Turing machines over infinite alphabets

Sławomir Lasota
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joint work with
Mikołaj Bojańczyk, Bartek Klin, Joanna Ochremiak, Szymon Toruńczyk

Dagstuhl seminar 14141, 2014.04.03

Turing machines over infinite alphabets
...but finite up to permutation (orbit-finite)

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computation in sets with atoms

computation in sets with atoms

sets with urelements

permutation models
[Mostowski '39]

sets with symmetry

nominal sets
[Gabbay, Pitts '99]

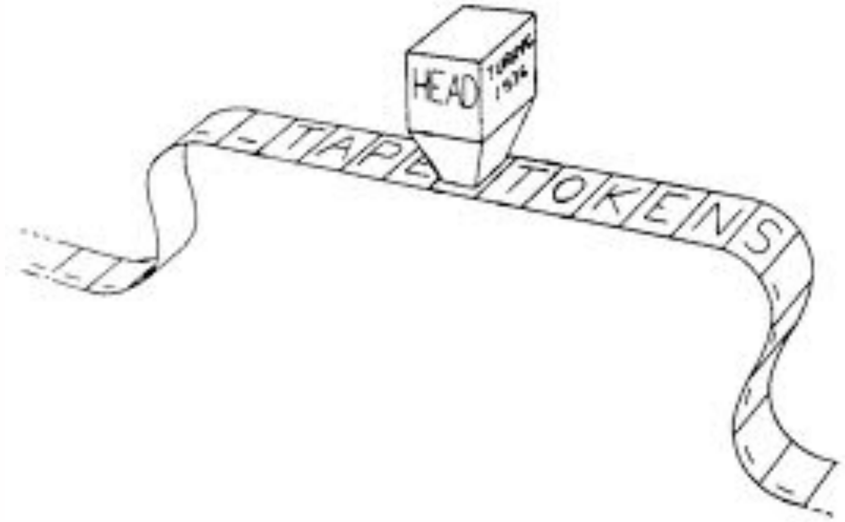
Fraenkel-Mostowski sets

named sets
[Pistore, Montanari '97]

nominal G-sets

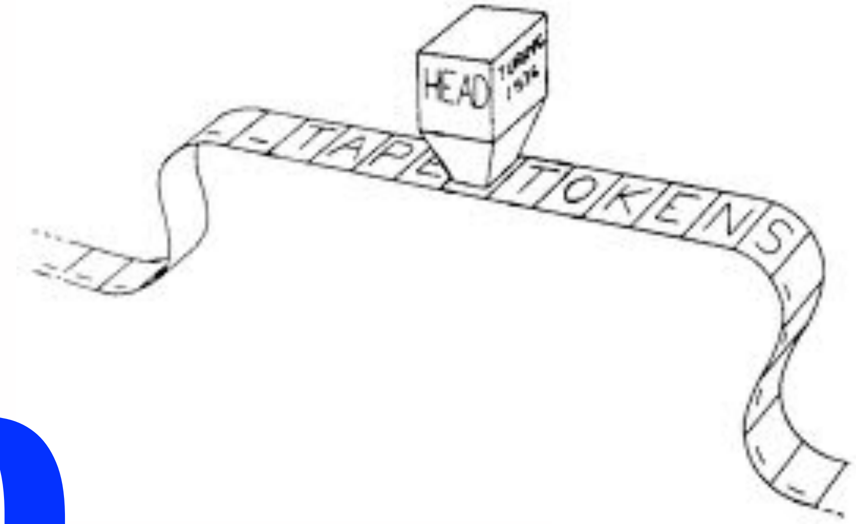
hereditarily finitely-supported sets

Turing machines
(any atoms)



- tape alphabet A
- states Q
- subset $\delta \subseteq Q \times A \times Q \times A \times \{\leftarrow, \rightarrow, \downarrow\}$
- subsets I, $F \subseteq Q$

Turing machines
(any atoms)



- tape alphabet A
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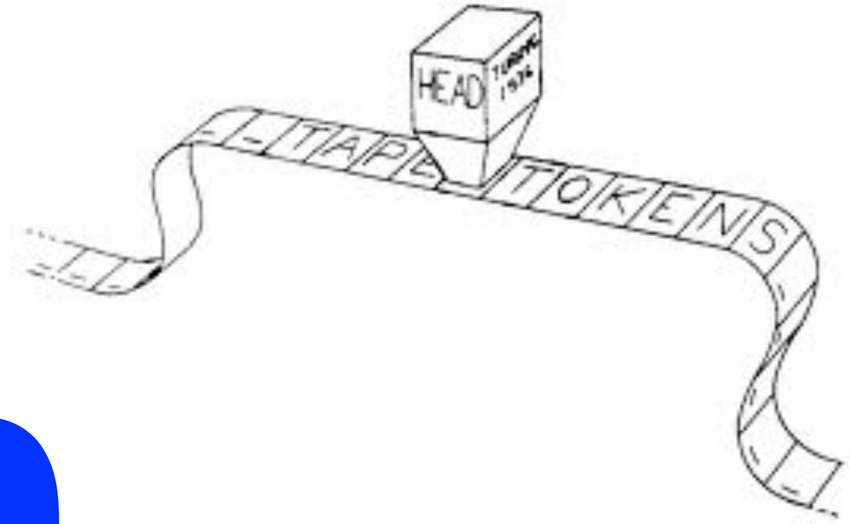
- subsets $I, F \subseteq Q$

} orbit-finite sets instead of finite ones

Turing machines
(any atoms)



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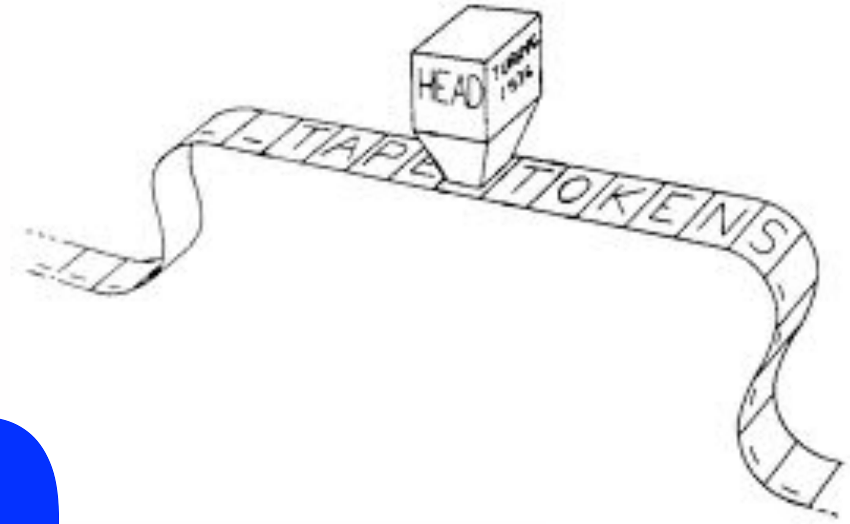
Configurations = $A^* \times Q \times A^*$

Turing machines
(any atoms)



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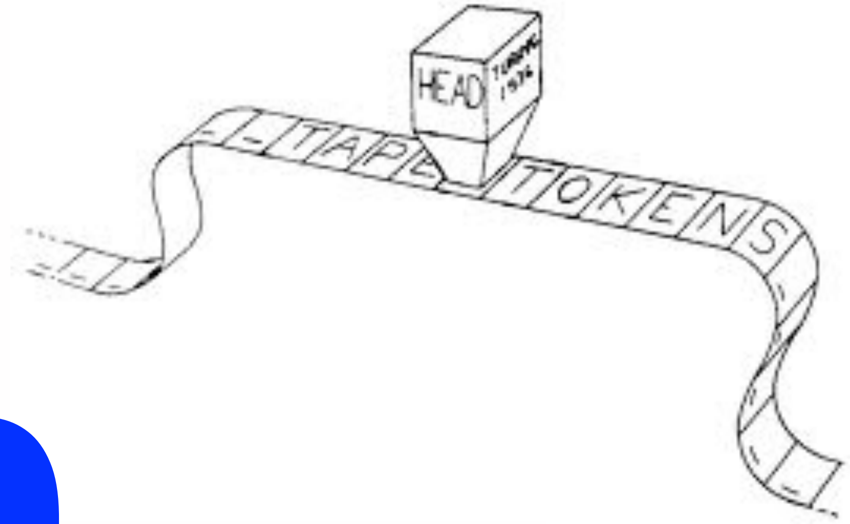
Configurations = $A^* \times Q \times A^*$

Deterministic machines:

- $\delta : Q \times A \rightarrow Q \times A \times \{\leftarrow, \rightarrow, \downarrow\}$

Turing machines
(any atoms)



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Configurations = $A^* \times Q \times A^*$

Deterministic machines:

- $\delta : Q \times A \rightarrow Q \times A \times \{\leftarrow, \rightarrow, \downarrow\}$

Consider only atoms (N, =) in this presentation

input alphabet: atoms

language:

tape alphabet:

states:

transitions:

input alphabet: atoms

atoms $\times$ (a finite set)

language:

tape alphabet:

states:

transitions:

input alphabet: atoms

language: "no atom appears twice":

$\{a_1 a_2 \ldots a_n \ : \ a_i \neq a_j \text{ when } i \neq j\}$

tape alphabet:

states:

transitions:

input alphabet: atoms

language: "no atom appears twice":

$\{a_1 a_2 \ldots a_n \ : \ a_i \neq a_j \text{ when } i \neq j\}$

tape alphabet: A = atoms ∪ {⊥}

states:

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tape alphabet: A = atoms ∪ {⊥}

states: Q = atoms ∪ {start, accept, ret}

transitions: δ : Q × A → Q × A × {←,→,↓}

δ(start, a) = (a, ⊥, →) a ∈ atoms

if in state start atom a is read from tape, goto state a, write ⊥ on tape, and move right

input alphabet: atoms

language: "no atom appears twice":

$\{a_1 a_2 \ldots a_n \ : \ a_i \neq a_j \text{ when } i \neq j\}$

tape alphabet: A = atoms ∪ {⊥}

states: Q = <u>atoms</u> ∪ {start, accept, ret}

transitions: δ : Q × A → Q × A × {←,→,↓}

δ(start, a) = (<u>a</u>, ⊥, →) a ∈ atoms

δ(<u>a</u>, b) = (<u>a</u>, b, →) a ≠ b, a, b ∈ atoms

if in state <u>a</u> atom b ≠ a is read from tape, stay in state <u>a</u>, write b on tape, and move right

input alphabet: atoms

language: "no atom appears twice":

$\{a_1 a_2 \ldots a_n \ : \ a_i \neq a_j \text{ when } i \neq j\}$

tape alphabet: A = atoms ∪ {⊥}

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δ(start, a) =	(a, ⊥, →)	a ∈ atoms
δ(a, b) =	(a, b, →)	a ≠ b, a, b ∈ atoms
δ(a, B) =	(ret, B, ←)	a ∈ atoms

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δ(<u>a</u>, B) =	(ret, B, ←)	a ∈ atoms
δ(ret, a) =	(ret, a, ←)	a ∈ atoms

input alphabet: atoms

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δ(a, B) = (ret, B, ←) a ∈ atoms

δ(ret, a) = (ret, a, ←) a ∈ atoms

δ(ret, ⊥) = (start, ⊥, →)

input alphabet: atoms

language: "no atom appears twice": $\{a_1 a_2 \ldots a_n \; : \; a_i \neq a_j \text{ when } i \neq j\}$

tape alphabet: $A = \text{atoms} \cup \{\bot\}$

states: $Q = \underline{\text{atoms}} \cup \{\text{start}, \text{accept}, \text{ret}\}$

transitions: $\delta : Q \times A \to Q \times A \times \{\leftarrow, \rightarrow, \downarrow\}$

$\delta(\text{start}, a) = (\underline{a}, \bot, \rightarrow)$ $a \in \text{atoms}$

$\delta(\underline{a}, b) = (\underline{a}, b, \rightarrow)$ $a \neq b, \; a, b \in \text{atoms}$

$\delta(\underline{a}, B) = (\text{ret}, B, \leftarrow)$ $a \in \text{atoms}$

$\delta(\text{ret}, a) = (\text{ret}, a, \leftarrow)$ $a \in \text{atoms}$

$\delta(\text{ret}, \bot) = (\text{start}, \bot, \rightarrow)$

$\delta(\text{start}, B) = (\text{accept}, B, \rightarrow)$

Do TMs determinize?

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Example alphabets:

- atoms
- ordered pairs of atoms
- unordered pairs of atoms
- unordered pairs of ordered pairs of atoms
- ordered triples of pairs of atoms modulo even number of flips

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Example alphabets:

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- ordered triples of pairs of atoms modulo even number of flips

} standard (first four)

Do TMs determinize?

Example alphabets:

- atoms
- ordered pairs of atoms
- unordered pairs of atoms
- unordered pairs of ordered pairs of atoms

} standard

- ordered triples of pairs of atoms modulo even number of flips

non-standard

alphabet: atoms

alphabet: atoms

a b a e d d c d f d g y h e u s e d f e r g f f e d s

alphabet: atoms

guess an atom
different than h

a b a e d d c d f d g y h e u s e d f e r g f f e d s

alphabet: atoms

guess an atom different than h

a b a e d d c d f d g y h e u s e d f e r g f f e d s

de-atomization:

alphabet: atoms

guess an atom
different than h

a b a e d d c d f d g y h e u s e d f e r g f f e d s

de-atomization:

- replacing atoms with binary encodings

a	1
b	101
e	1001
d	10001
c	...

alphabet: atoms

guess an atom different than h

a b a e d d c d f d g y h e u s e d f e r g f f e d s

de-atomization:

- replacing atoms with binary encodings
- atom-less simulation of computation of a machine

a	1
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d	10001
c	...

alphabet: atoms

guess an atom different than h

a b a e d d c d f d g y h e u s e d f e r g f f e d s

de-atomization:

- replacing atoms with binary encodings
- atom-less simulation of computation of a machine

a	1
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Fact: TMs over this alphabet do determinize

alphabet: ordered pairs of atoms

$$(a, b) \in \text{atoms}^2 \qquad a \neq b$$

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- input word represents a directed graph

alphabet: ordered pairs of atoms

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- input word represents a directed graph
- nodes (atoms) can be computed using projections

$$(a, b) \mapsto a \qquad\qquad (a, b) \mapsto b$$

  and stored on the tape

- then any decidable property of directed graphs can be decided deterministically

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$$\{a, b\} \in \mathcal{P}_2(\text{atoms})$$

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alphabet: unordered pairs of atoms

$\{a, b\} \in \mathcal{P}_2(\text{atoms})$

- input word represents an undirected graph
- can nodes (atoms) be computed?

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alphabet: unordered pairs of atoms

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- input word represents an undirected graph
- can nodes (atoms) be computed?

$$\{a, b\} \mapsto a$$

$$(\{a, b\}, \{b, c\}) \mapsto b$$

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alphabet: unordered pairs of atoms

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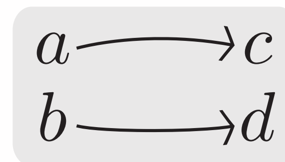
Fact: TMs over this alphabet do determinize

alphabet: unordered pairs of ordered pairs of atoms

$$\{(a,c),(b,d)\}$$

alphabet: unordered pairs of ordered pairs of atoms

$$\{(a, c), (b, d)\}$$



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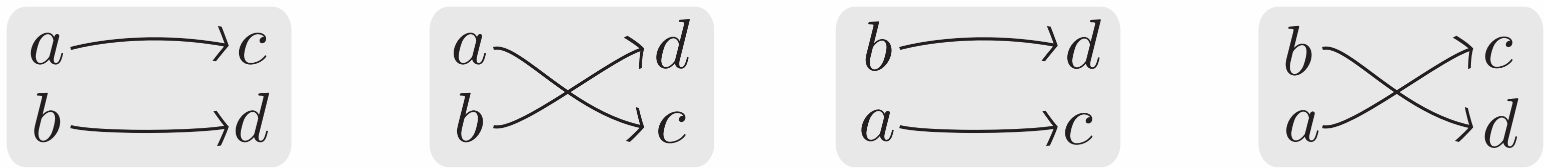

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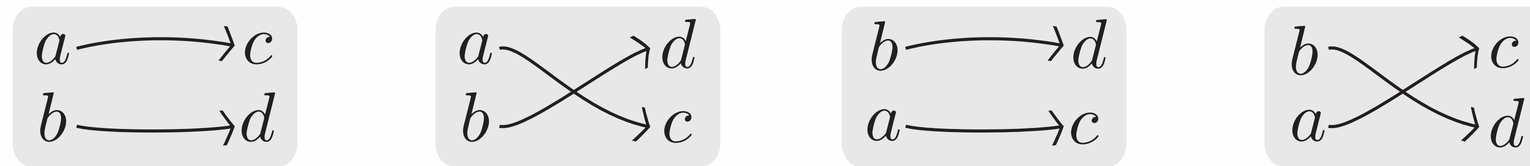
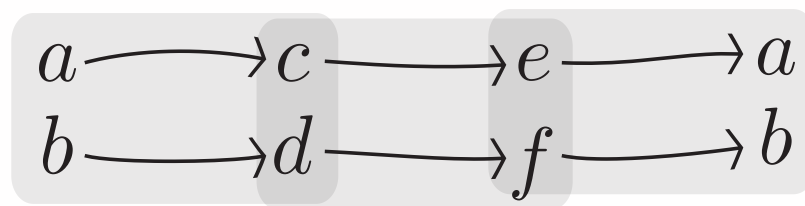


simple strips:



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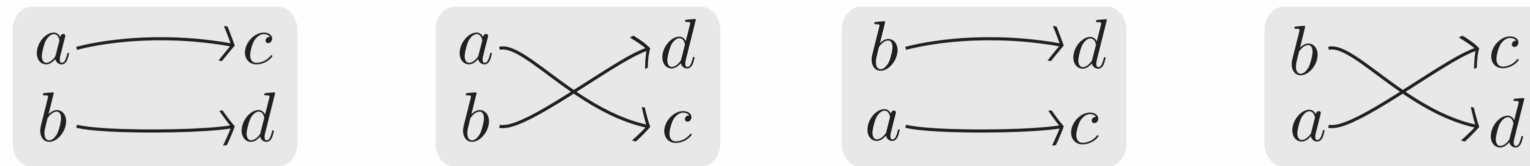

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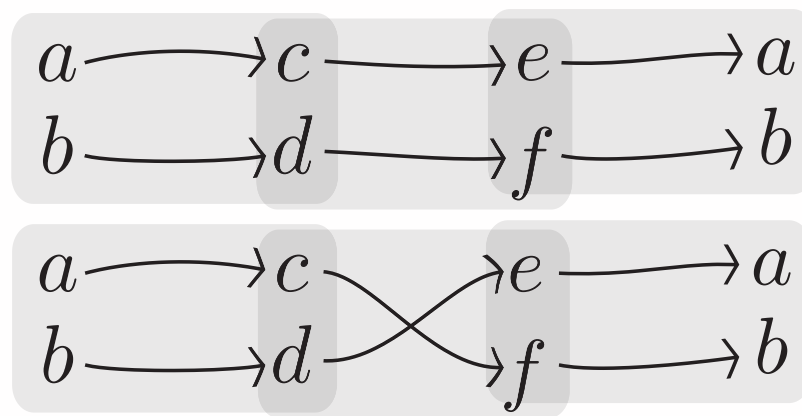

is not a simple strip

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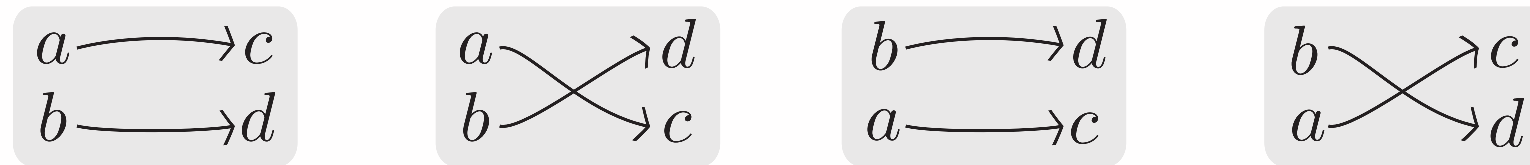

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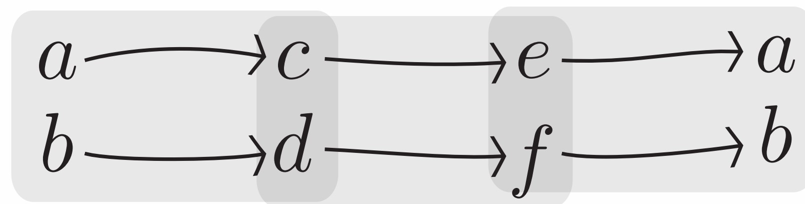

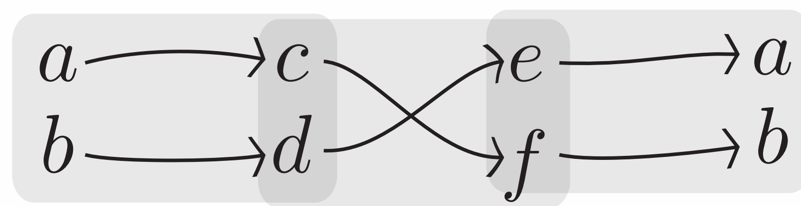

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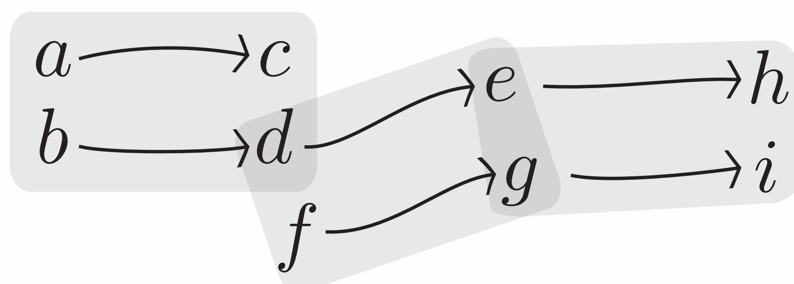
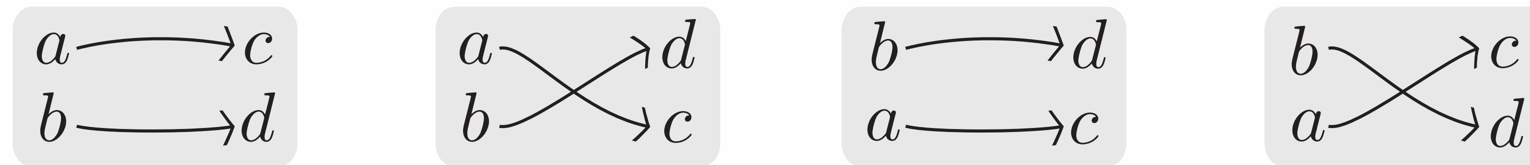


neither

alphabet: unordered pairs of ordered pairs of atoms

$$\{(a,c),(b,d)\}$$



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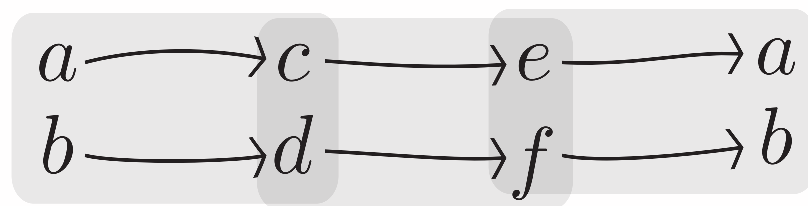

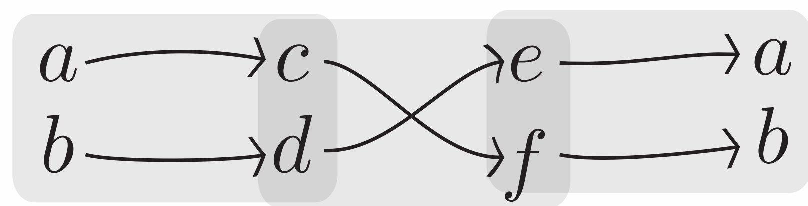

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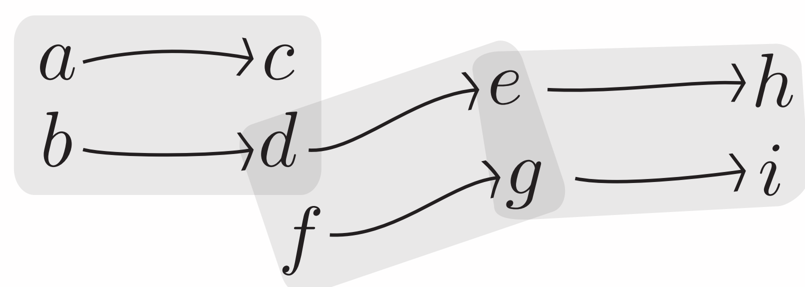
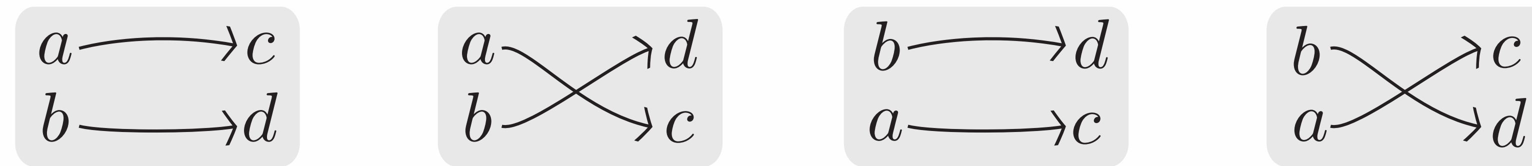


neither

Are simple strips recognized by a nondeterministic TM?

alphabet: unordered pairs of ordered pairs of atoms

$$\{(a,c),(b,d)\}$$



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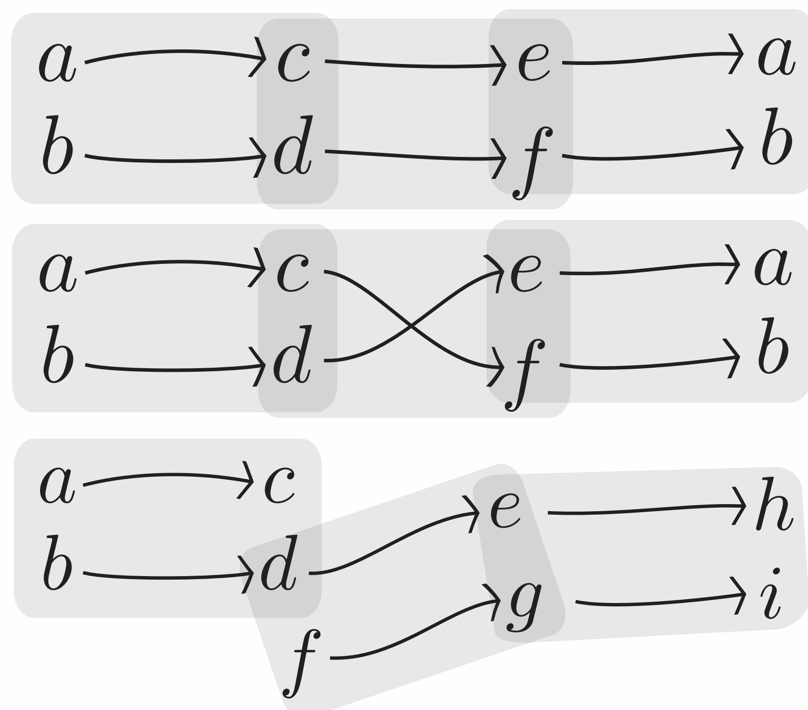

is not a simple strip

neither

Are simple strips recognized by a deterministic TM?

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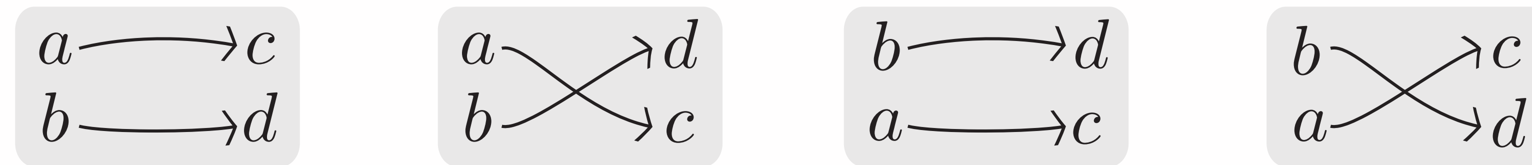

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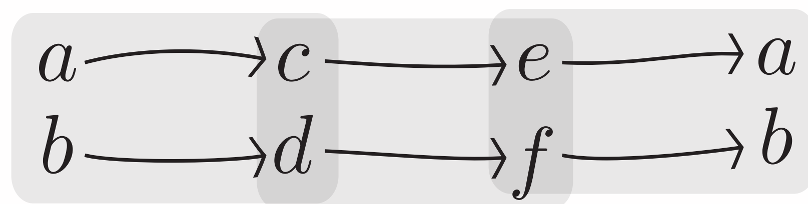

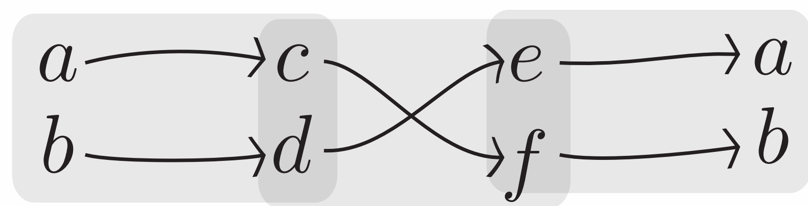

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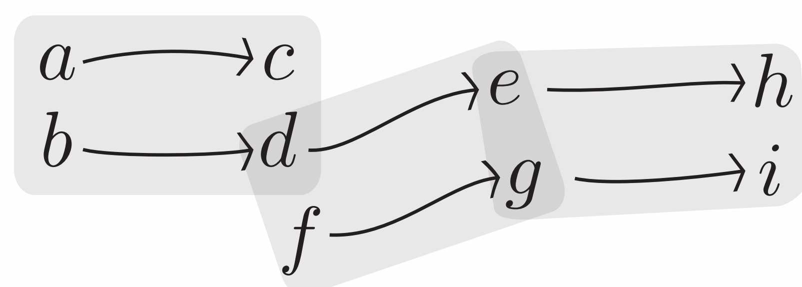

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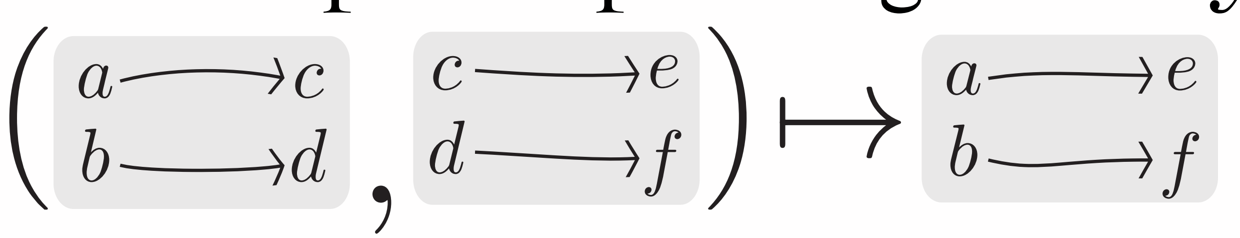

alphabet: unordered pairs of ordered pairs of atoms

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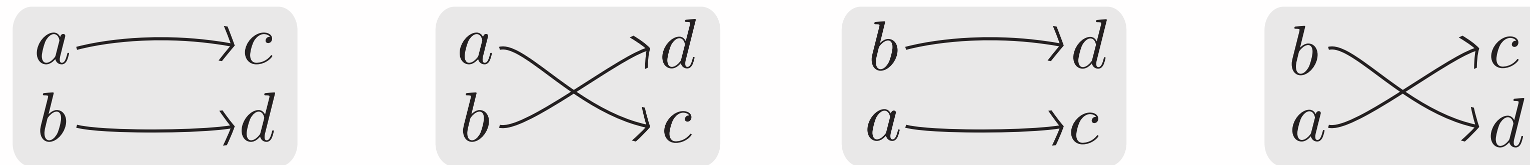

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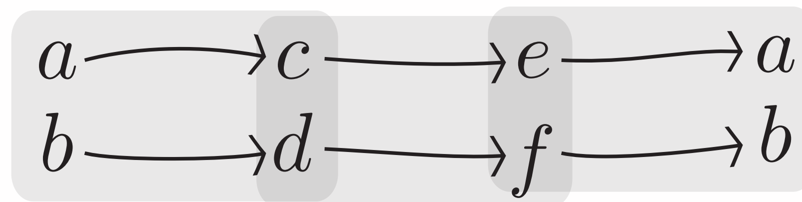

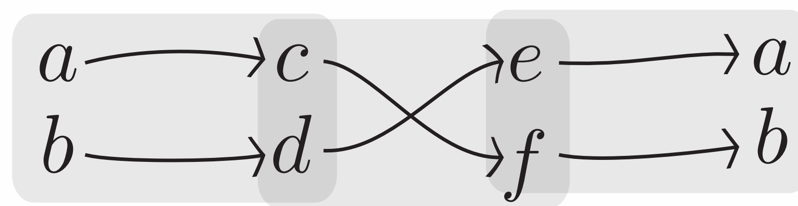

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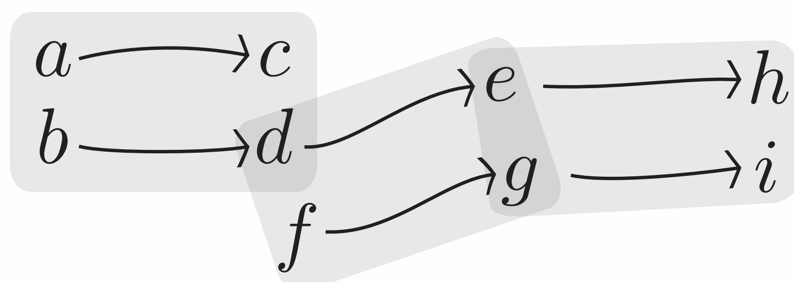
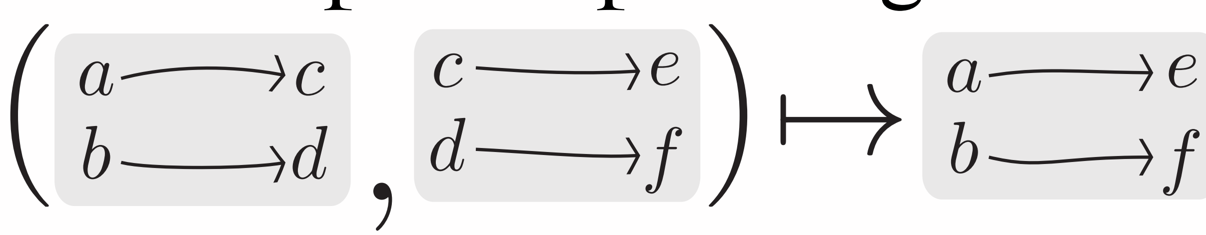
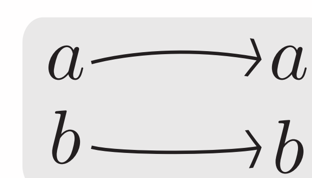


neither

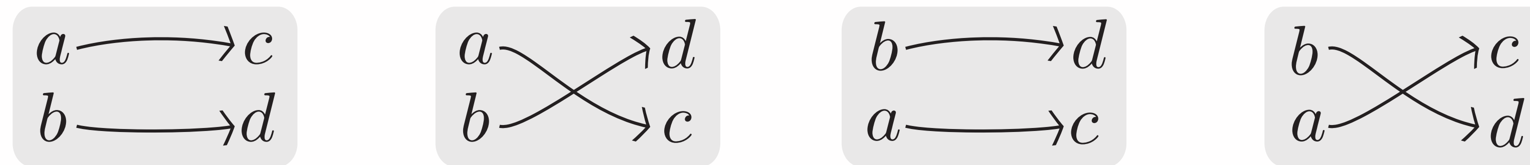
Are simple strips recognized by a deterministic TM?





alphabet: unordered pairs of ordered pairs of atoms

$$\{(a,c),(b,d)\}$$



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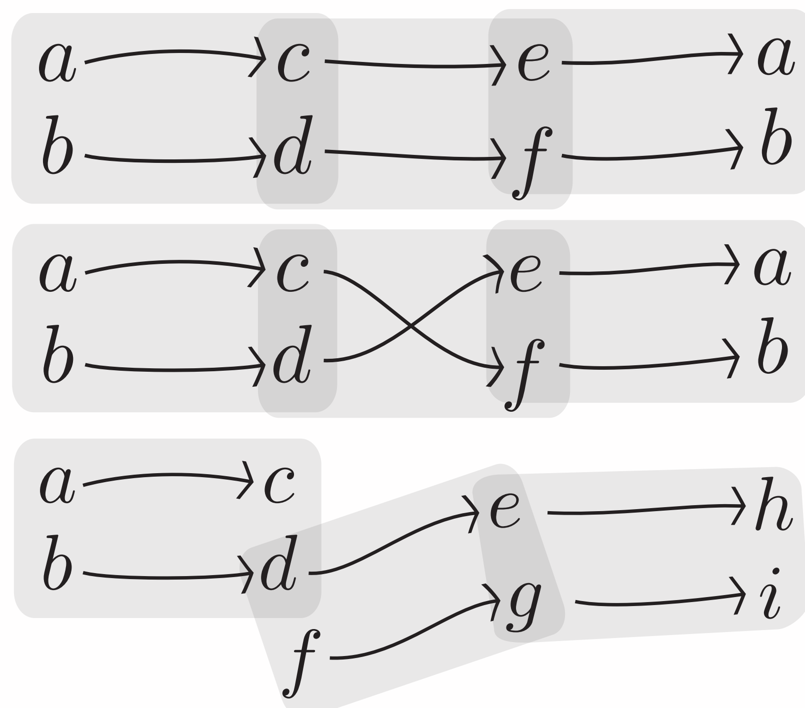

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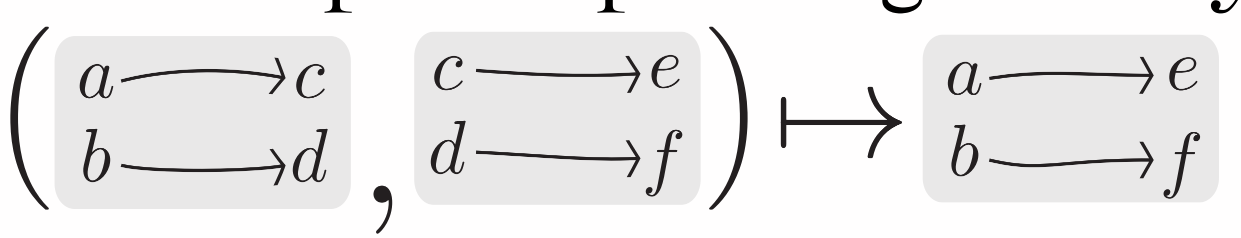


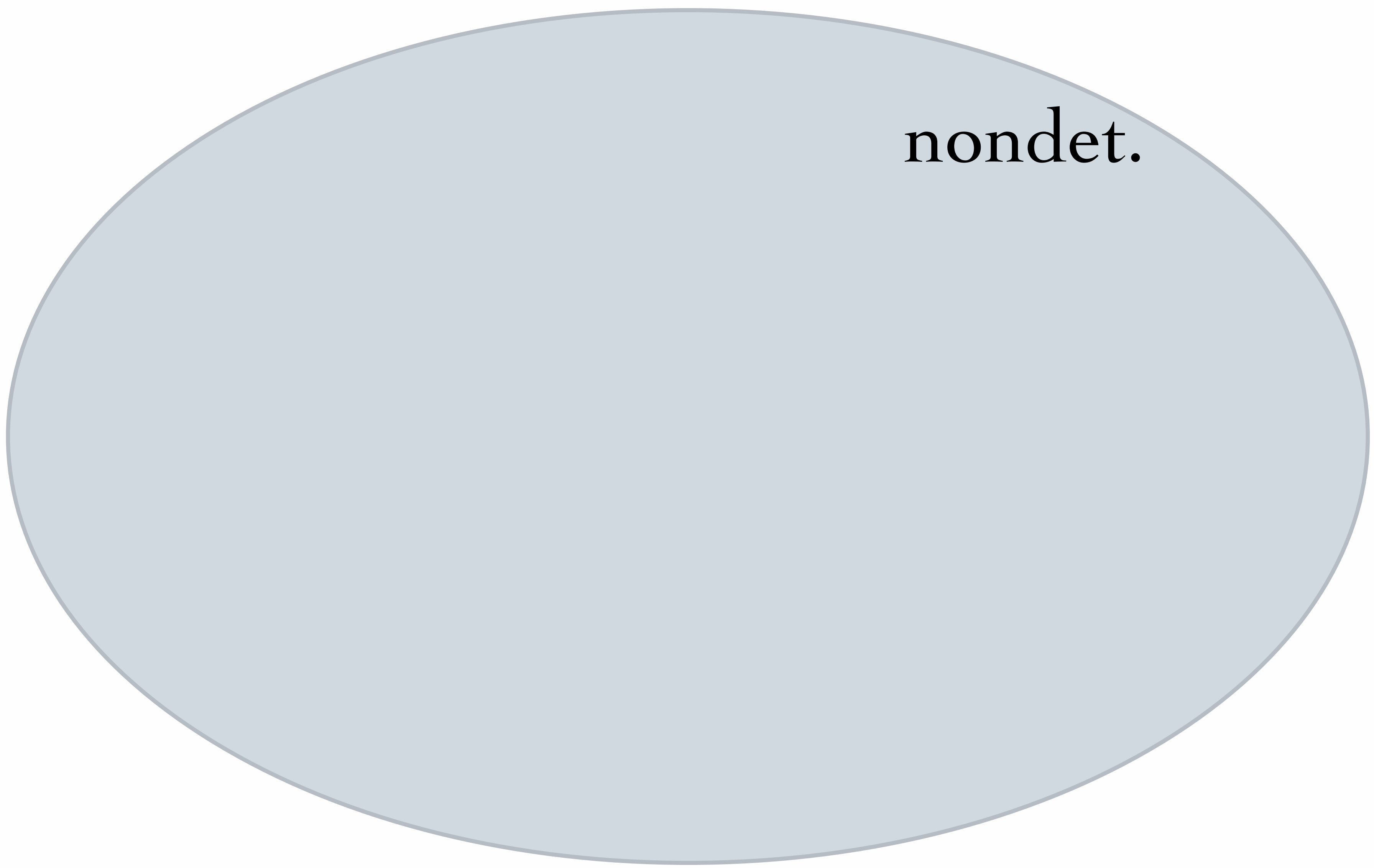
Fact: TMs over this alphabet do determinize

Theorem:

There is an alphabet A, and a language over A that is in NP but is not recognizable by a deterministic TM.

Theorem:

There is an alphabet A, and a language over A that is in NP but is not recognizable by a deterministic TM.



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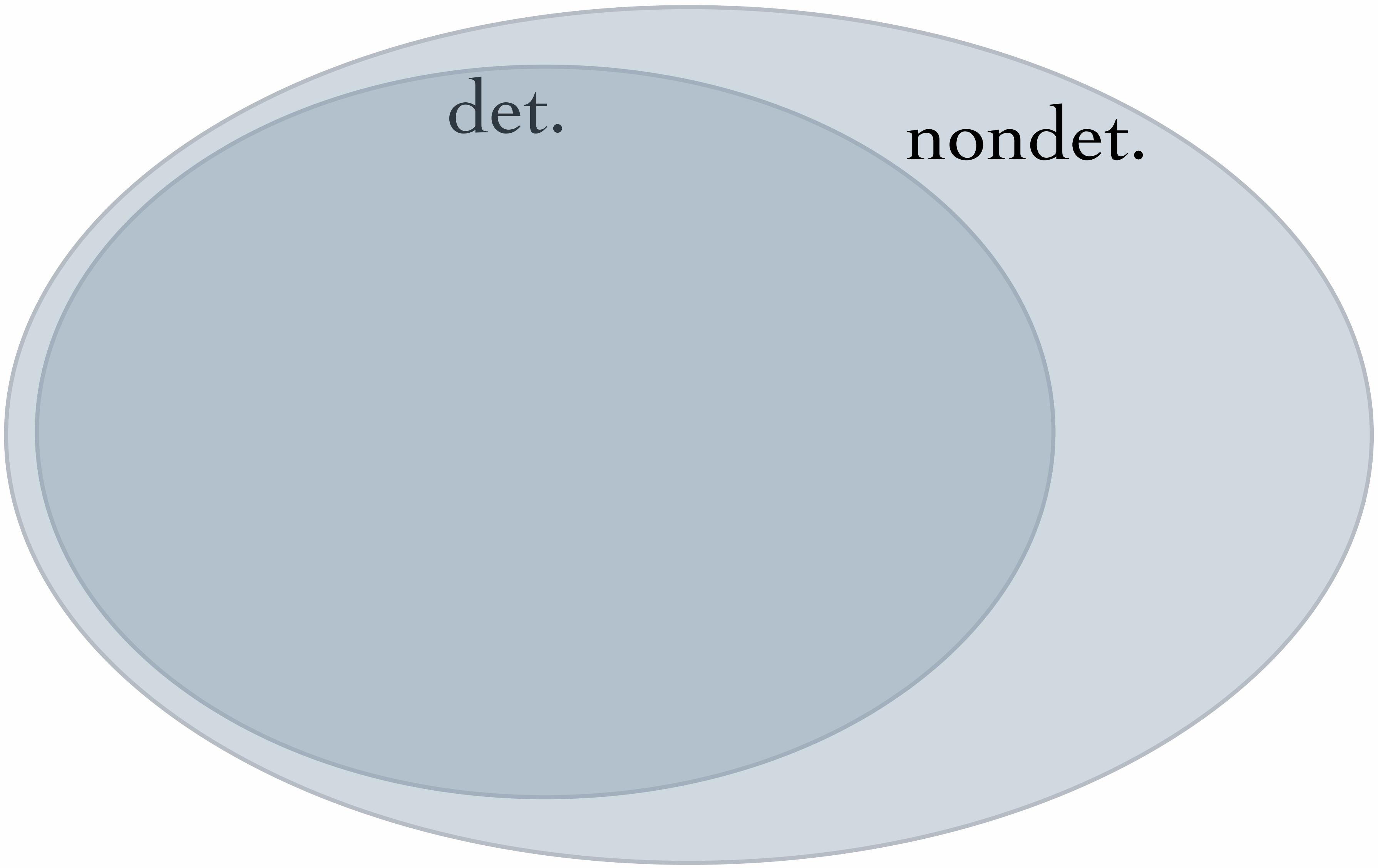

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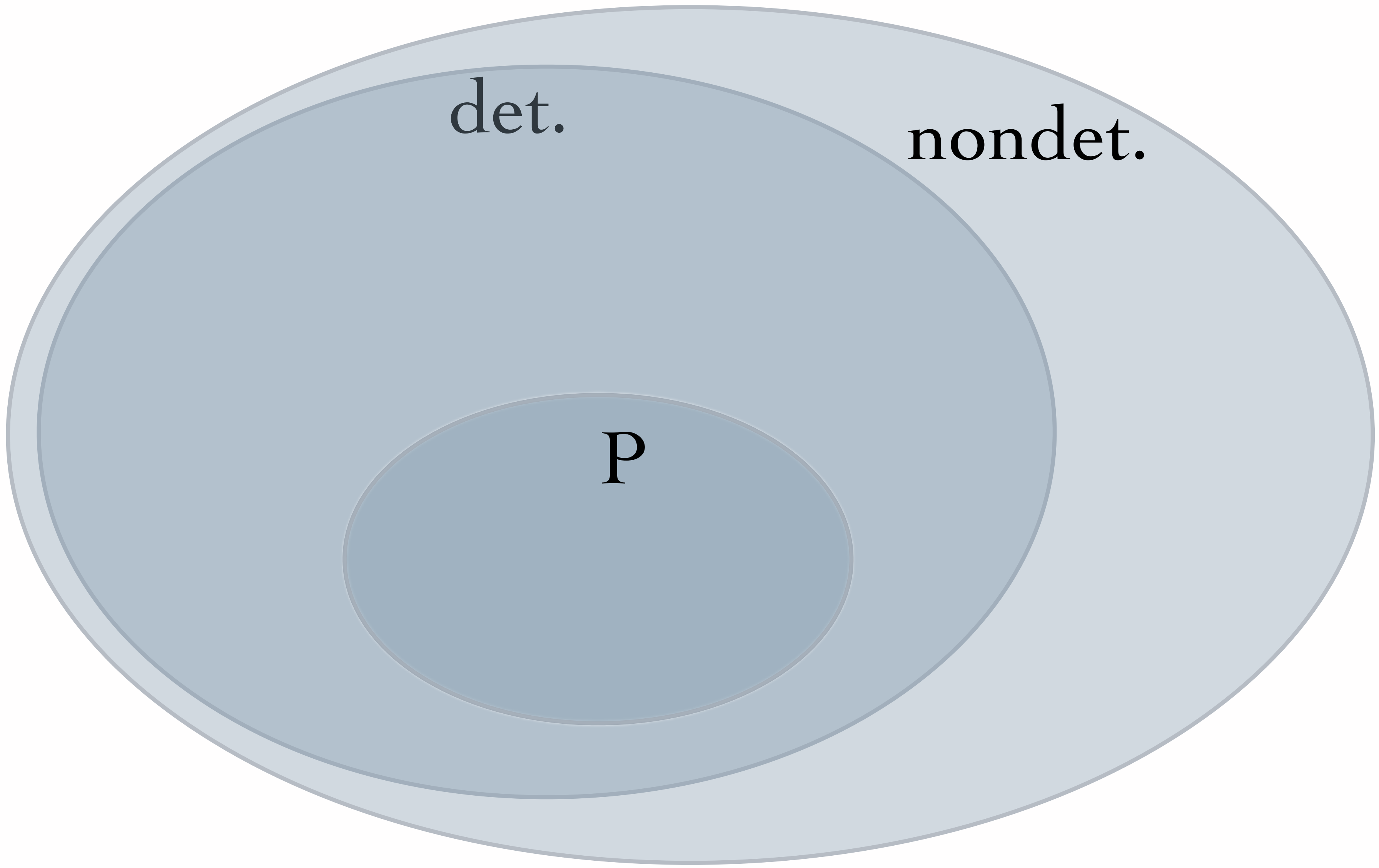

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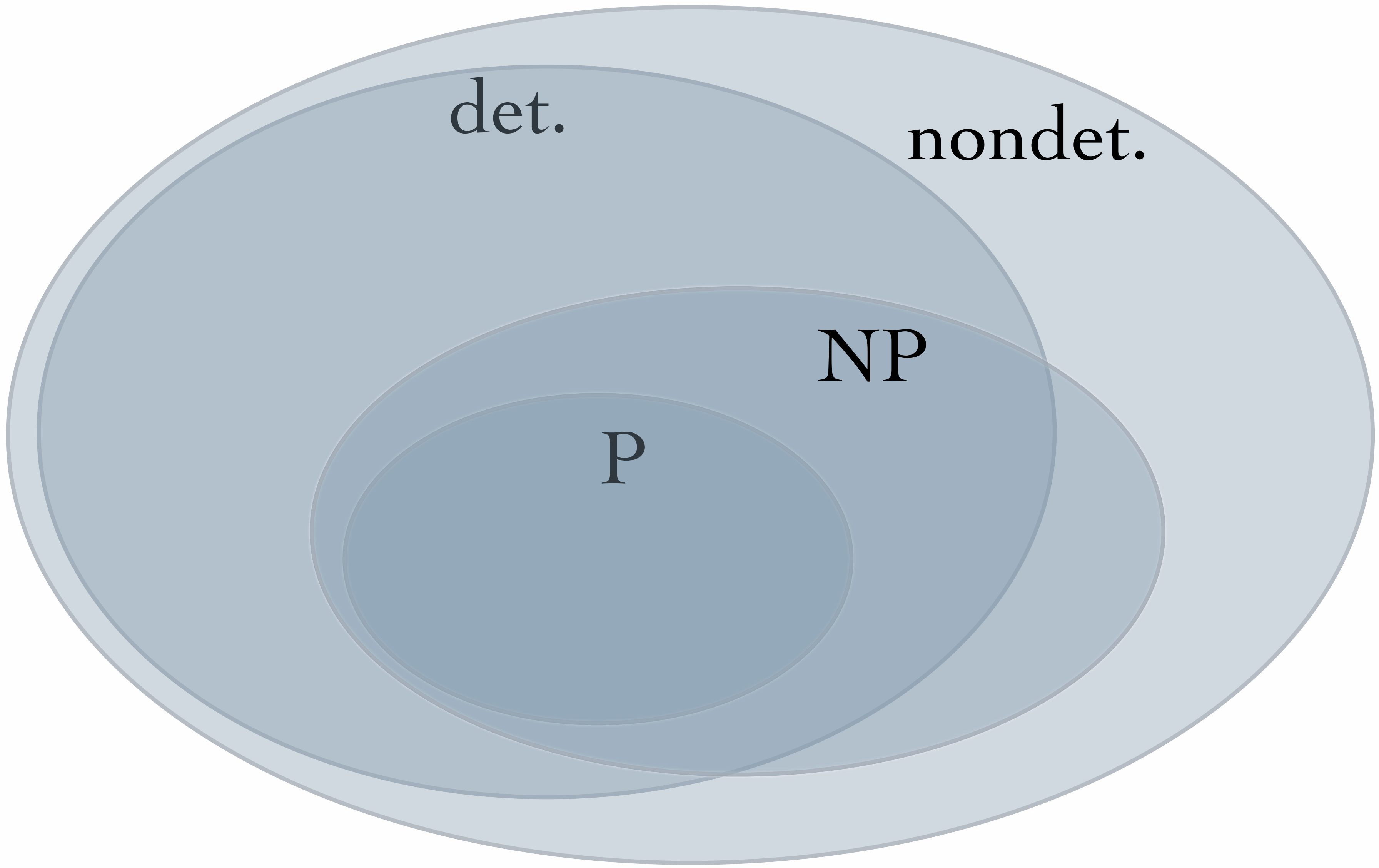

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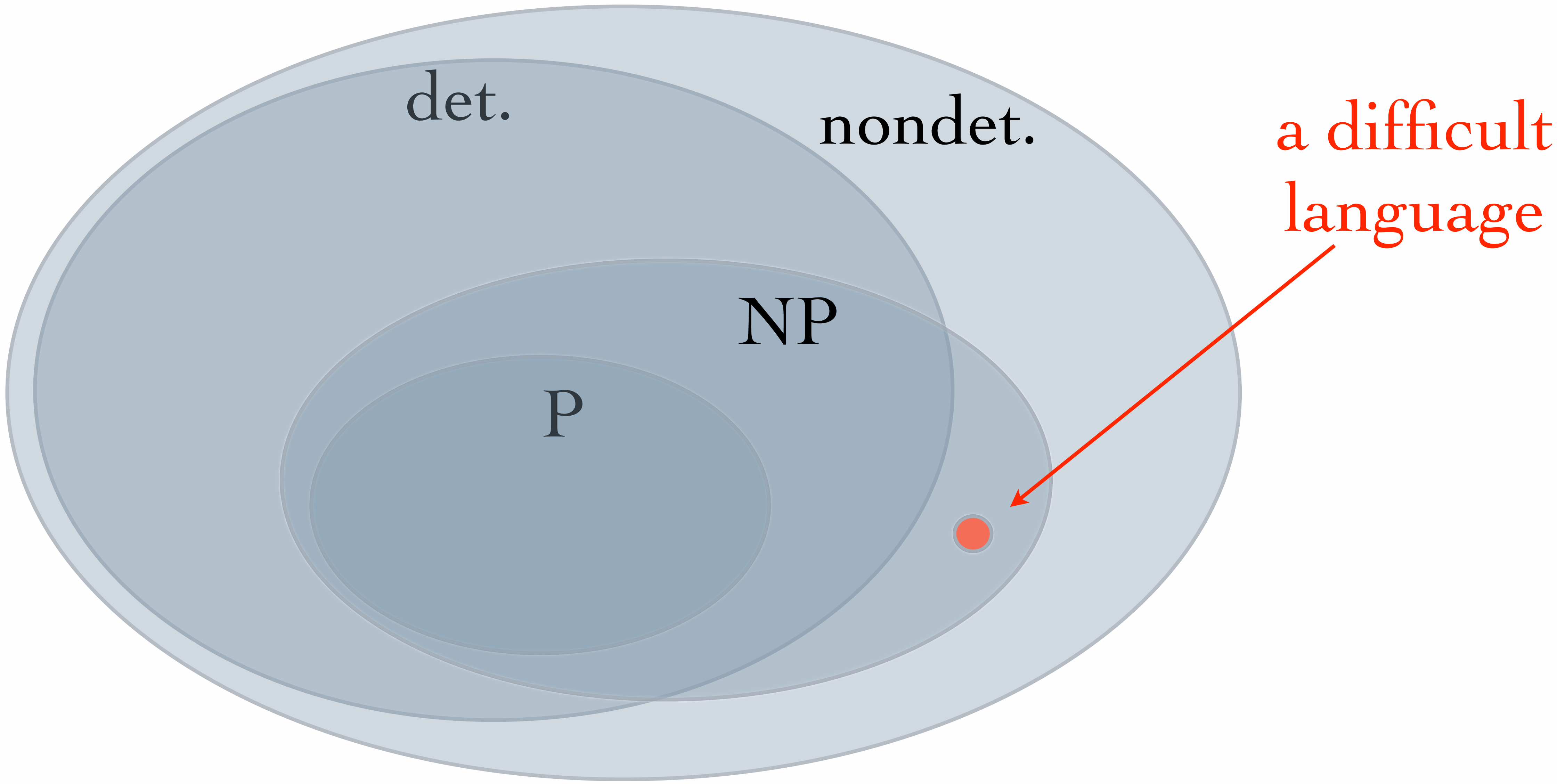

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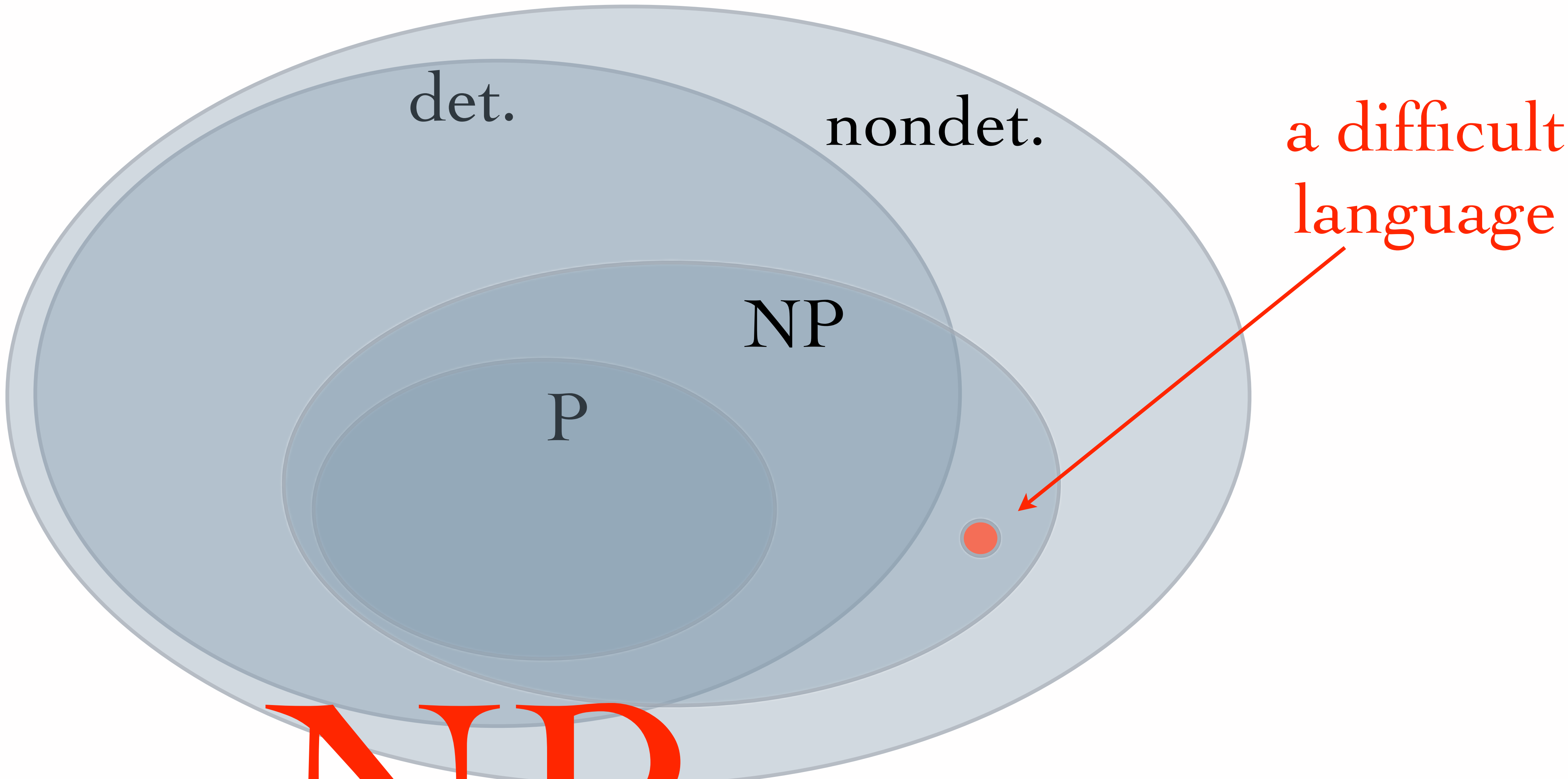

P ≠ NP

alphabet: ordered triples of ordered pairs of atoms modulo even number of flips

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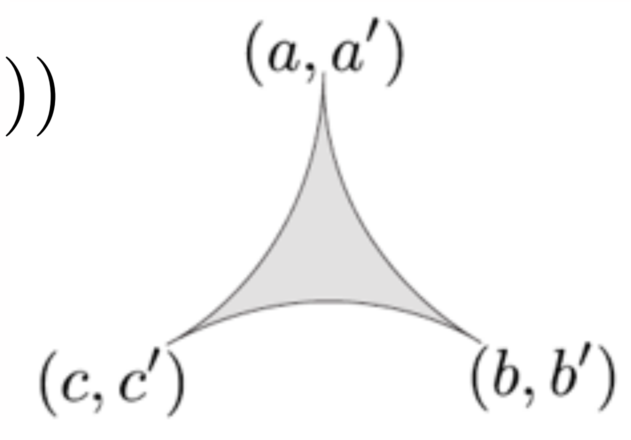
Triangle = $((a, a'), (b, b'), (c, c'))$



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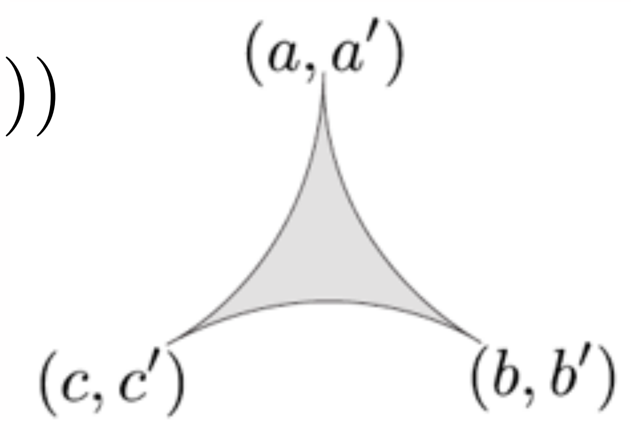

Let triangles with same pairs be equivalent if exactly two pairs are flipped:

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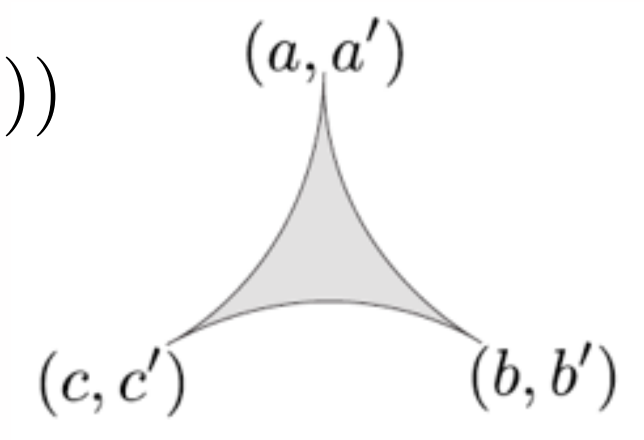

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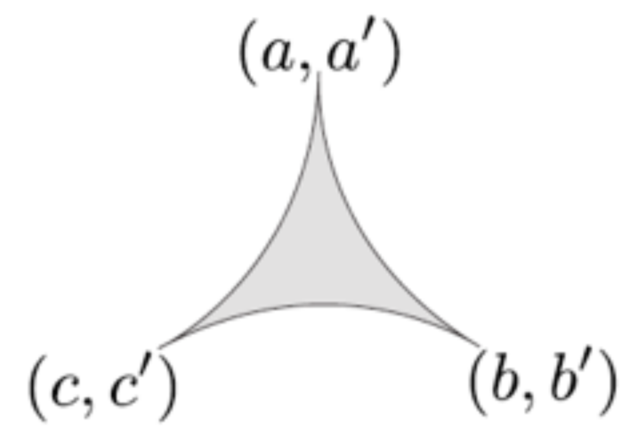
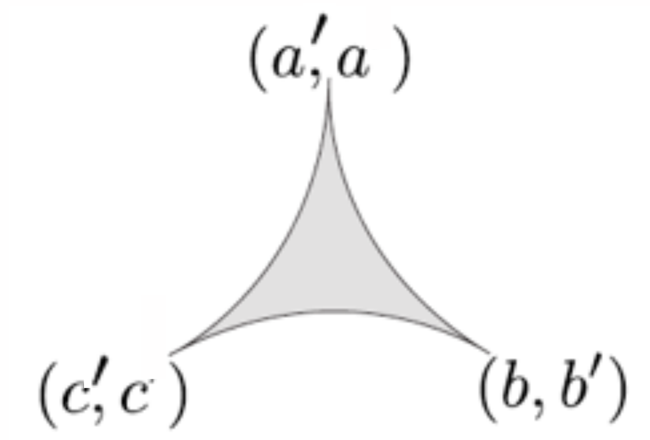


$\approx$



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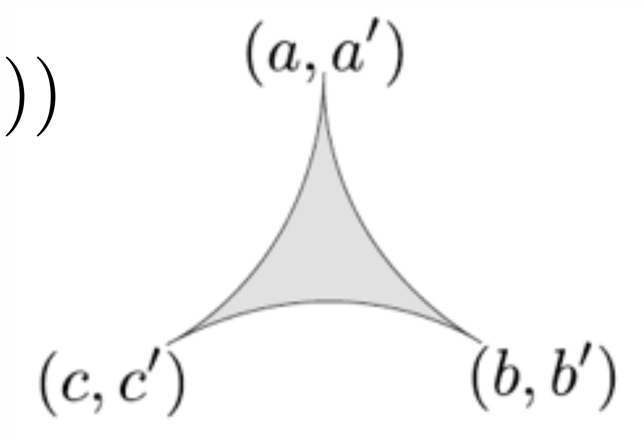

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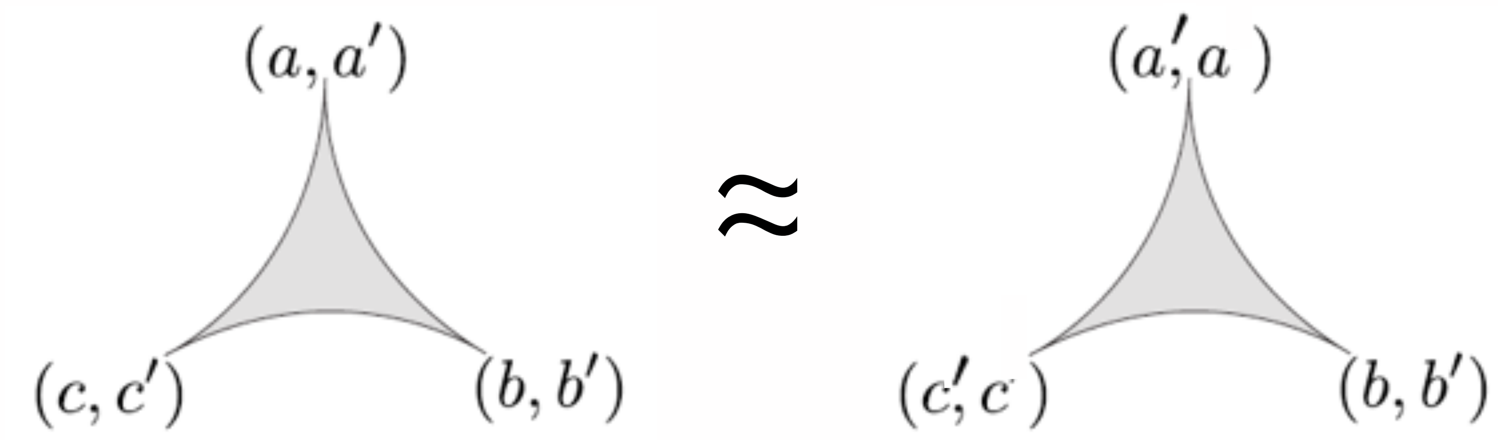
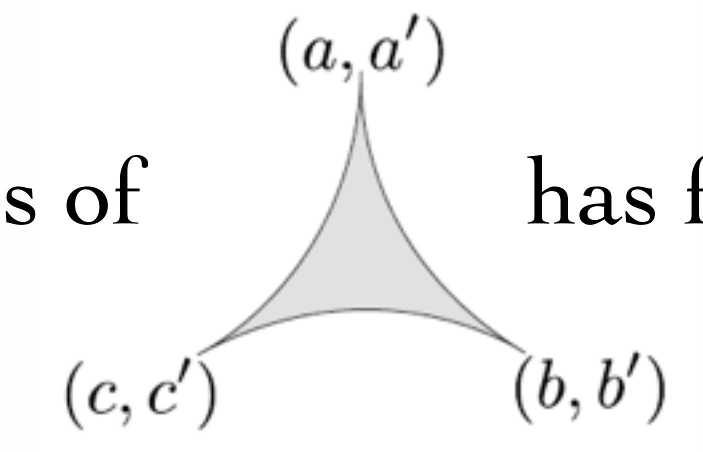


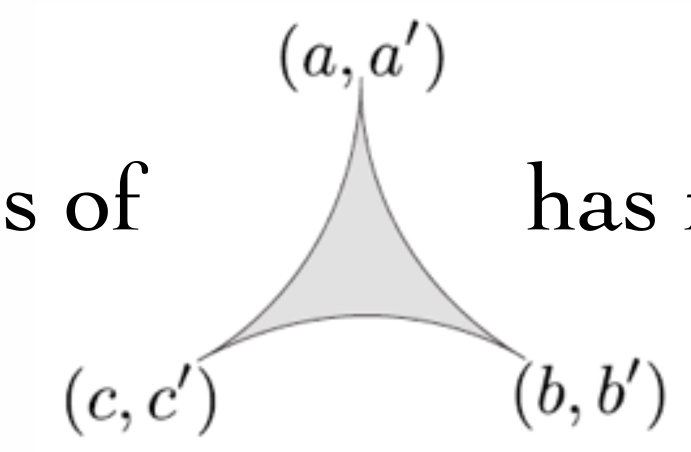
Alphabet = equivalence classes of triangles

alphabet: ordered triples of ordered pairs of atoms modulo even number of flips

equivalence class of  has four elements:

alphabet: ordered triples of ordered pairs of atoms modulo even number of flips

equivalence class of



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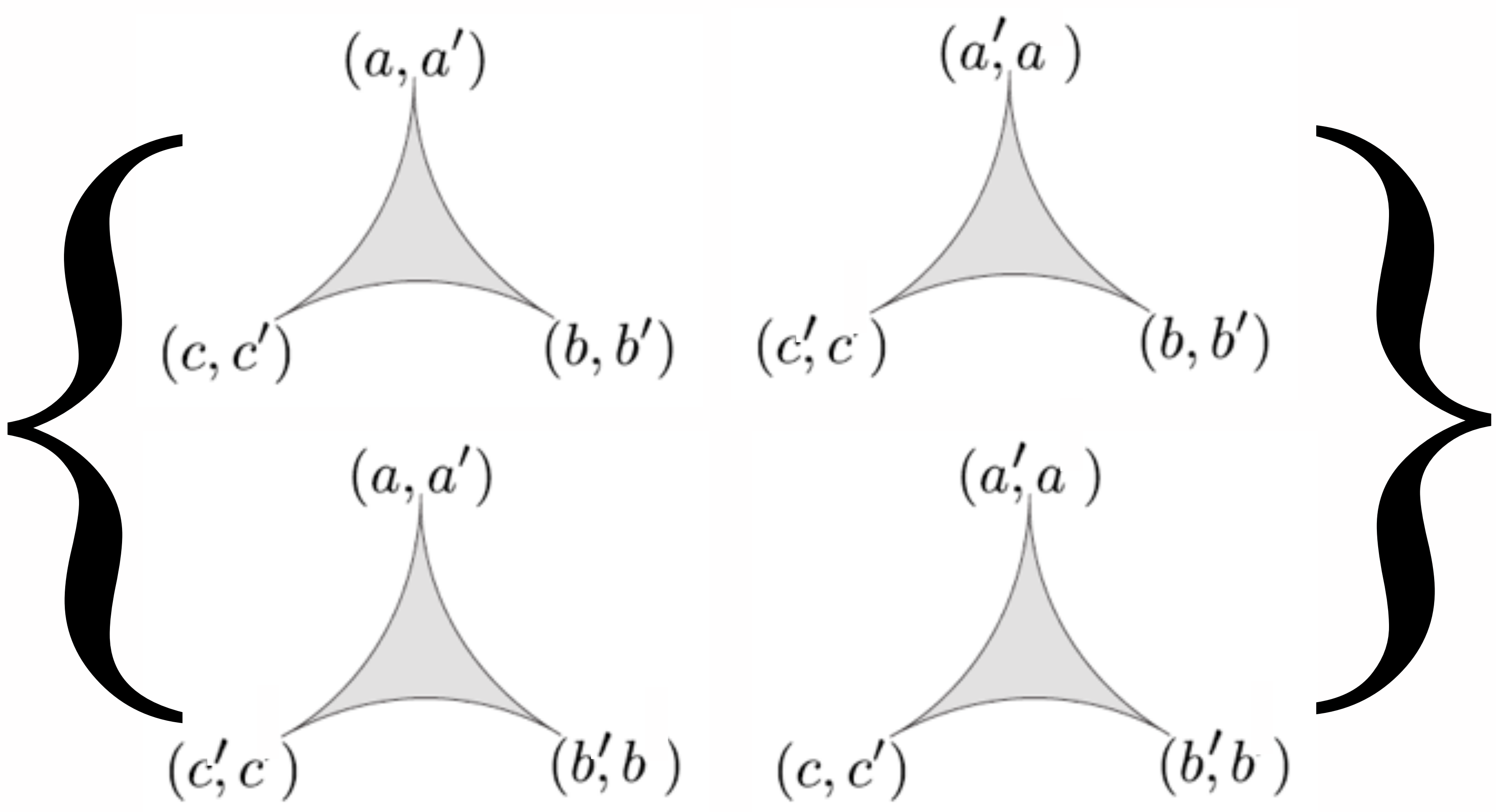

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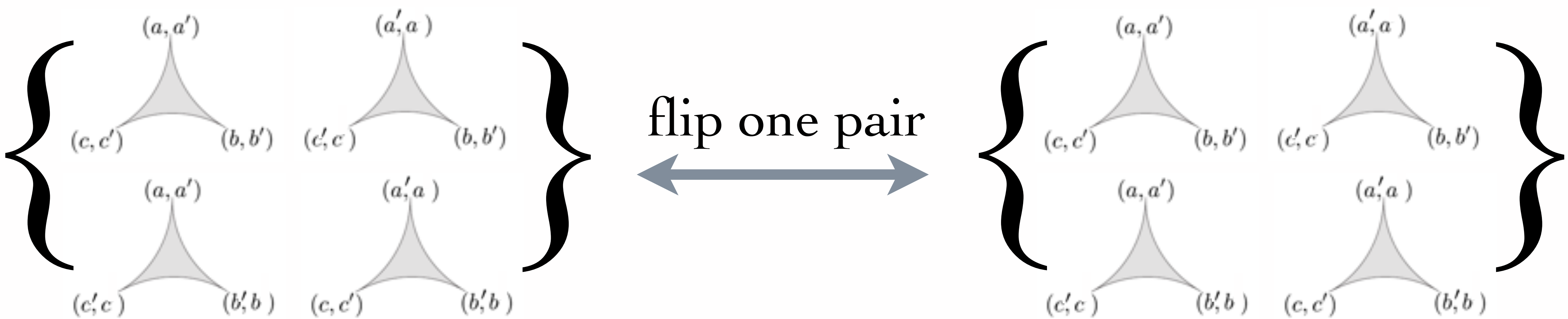
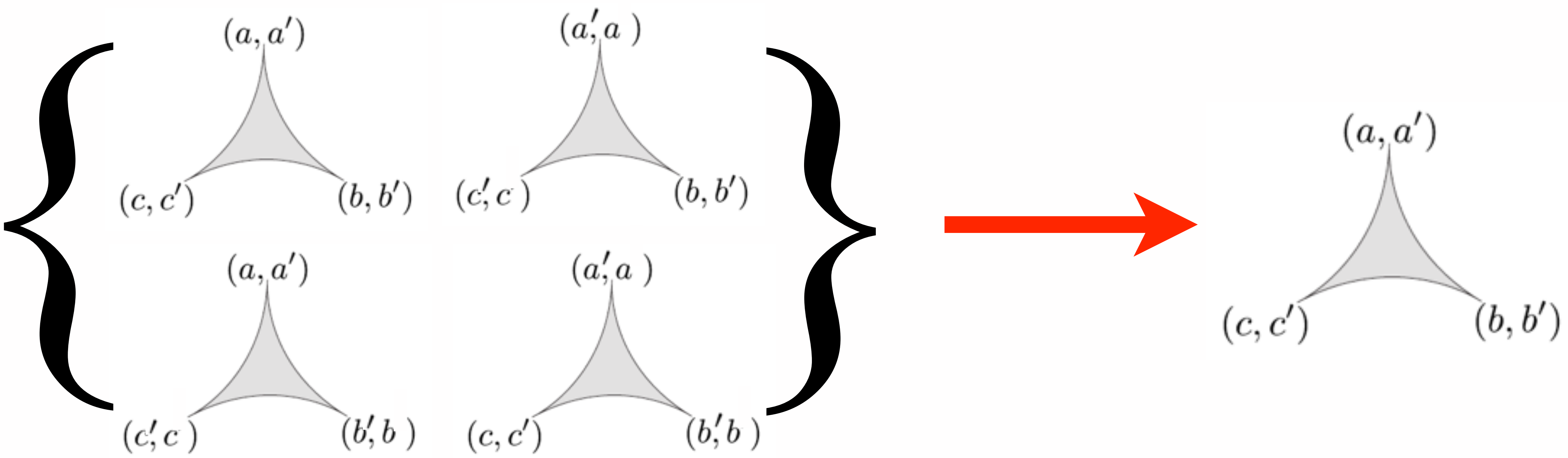
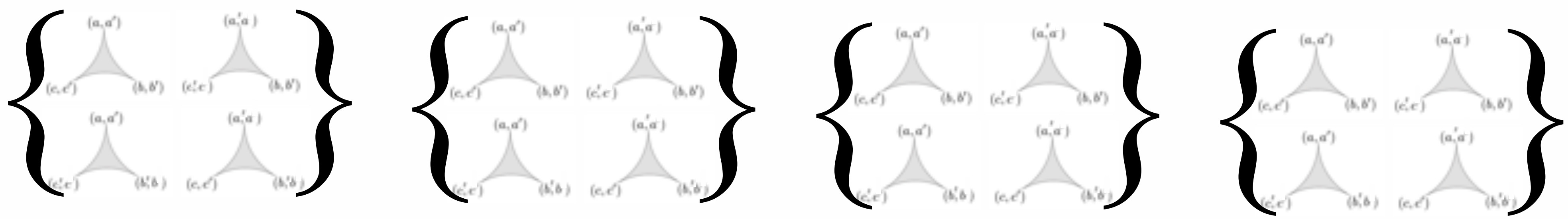


alphabet: ordered triples of ordered pairs of atoms modulo even number of flips

there is no function!



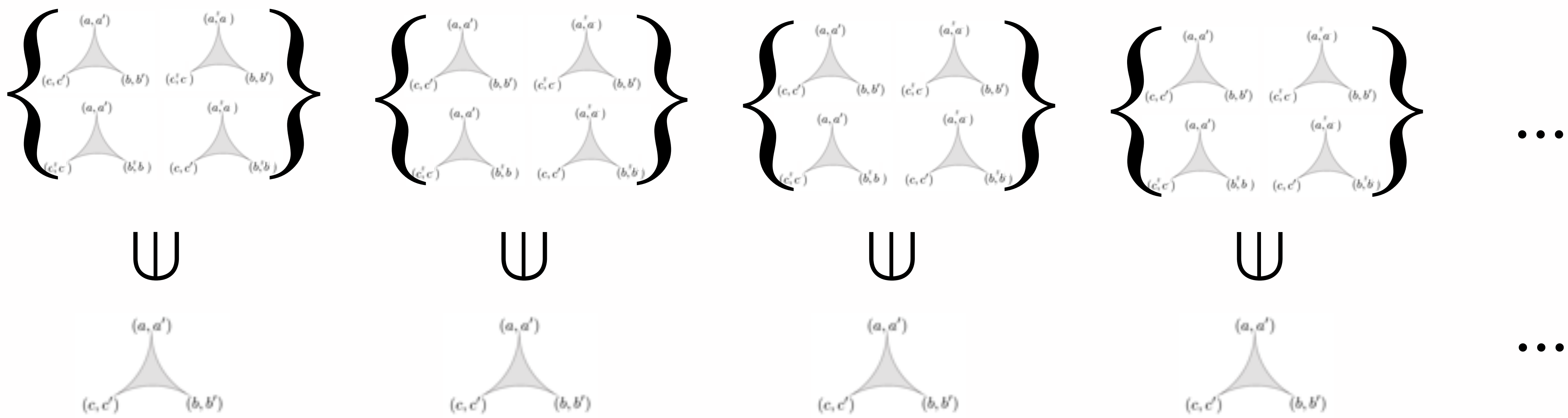
difficult language



...

difficult language

sequence of elements



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of
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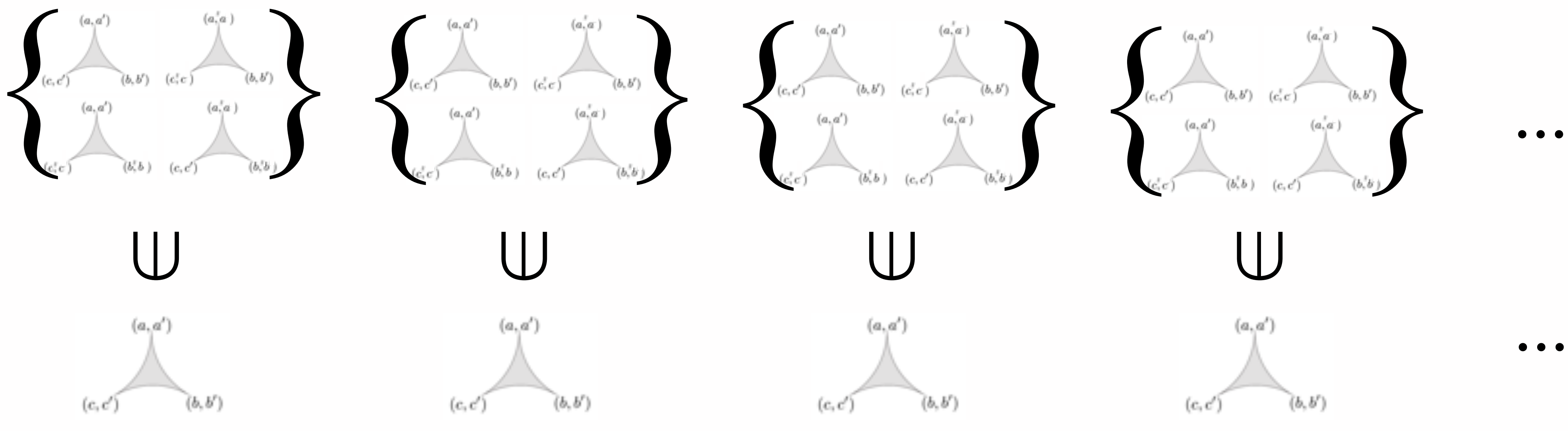
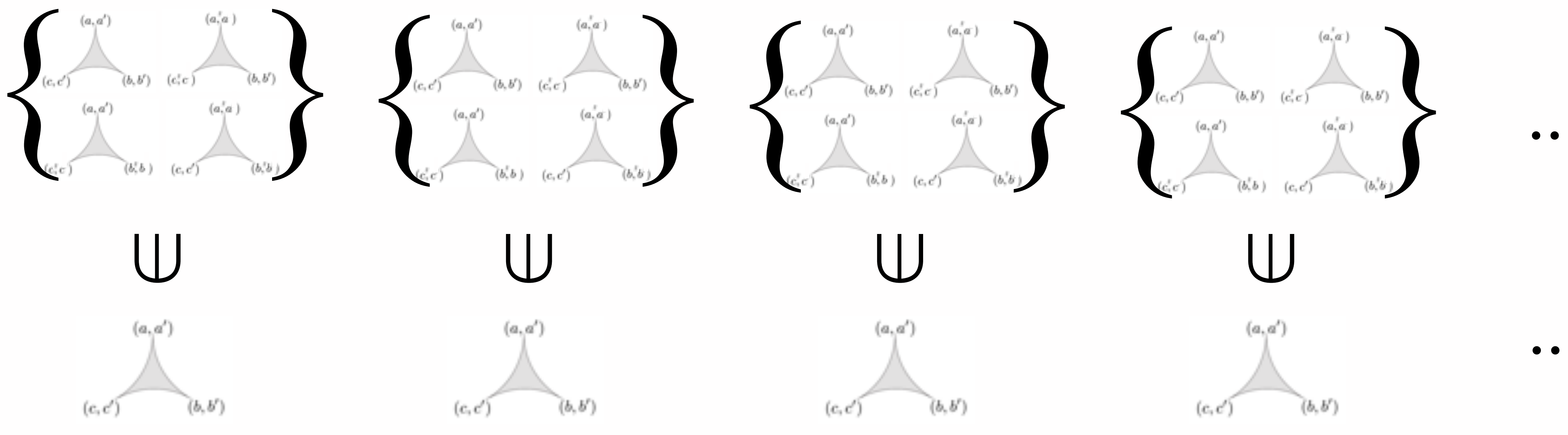


Language: a word is in the language iff
some sequence of elements is conflict-free

difficult language

sequence of elements



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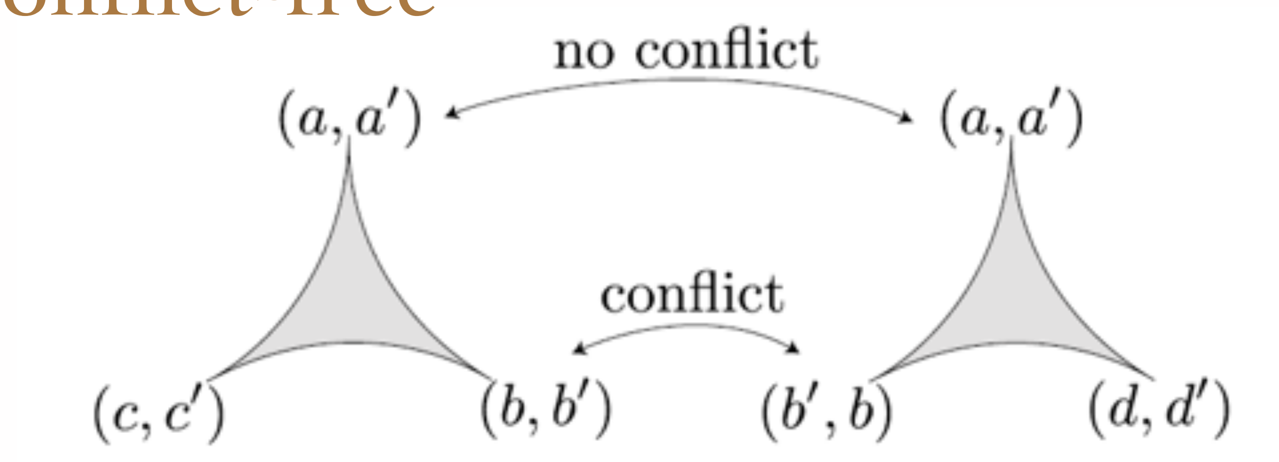

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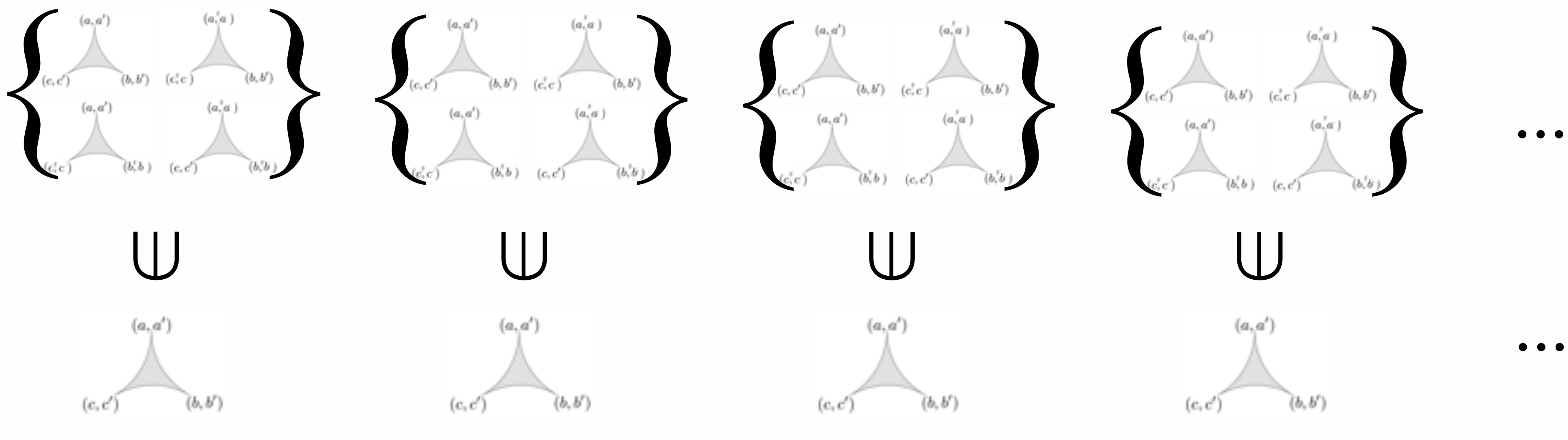

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closely related to Cai-Fuerer-Immerman graphs (1992)

difficult language

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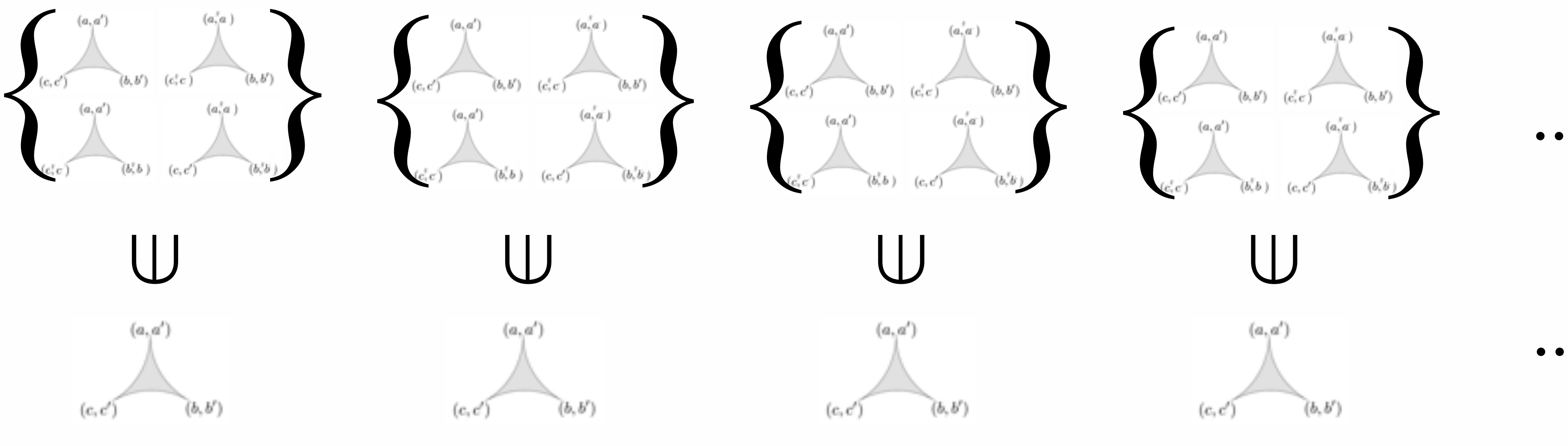

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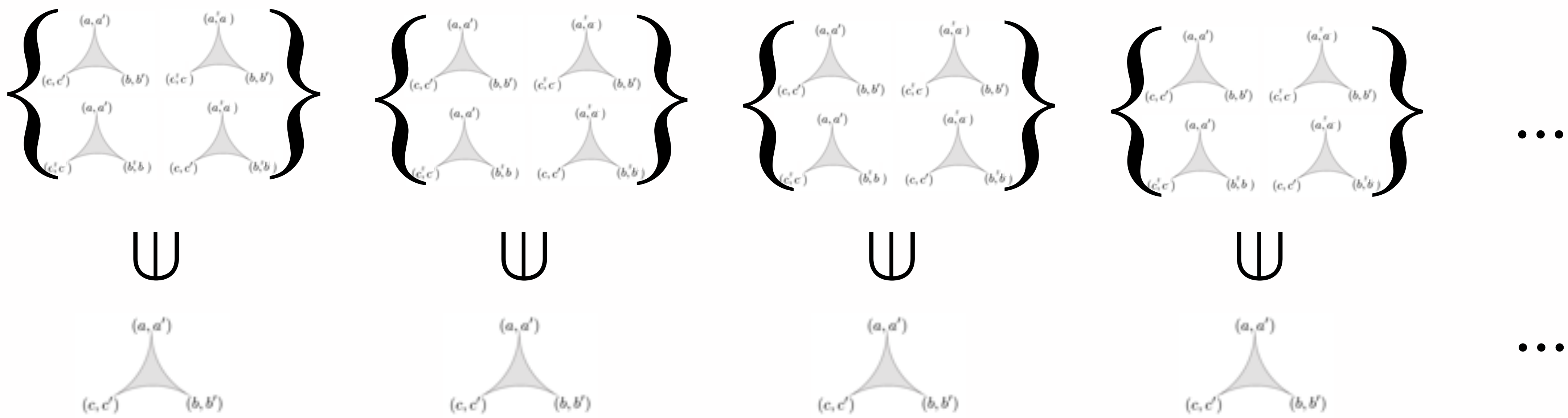
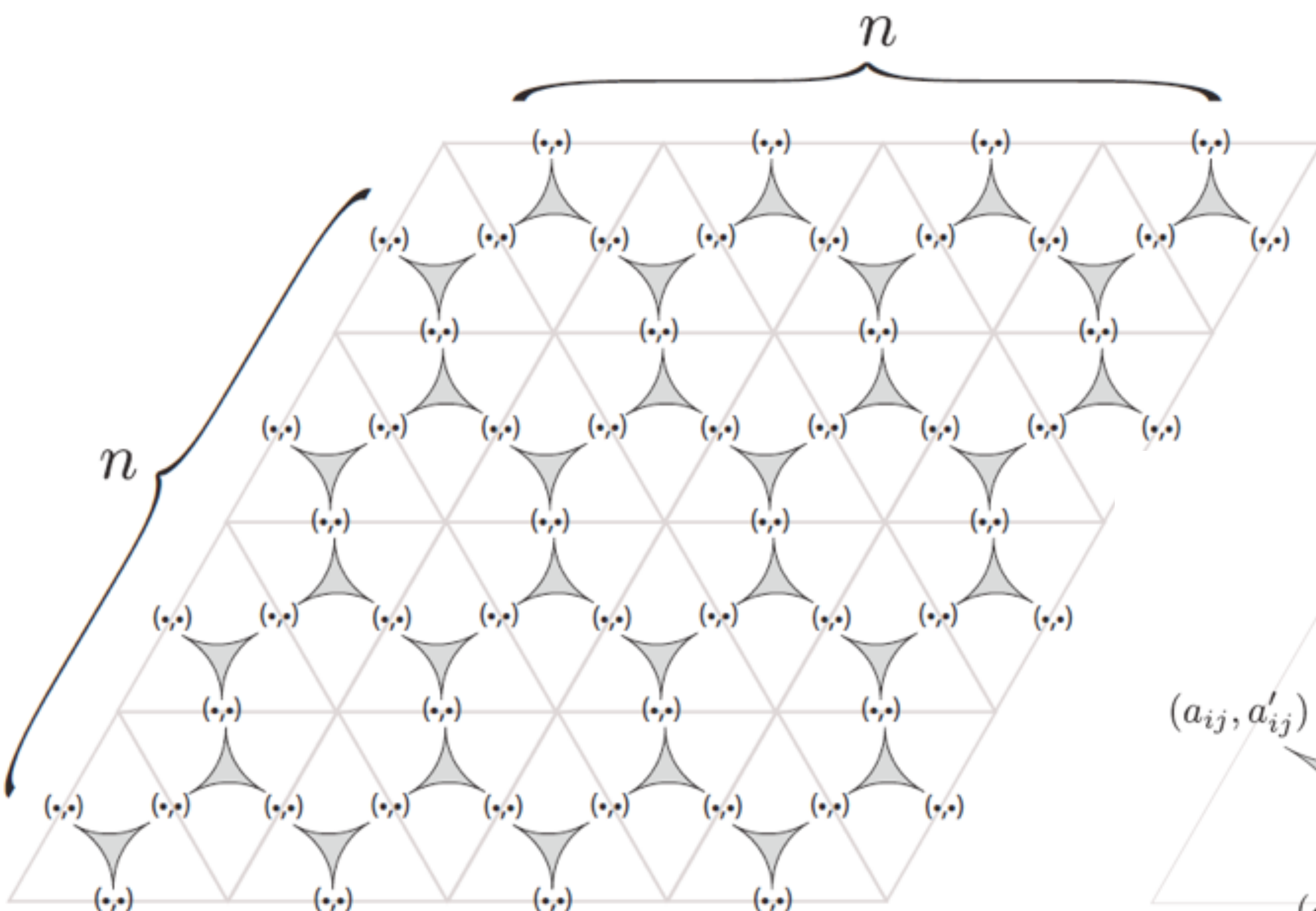


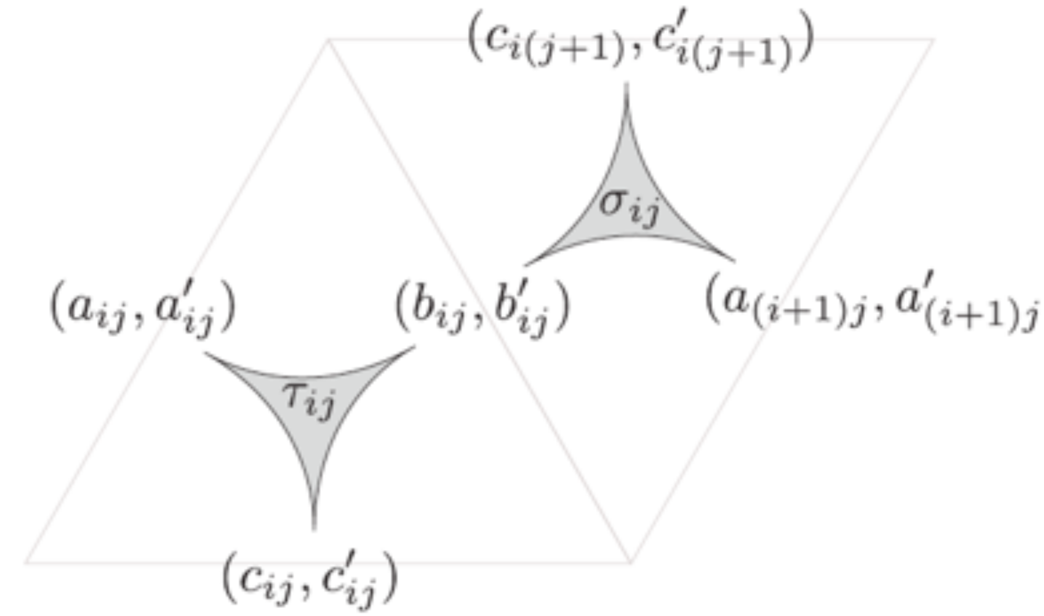
Language: a word is in the language iff some sequence of elements is conflict-free

recognized in NP: guess a sequence of elements

not recognized by a deterministic machine: enumeration of sequences of elements is not doable by a deterministic machine

difficult language





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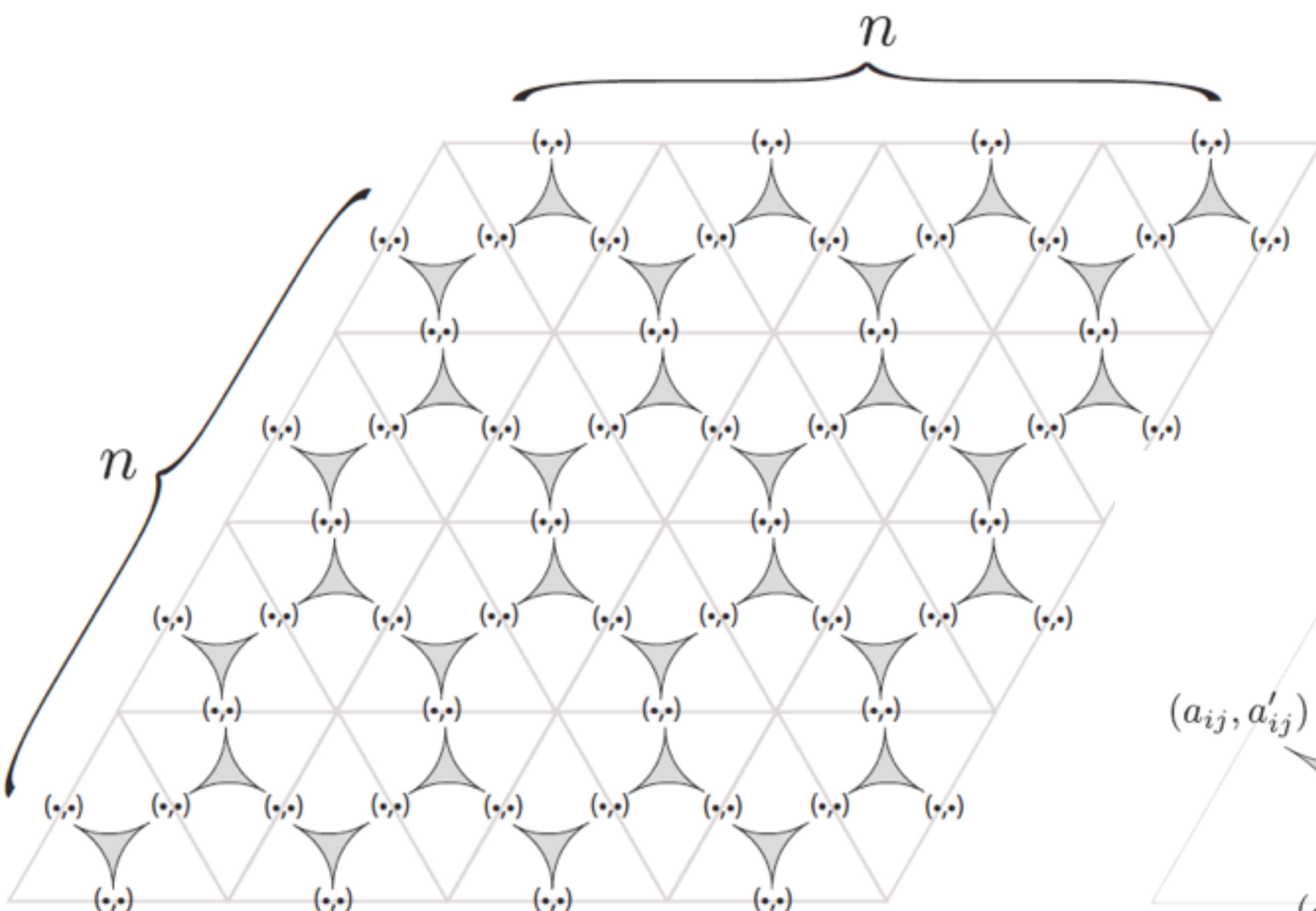

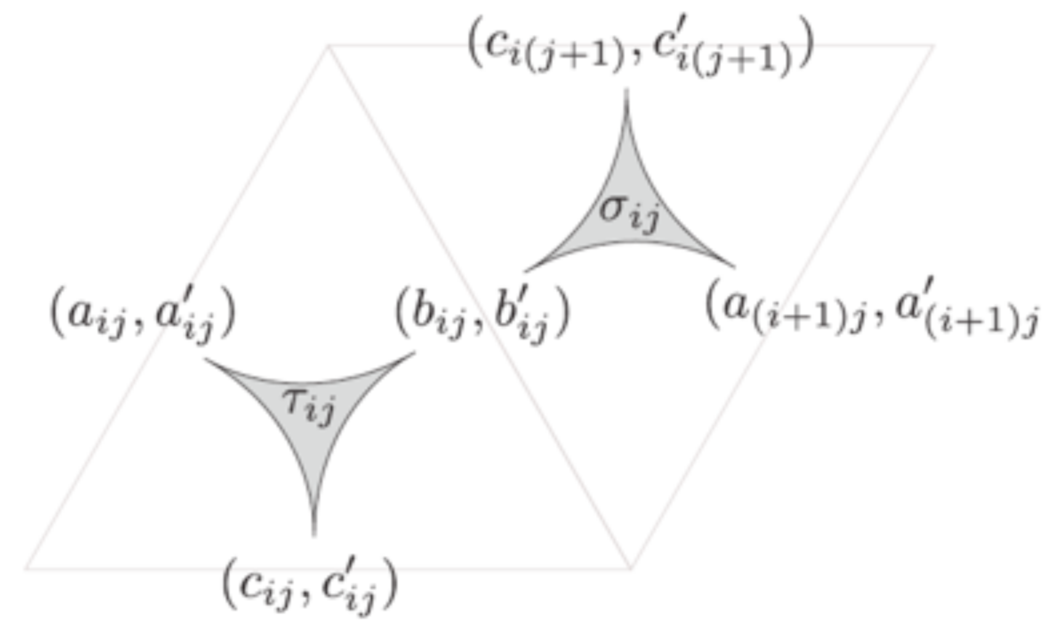

For sufficiently large size, deterministic machine can not distinguish an input torus from a "flipped" one

recent and on-going work

- effective characterization of standard alphabets using CSP theory
- model-theoretic characterization of standard alphabets (homogenizability)
- generalization to other well-behaved atoms
- applications to descriptive complexity
- characterization of standard atoms
- ...

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many interesting links to other fields

open problems

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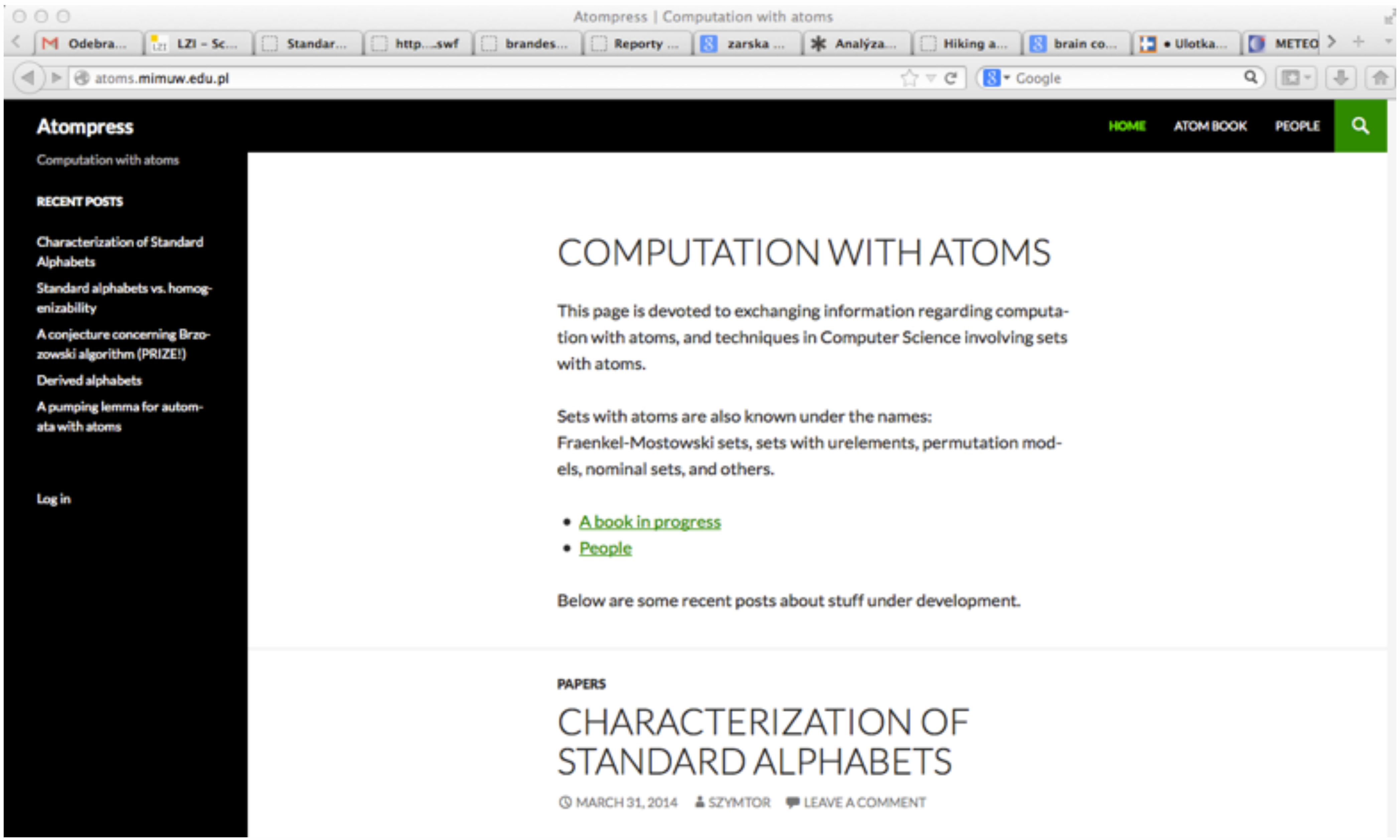

thank you!