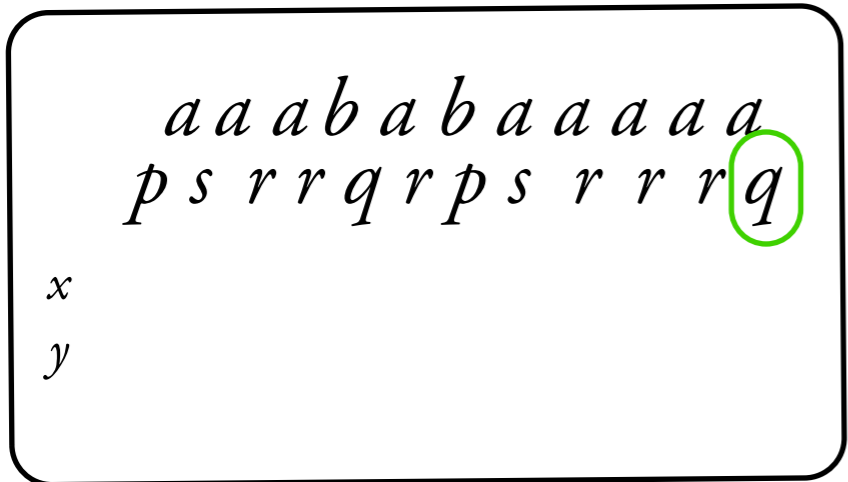
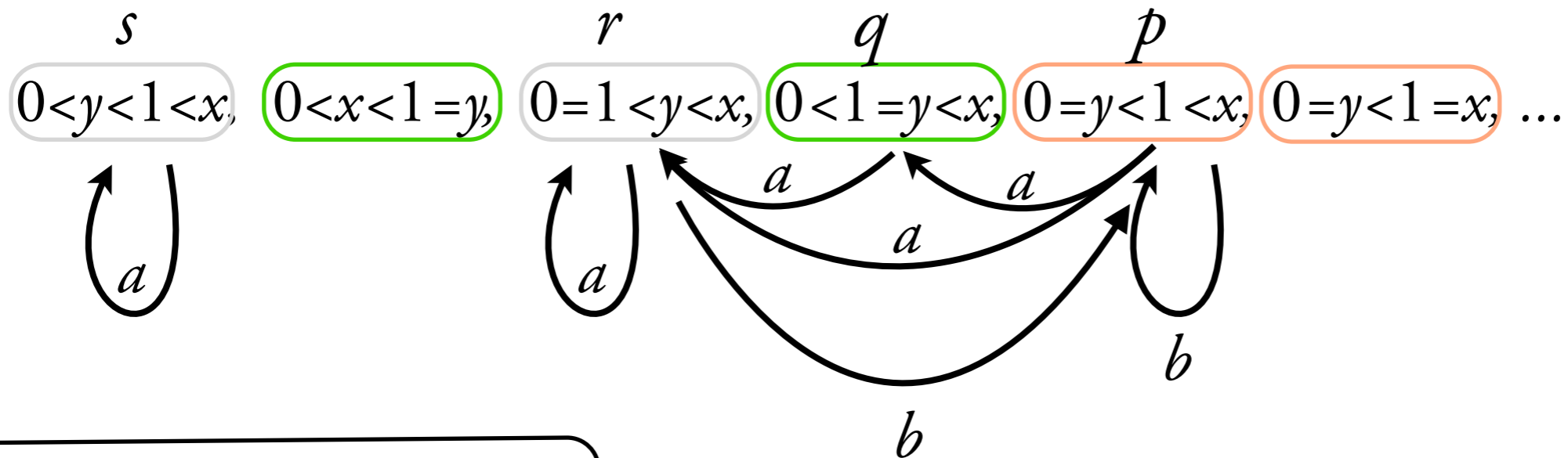
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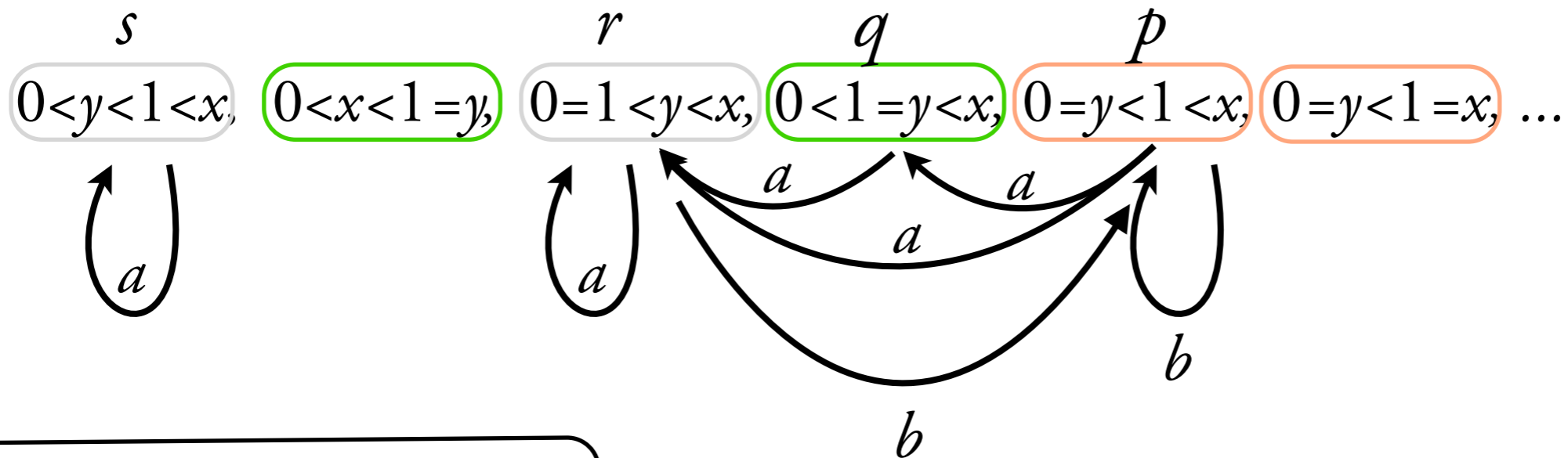
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>s</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>p</i>	<i>s</i>	<i>r</i>	<i>r</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5
<i>y</i>	0	1	2	4	0	4	0	1			

~~bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}~~

Discrete case

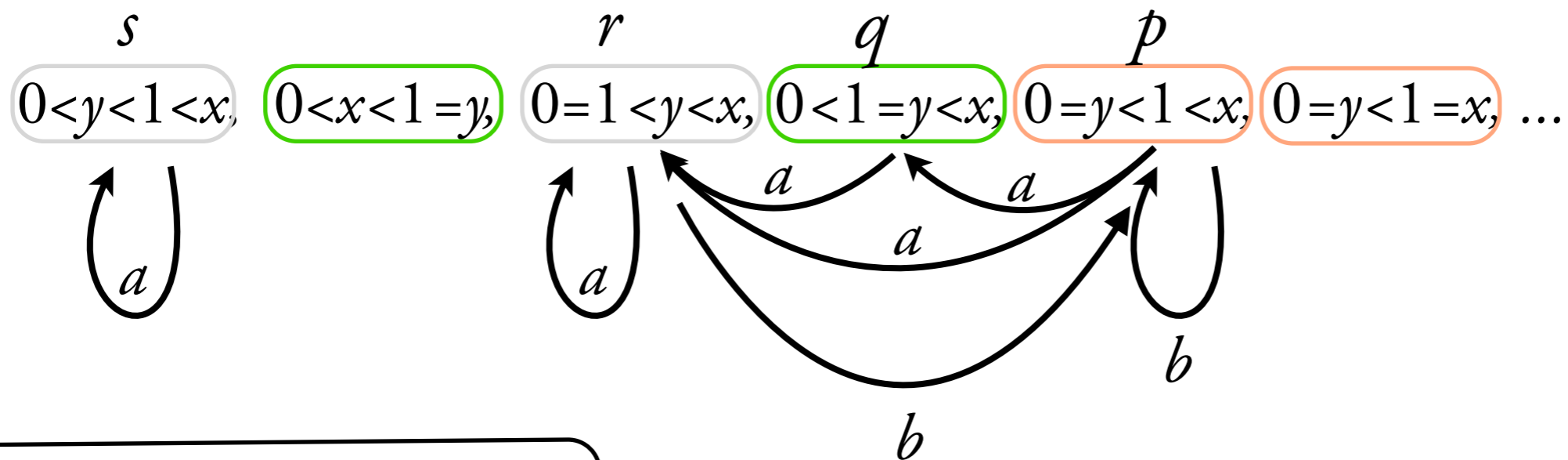
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>s</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>p</i>	<i>s</i>	<i>r</i>	<i>r</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	
<i>y</i>	0	1	2	4	0	4	0	1	4		

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Discrete case

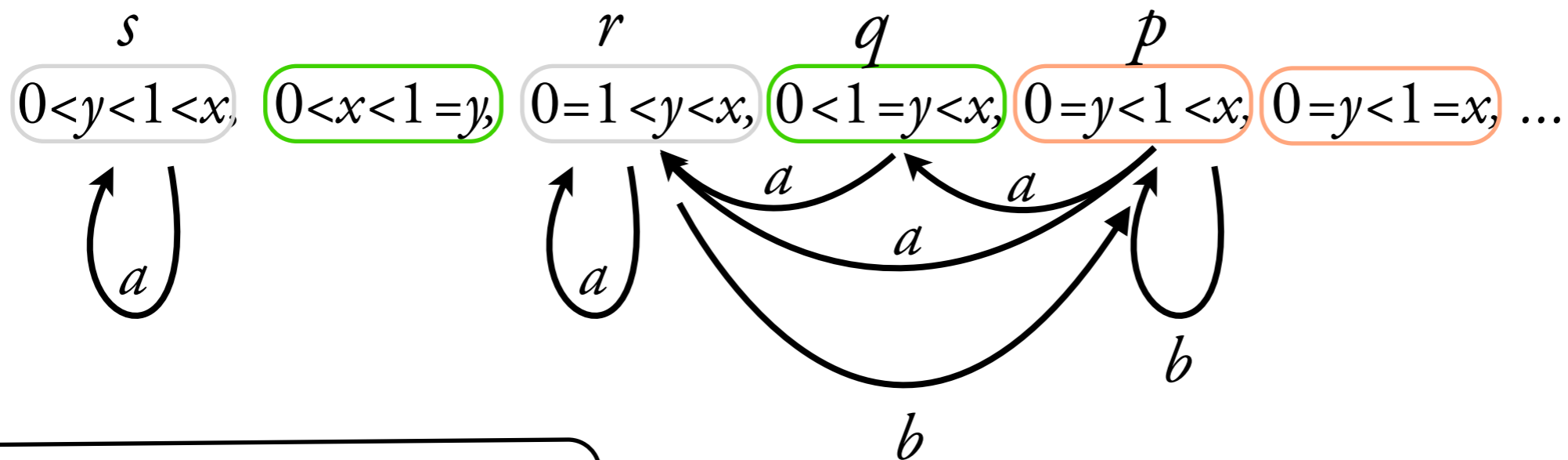
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>		
	<i>p</i>	<i>s</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>p</i>	<i>s</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	?		
<i>y</i>	0	1	2	4	0	4	0	1	4	?		

~~bisimulation \rightarrow runs of the region automaton correspond to runs of \mathcal{A}~~

Discrete case

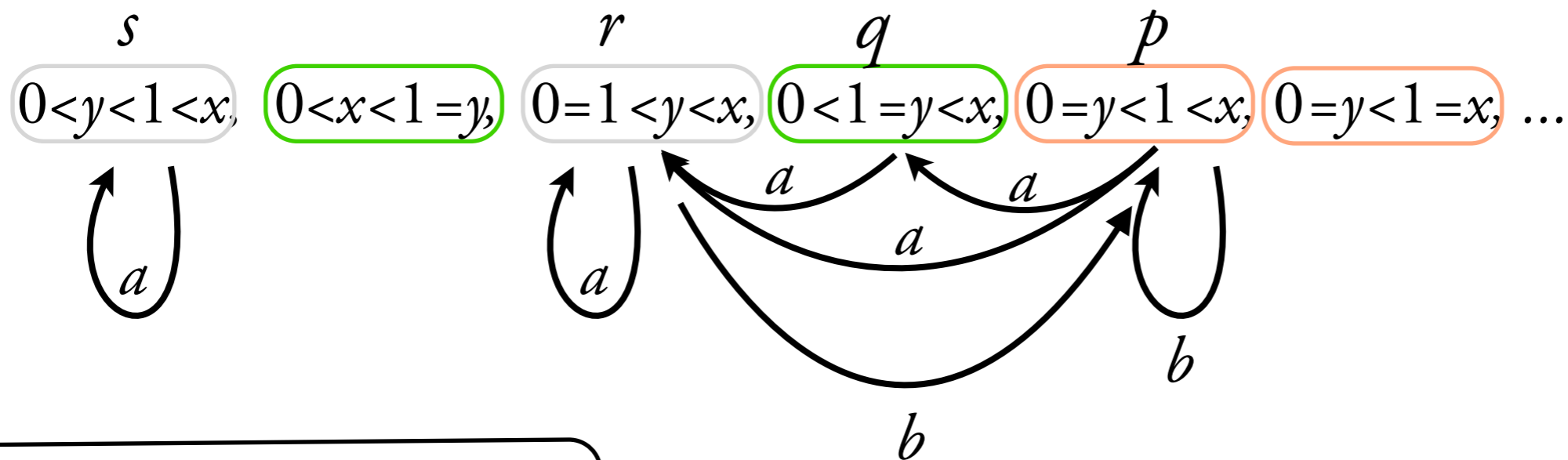
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	<i>a</i>	<i>a</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>b</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	
	<i>p</i>	<i>s</i>	<i>r</i>	<i>r</i>	<i>q</i>	<i>r</i>	<i>p</i>	<i>s</i>	<i>r</i>	<i>r</i>	<i>r</i>	<i>q</i>
<i>x</i>	5	5	5	5	5	5	5	5	5	5	5	?
<i>y</i>	0	1	2	4	0	4	0	1	4	4	4	?

but: in any cell and any n we can find configurations which are n -bisimilar
 → finite runs of the region automaton correspond to runs of \mathcal{A}

An elaboration of these ideas yields the results for infinite runs and database constraints and unary predicates in \mathcal{D} .

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Theorem. For any reasonable structure \mathcal{D} , emptiness of \mathcal{D} -automata is decidable and in PSPACE.

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Theorem. For any reasonable structure \mathcal{D} , emptiness of \mathcal{D} -automata is decidable and in PSPACE.

Corollary. Deciding LTL+data tests properties of \mathcal{D} -automata is PSPACE-complete.

Thank you!

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