Homomorphism problems for FO definable structures

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joint work with:
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FSTTCS 2016, Chennai

Plan

- **homomorphism problem**
- orbit-finite homomorphism problem
- decidability/undecidability results
- open problem

homomorphism problem

homomorphism problem

For two finite relational structures over the same signature

$\mathbb{A}$ $\mathbb{B}$

homomorphism problem

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$$\mathbb{A} \xrightarrow{\;?\;} \mathbb{B}$$

is there a homomorphism between them?

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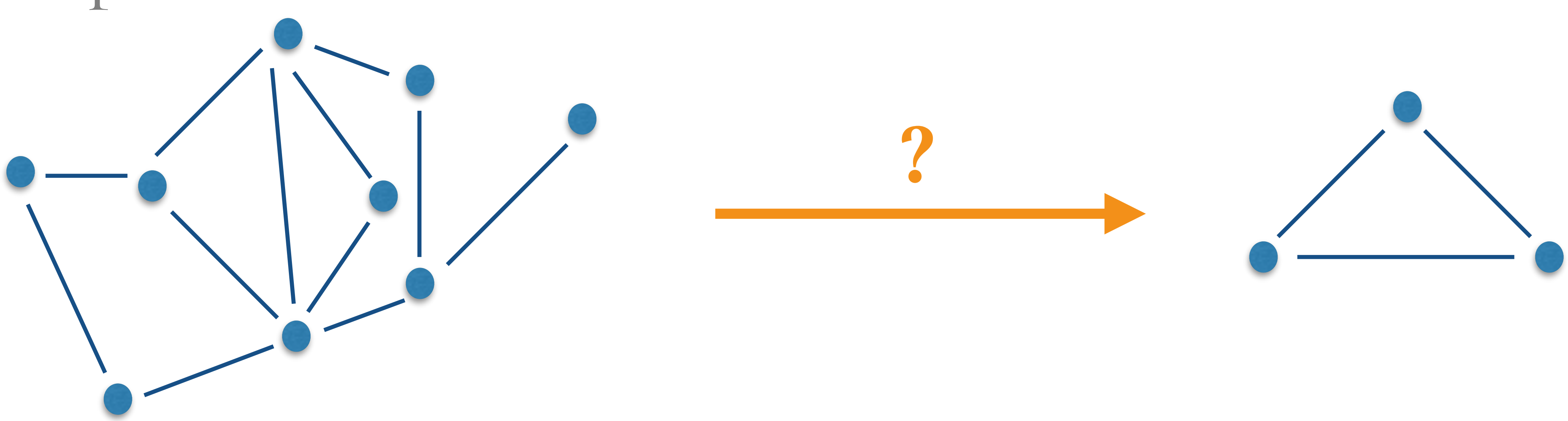
Example:



constraint satisfaction problem (CSP)

$$\mathbb{A} \xrightarrow{?} \mathbb{B}$$

constraint satisfaction problem (CSP)

fixed (template)

$$\mathbb{A} \xrightarrow{?} \mathbb{B}$$

constraint satisfaction problem (CSP)

instance

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Conjecture: [Feder, Vardi 1998]

For every template $\mathbb{B}$, CSP($\mathbb{B}$) is either in P, or NP-c.

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orbit-finite signatures allowed!

possibly infinite but orbit-finite

orbit-finite computation theory

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orbit-finite automata

[Bojańczyk, Klin, L. 2011, 2014]

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orbit-finite CSP: orbit-finite instances, finite templates

[Klin, Kopczyński, Ochremiak, Toruńczyk 2015]

infinite CSP

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finite instances, infinite templates (= evaluation of existential FO)

[Bodirsky 2007, 2012]

[Bodirsky, Nesetril 2006]

[Bodirsky, Kara 2010]

[Bodirsky, Pinsker 2010, 2011]

[Bodirsky, Dalmau 2013]

[Bodirsky, Wrona 2012]

[Bodirsky, Martin, Pinsker, Pongracz 2016]

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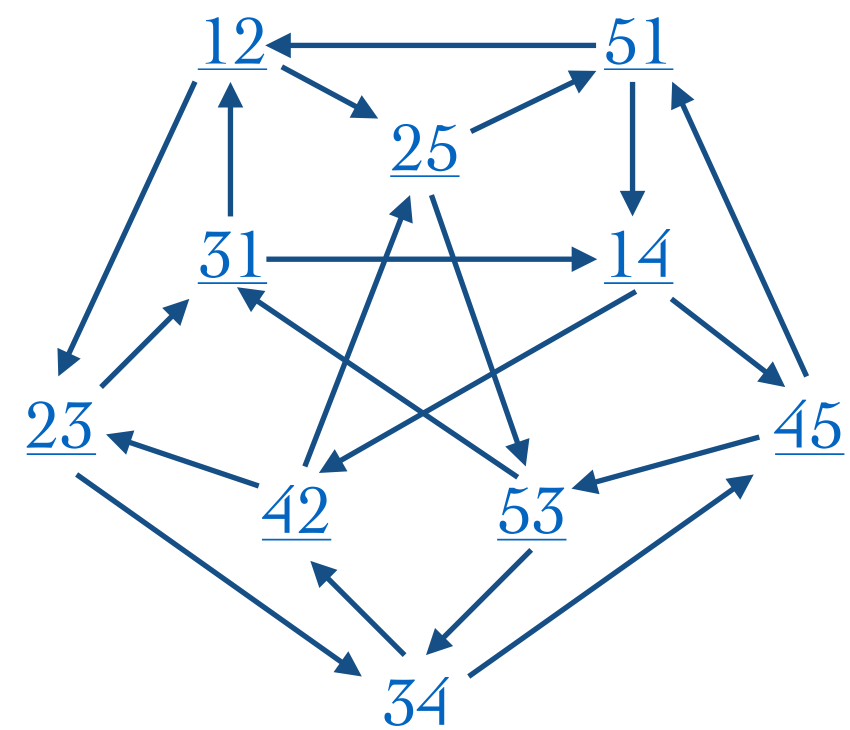

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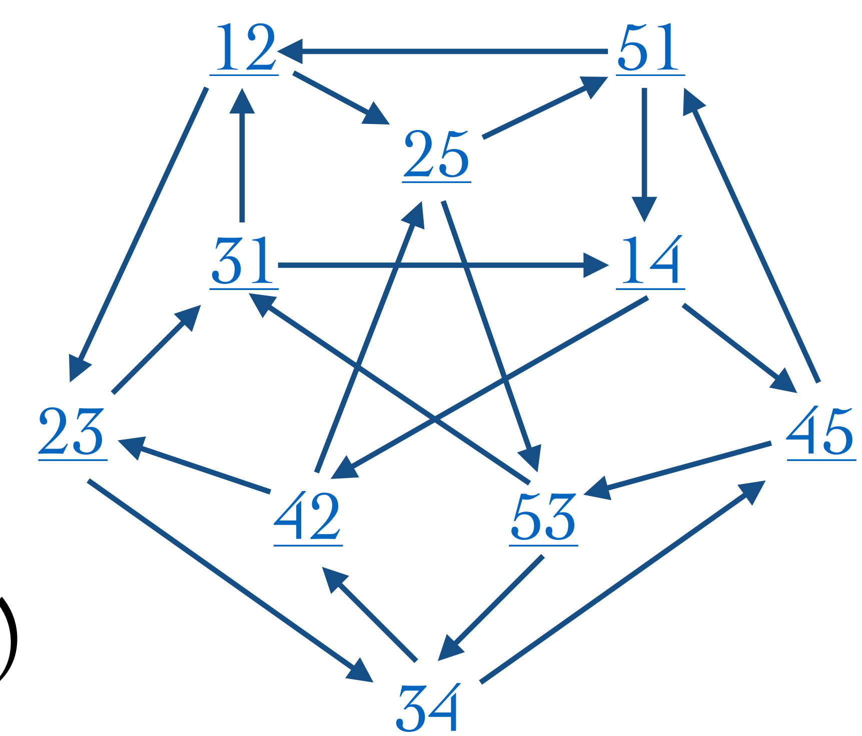

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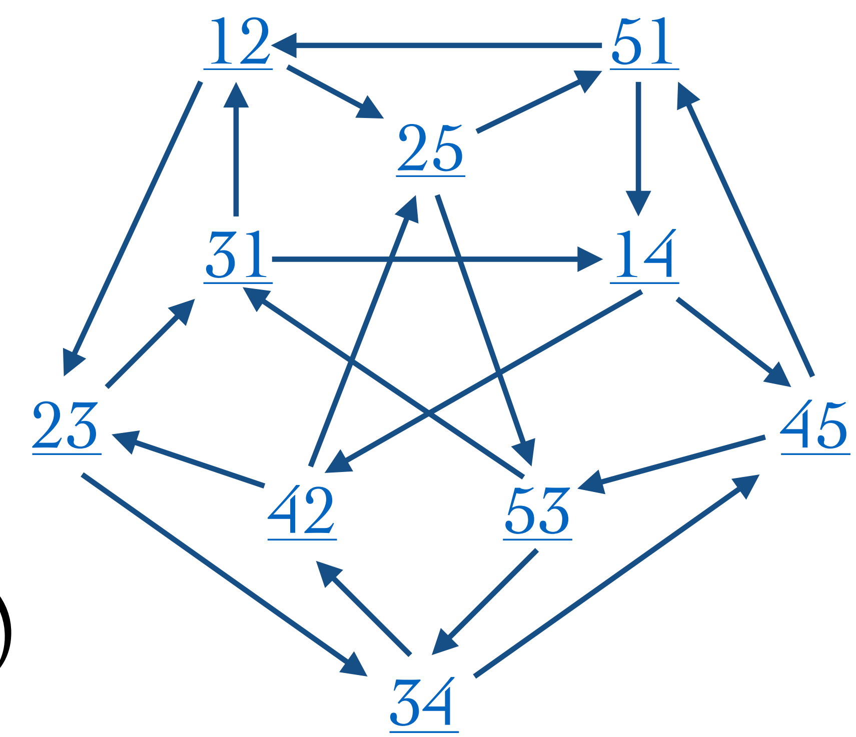

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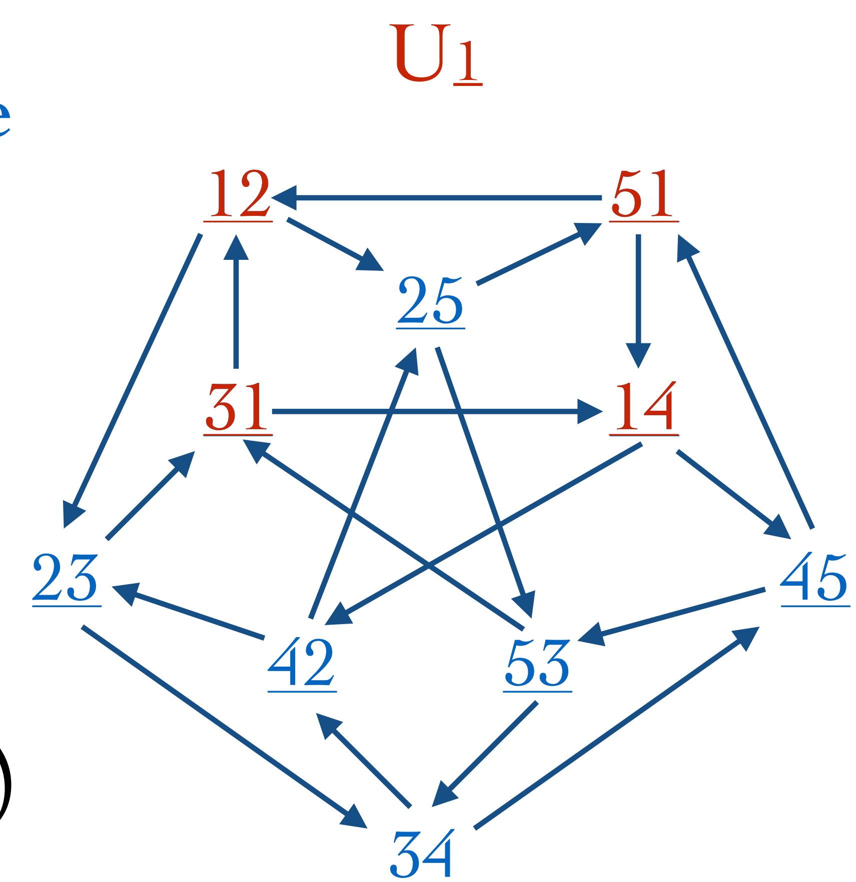

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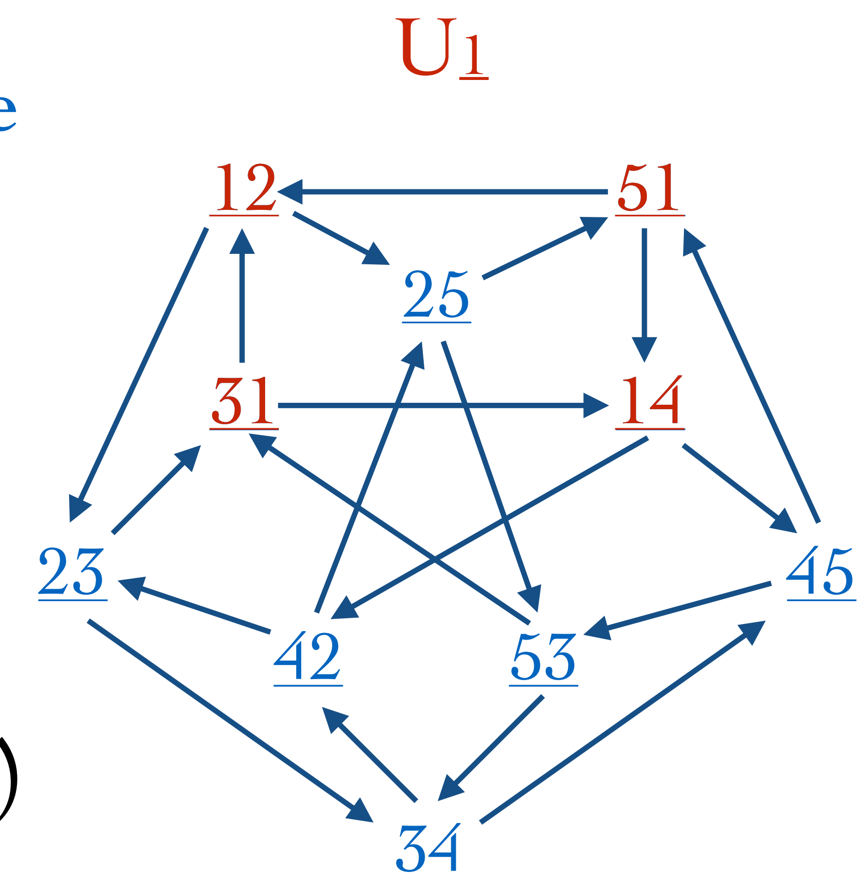

definable structures
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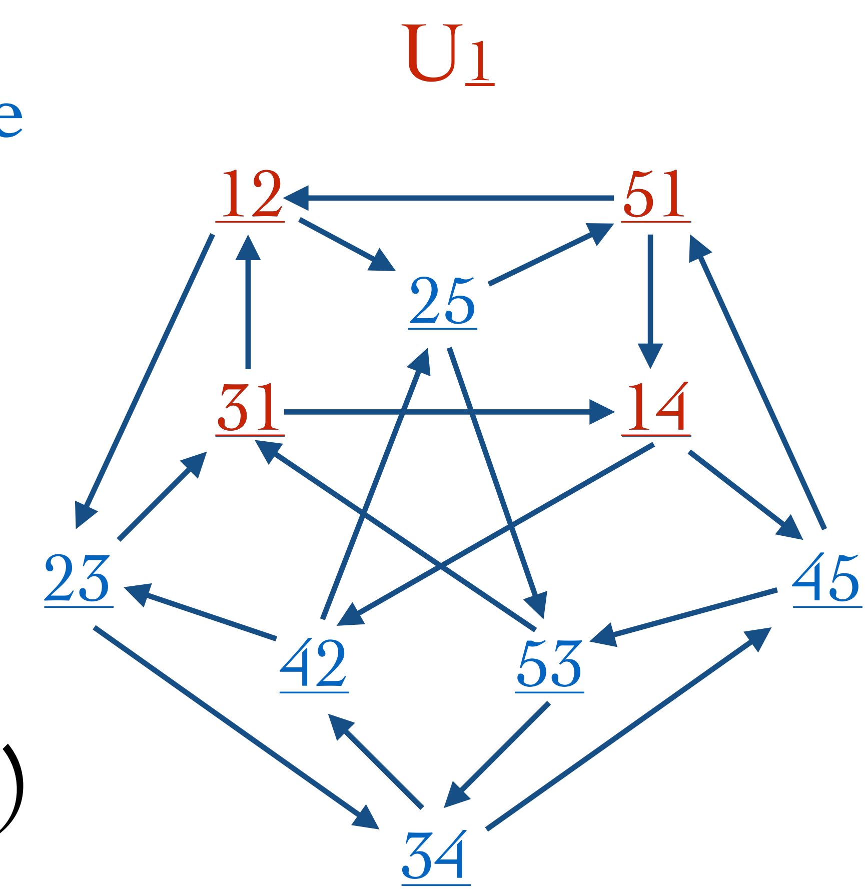

definable structures over a finite signature **=** FO interpretations (with constants) in $\mathcal{A}$

(up to iso)

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definable (definable signature)

$$\mathbb{A} \xrightarrow{?} \mathbb{B}$$

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$$(V, E) \xrightarrow{\;?\;} \{\} \text{ — } \{\{\}\}$$

there is a homomorphism
but no definable one

$$\mathbb{A} \xrightarrow{\ ?\ } \mathbb{B}$$

definable

- definable homomorphism problem is **semi-decidable**

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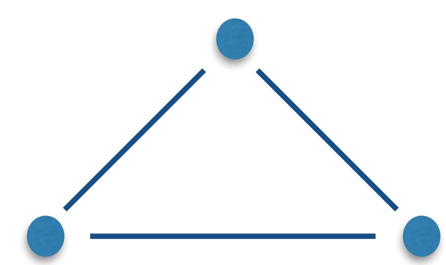

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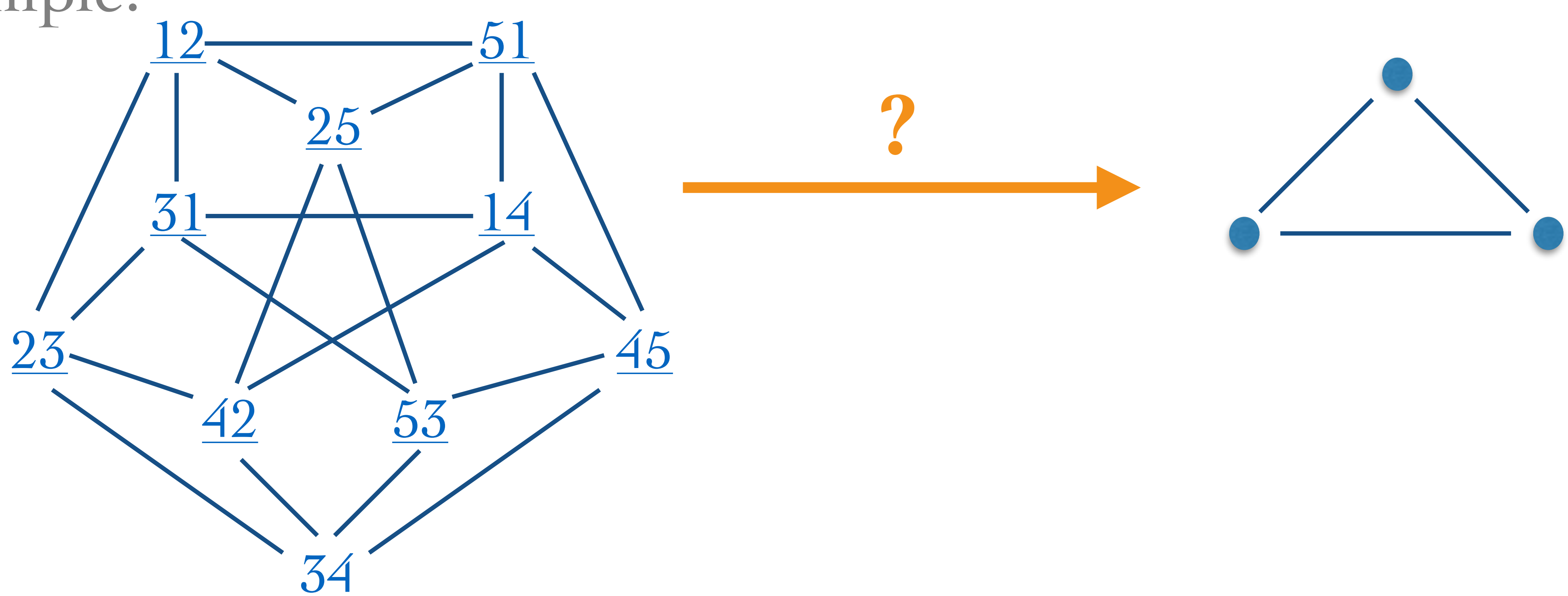

Plan

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- orbit-finite homomorphism problem
- **decidability/undecidability results**
- open problem

our results

Atoms $\mathcal{A}$ = ($\{\underline{0}, \underline{1}, \underline{2}, \ldots\}$, =).

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reduction to
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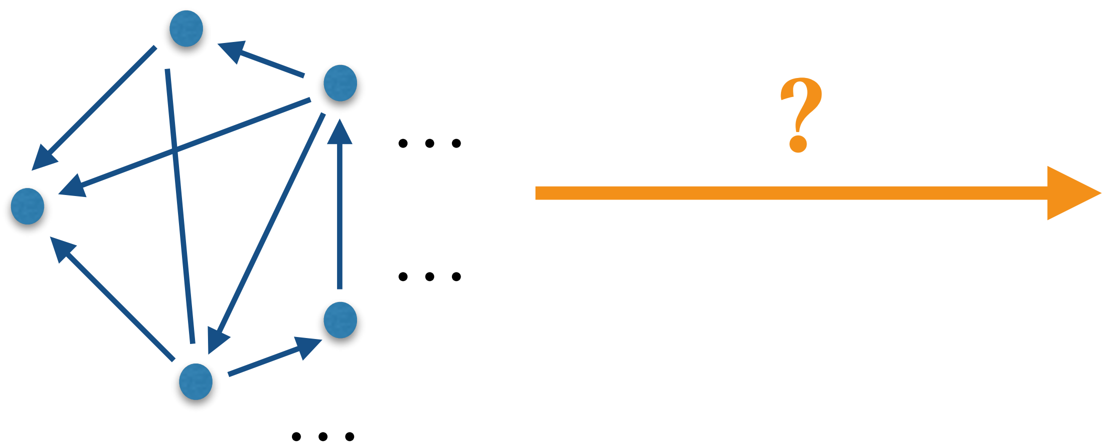

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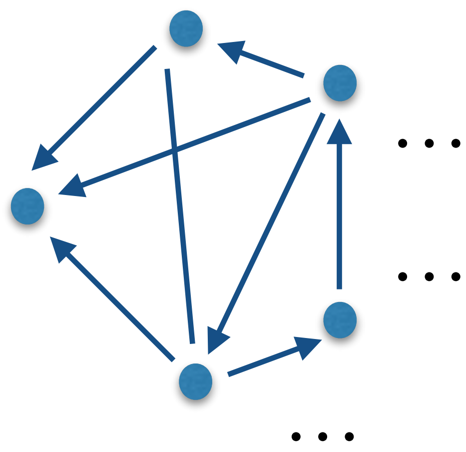
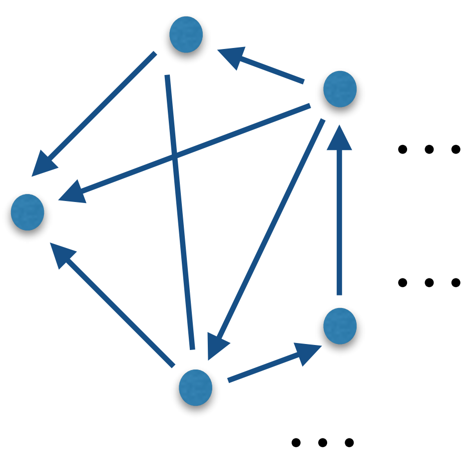


V = { {a,b} : a, b in $\mathcal{A}$, a ≠ b }

E = { ({a,b}, {b,c}) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a }

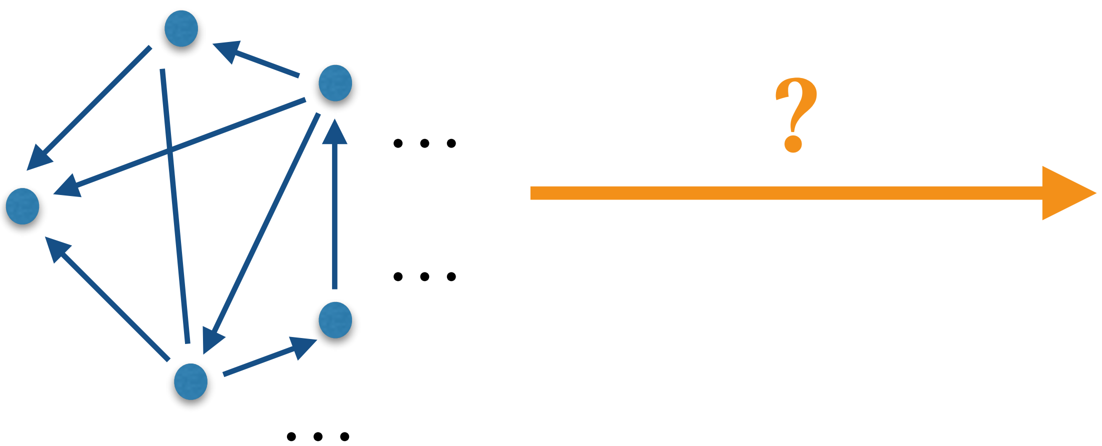


$\xrightarrow{?}$

V = { {a,b} : a, b in $\mathcal{A}$, a ≠ b }

E = { ({a,b}, {b,c}) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a }

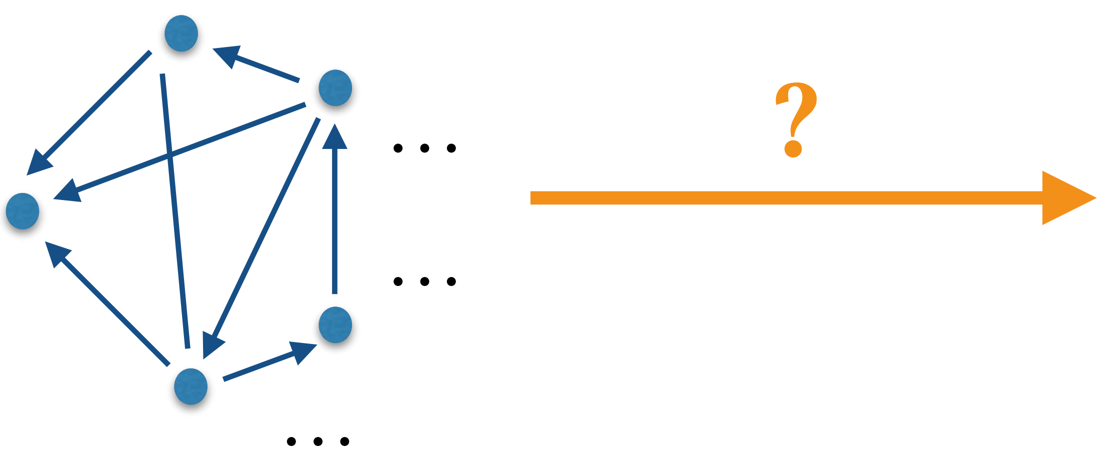
iff



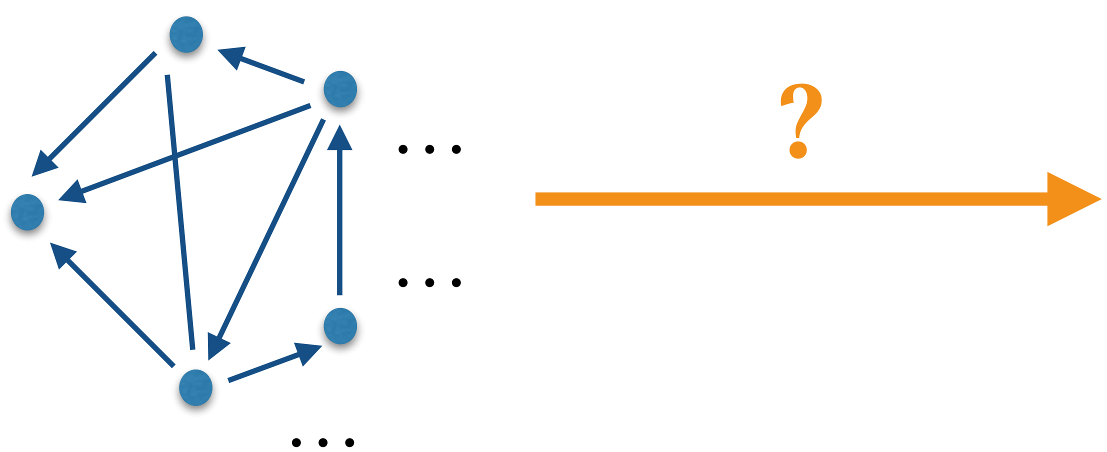
$V = \{ \{a,b\} : \ a, b \text{ in } \mathcal{A}, \ a \neq b \}$

$E = \{ (\{a,b\}, \{b,c\}) : \ a,b,c \text{ in } \mathcal{A}, \ a\neq b, \ b\neq c, \ c\neq a \}$

iff



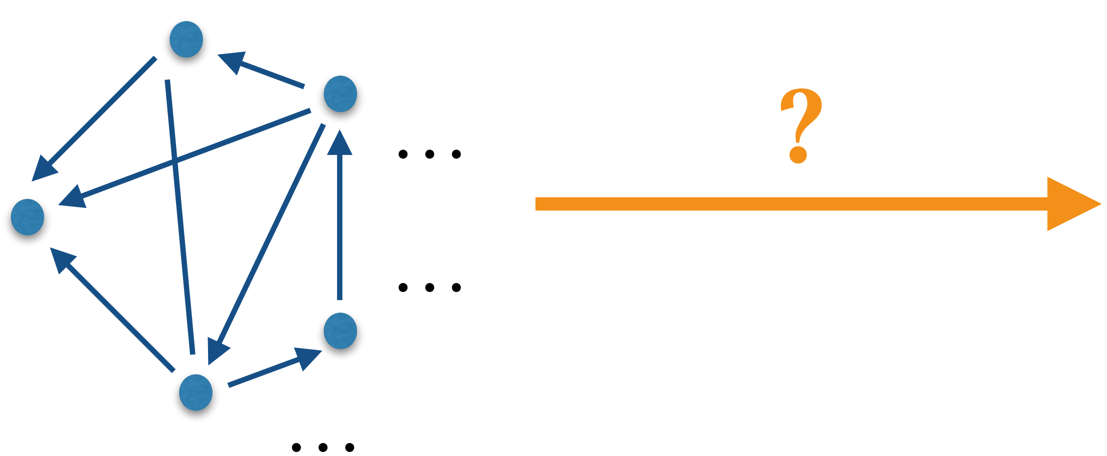
$V = \{ ab : \ a, b \text{ in } \mathcal{A}, \ a \neq b \}$



$V = \{ \{a,b\} : a, b \text{ in } \mathcal{A}, a \neq b \}$

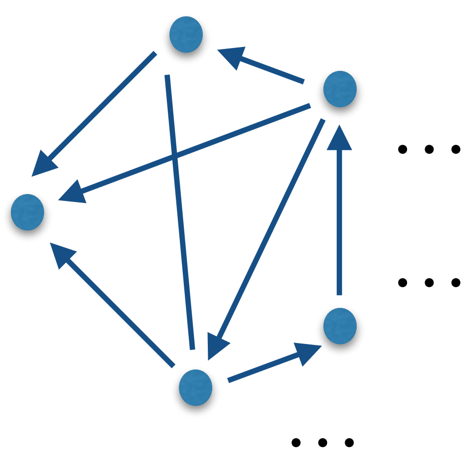
$E = \{ (\{a,b\}, \{b,c\}) : a,b,c \text{ in } \mathcal{A}, a \neq b, b \neq c, c \neq a \}$

iff



$V = \{ ab : a, b \text{ in } \mathcal{A}, a \neq b \}$

$E = \{ (ab, bc) : a,b,c \text{ in } \mathcal{A}, a \neq b, b \neq c, c \neq a \} \cup$

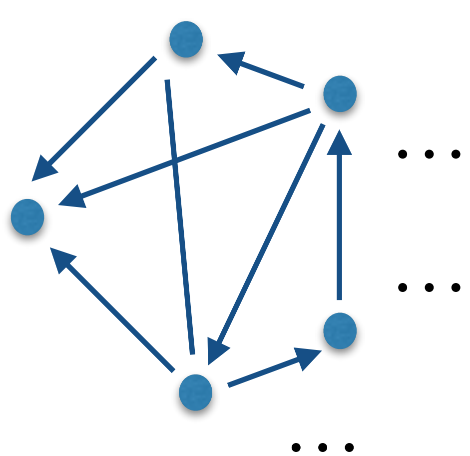


?

V = { {a,b} : a, b in $\mathcal{A}$, a $\neq$ b }

E = { ({a,b}, {b,c}) : a,b,c in $\mathcal{A}$, a$\neq$b, b$\neq$c, c$\neq$a }

iff

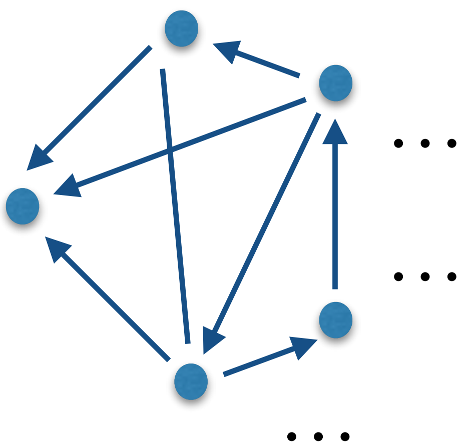


?

V = { ab : a, b in $\mathcal{A}$, a $\neq$ b }

E = { (ab, bc) : a,b,c in $\mathcal{A}$, a$\neq$b, b$\neq$c, c$\neq$a } $\cup$

{ (ab, cb) : a,b,c in $\mathcal{A}$, a$\neq$b, b$\neq$c, c$\neq$a } $\cup$

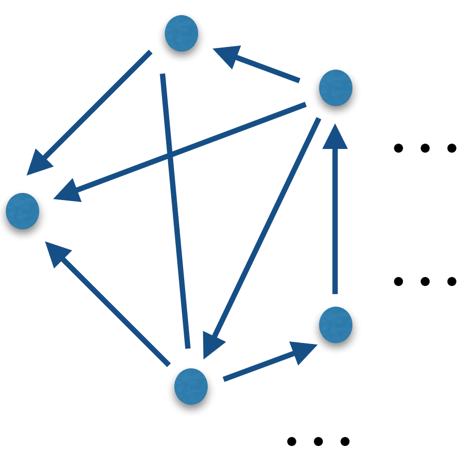


$$\xrightarrow{\quad ? \quad}$$

$V = \{\ \{a,b\} : a, b \text{ in } \mathcal{A}, a \neq b\ \}$

$E = \{\ (\{a,b\}, \{b,c\}) : a,b,c \text{ in } \mathcal{A}, a{\neq}b, b{\neq}c, c{\neq}a\ \}$

iff



$$\xrightarrow{\quad ? \quad}$$

$V = \{\ ab : a, b \text{ in } \mathcal{A}, a \neq b\ \}$

$E = \{\ (ab, bc) : a,b,c \text{ in } \mathcal{A}, a{\neq}b, b{\neq}c, c{\neq}a\ \} \cup$
$\{\ (ab, cb) : a,b,c \text{ in } \mathcal{A}, a{\neq}b, b{\neq}c, c{\neq}a\ \} \cup$
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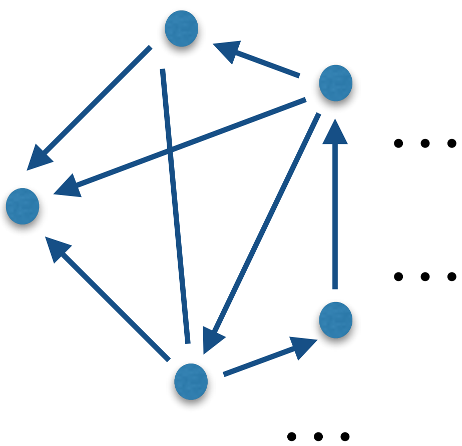

$\xrightarrow{?}$

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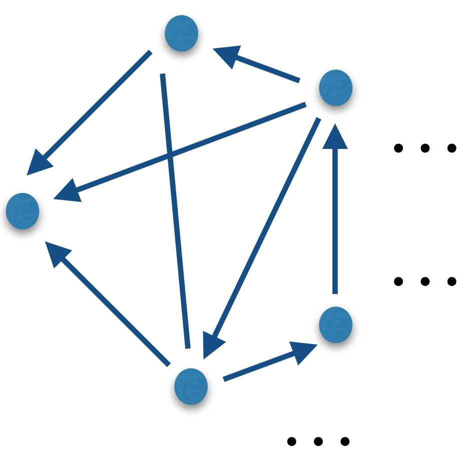
E = { ({a,b}, {b,c}) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a }

iff



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V = { ab : a, b in $\mathcal{A}$, a ≠ b }

E = { (ab, bc) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a } ∪
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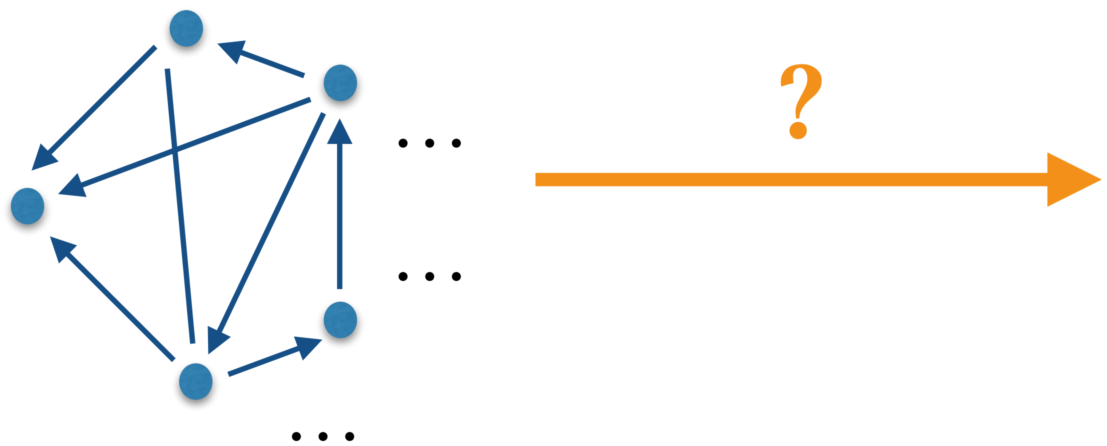
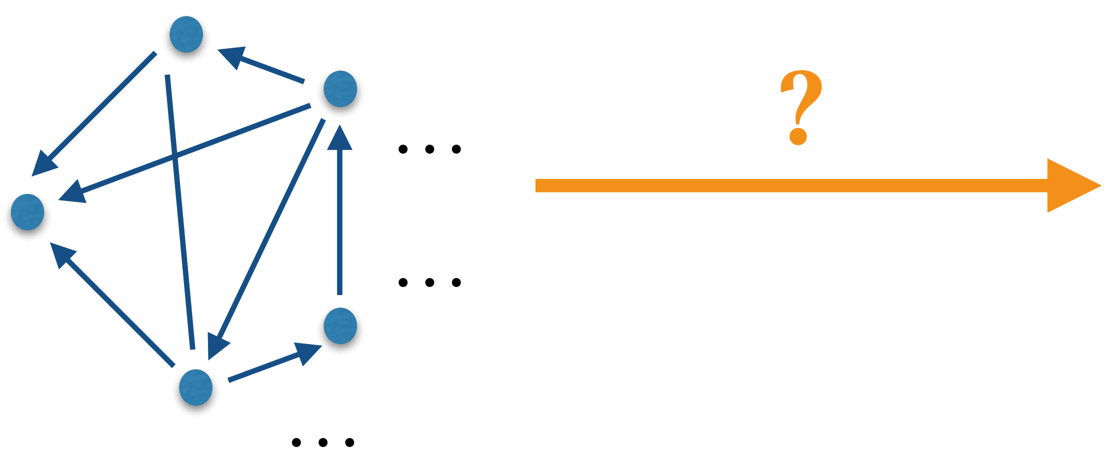
{ (ba, bc) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a } ∪
{ (ba, cb) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a }



$V = \{ ab : a, b \text{ in } \mathcal{A}, a \neq b \}$

$E = \{ (ab, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$
$\{ (ab, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$
$\{ (ba, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$
$\{ (ba, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \}$



$V = \{ ab : a, b \text{ in } \mathcal{A}, a \neq b \}$

$E = \{ (ab, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$
$\{ (ab, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$
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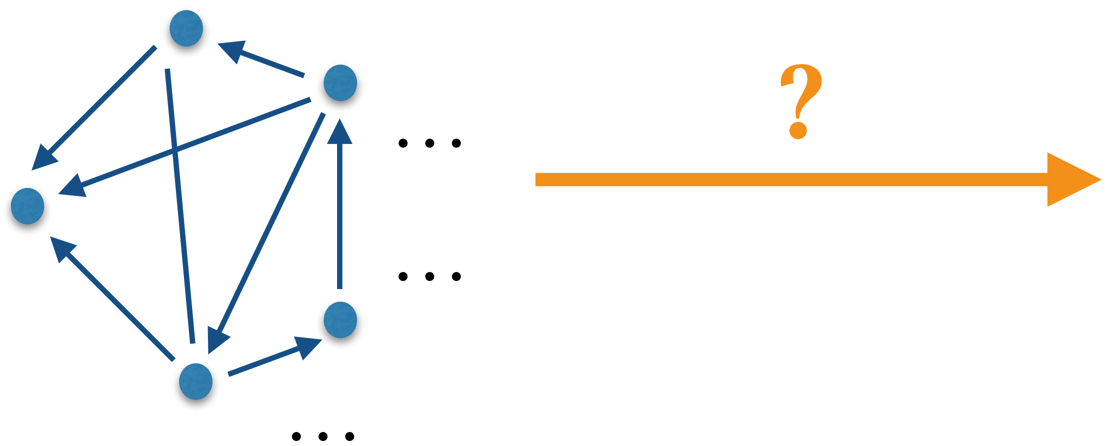
$\{ (ba, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \}$



go to orbits?

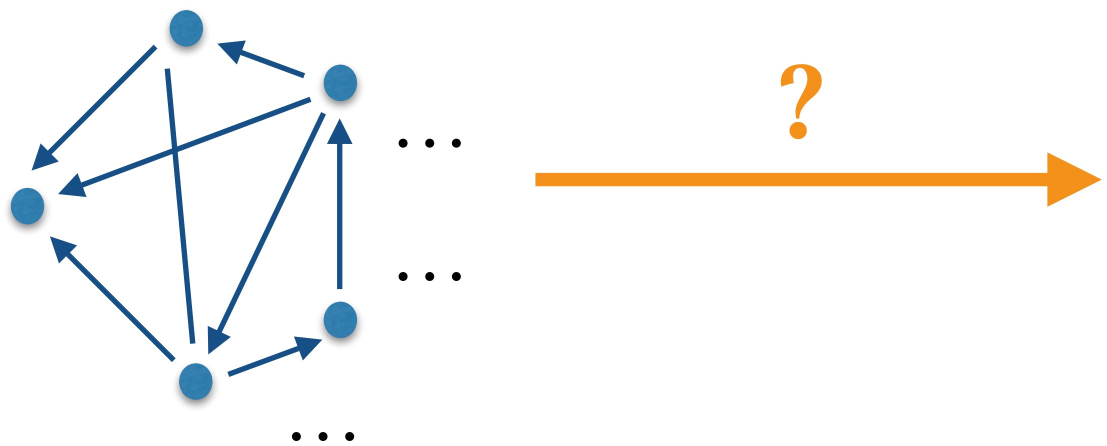
V = { ab : a, b in $\mathcal{A}$, a ≠ b }

take V×V as elements

E = { (ab, bc) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a } ∪
{ (ab, cb) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a } ∪
{ (ba, bc) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a } ∪
{ (ba, cb) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a }

E is a unary relation now

go to orbits?



V = { ab : a, b in $\mathcal{A}$, a ≠ b }

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{ (ba, bc) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a } ∪
{ (ba, cb) : a,b,c in $\mathcal{A}$, a≠b, b≠c, c≠a }

E is a unary relation now

orbits in V×V:

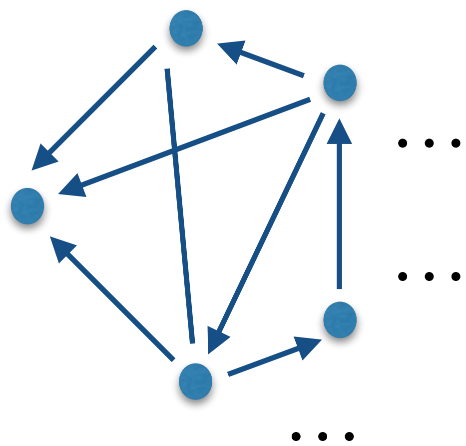
$$\left.\begin{array}{l}
\{\, (ab,\ bc) :\ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
\{\, (ab,\ cb) :\ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
\{\, (ba,\ bc) :\ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
\{\, (ba,\ cb) :\ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\}
\end{array}\right\}\ \mathrm{E}$$

{ (ab, cd) : a,b,c,d in $\mathcal{A}$, a≠b, b≠c, c≠a, … }

{ (ab, ba) : a,b in $\mathcal{A}$, a≠b }

{ (ab, ab) : a,b in $\mathcal{A}$, a≠b }

go to orbits?



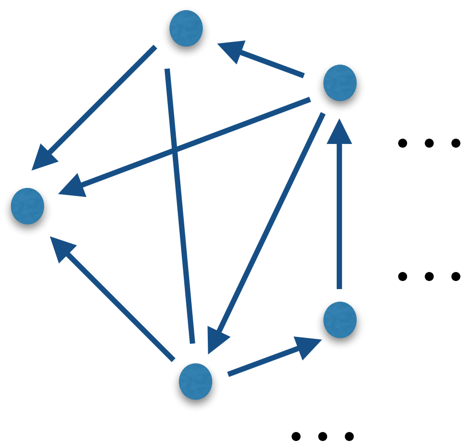
?

$V = \{ ab : a, b \text{ in } \mathcal{A}, a \neq b \}$

take V×V as elements

$$E = \{ (ab, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$$
$$\{ (ab, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$$
$$\{ (ba, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$$
$$\{ (ba, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \}$$

E is a unary relation now



?

go to orbits?

$V = \{ ab : a, b \text{ in } \mathcal{A}, a \neq b \}$

take $V \times V$ as elements

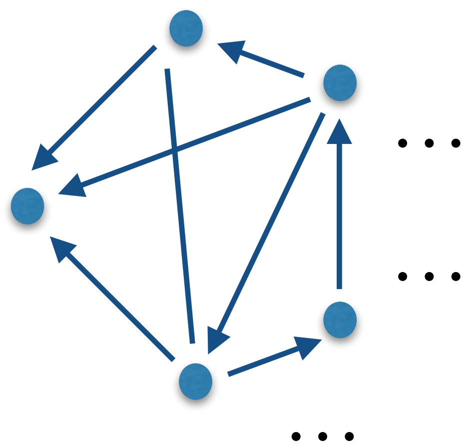
$$\begin{aligned} E = \{ (ab, bc) :\ & a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup \\ \{ (ab, cb) :\ & a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup \\ \{ (ba, bc) :\ & a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup \\ \{ (ba, cb) :\ & a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \end{aligned}$$

E is a unary relation now

iff

orbits in $V \times V$

go to orbits?



?

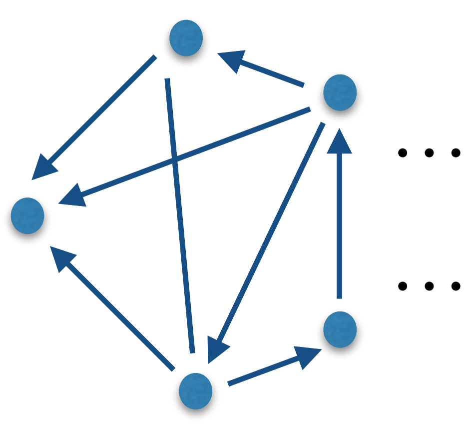
$V = \{ ab : a, b \text{ in } \mathcal{A}, a \neq b \}$

take V×V as elements

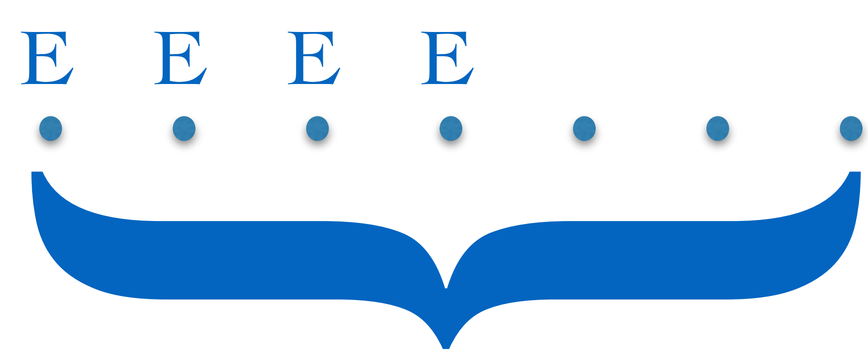
$$E = \{ (ab, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$$
$$\{ (ab, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$$
$$\{ (ba, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$$
$$\{ (ba, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \}$$

E is a unary relation now

iff

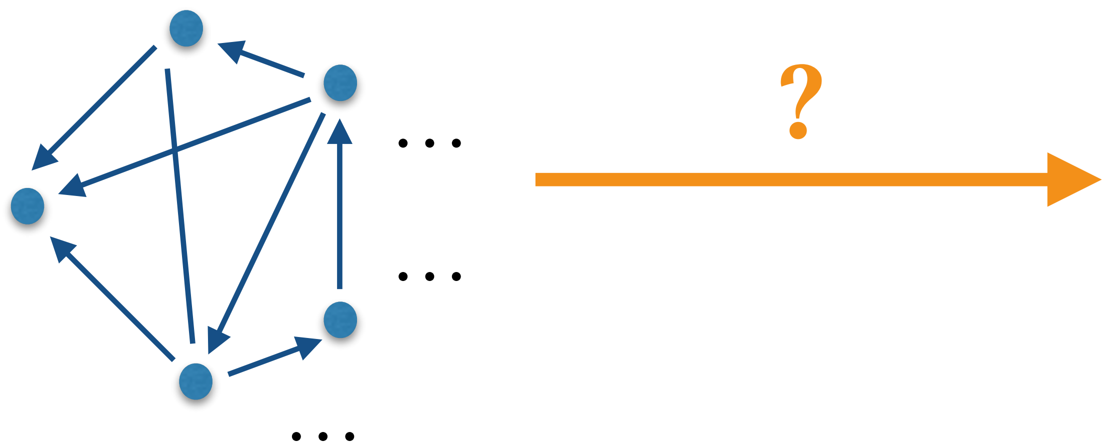


?



orbits in V×V

go to orbits?



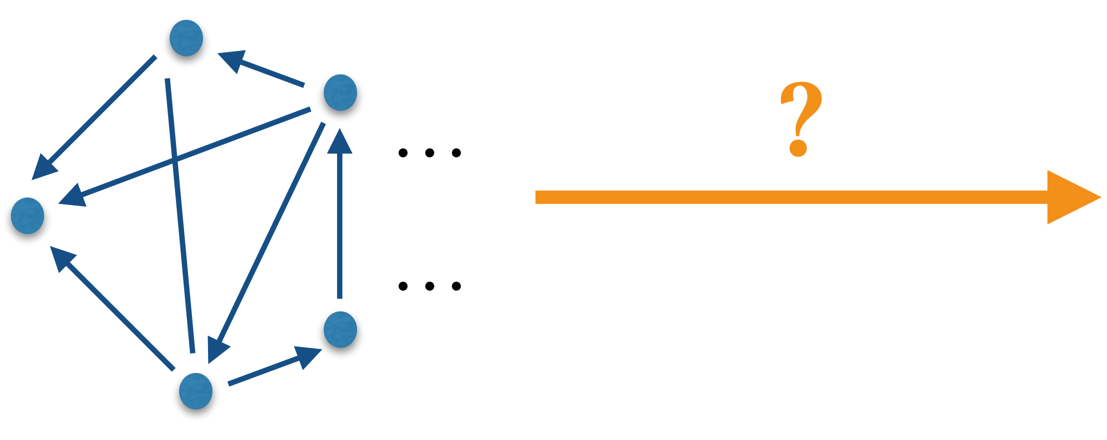
$V = \{ ab : a, b \text{ in } \mathcal{A}, a \neq b \}$

take V×V as elements

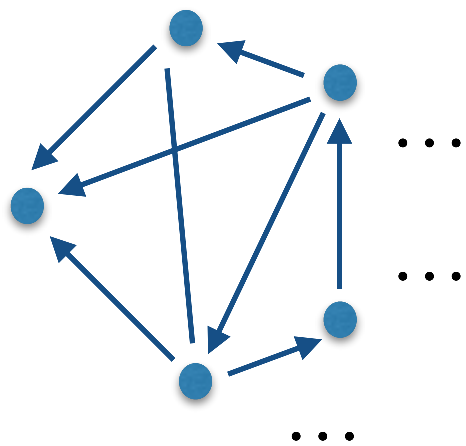
$E = \{ (ab, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$
$\{ (ab, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$
$\{ (ba, bc) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \} \cup$
$\{ (ba, cb) : a,b,c \text{ in } \mathcal{A}, a\neq b, b\neq c, c\neq a \}$

E is a unary relation now

$\Downarrow$



go to orbits?



?

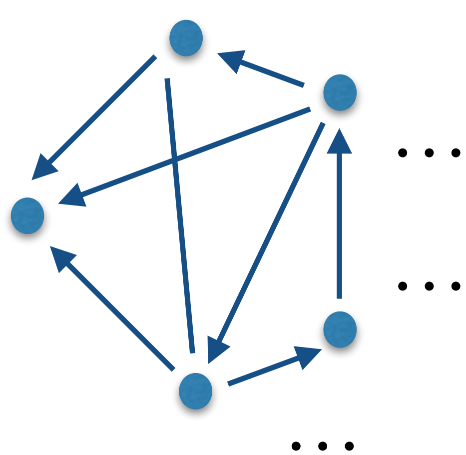
$V = \{\, ab : a, b \text{ in } \mathcal{A},\ a \neq b \,\}$

take V×V as elements

$$
\begin{aligned}
E = &\{\, (ab, bc) : a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \cup \\
&\{\, (ab, cb) : a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \cup \\
&\{\, (ba, bc) : a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \cup \\
&\{\, (ba, cb) : a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\}
\end{aligned}
$$

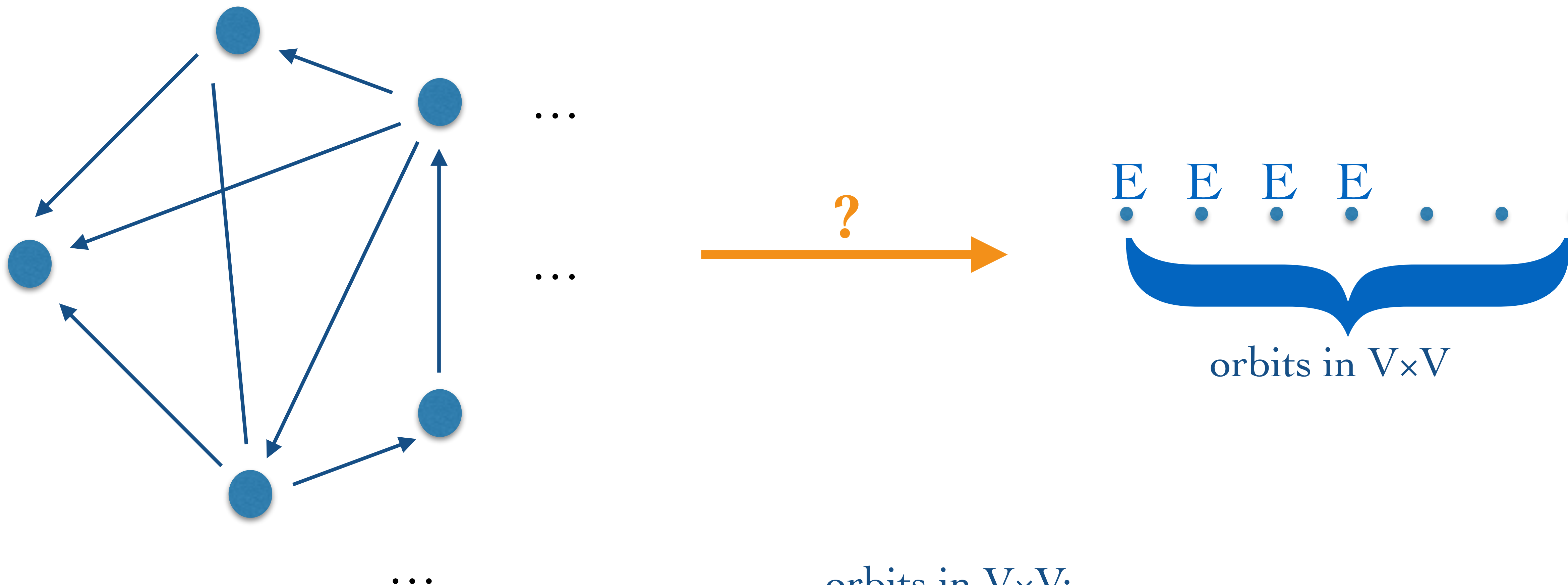
E is a unary relation now

$\Uparrow$?



?

$V = \{\, ab : \ a, b \text{ in } \mathcal{A},\ a \neq b \,\}$



orbits in V×V:

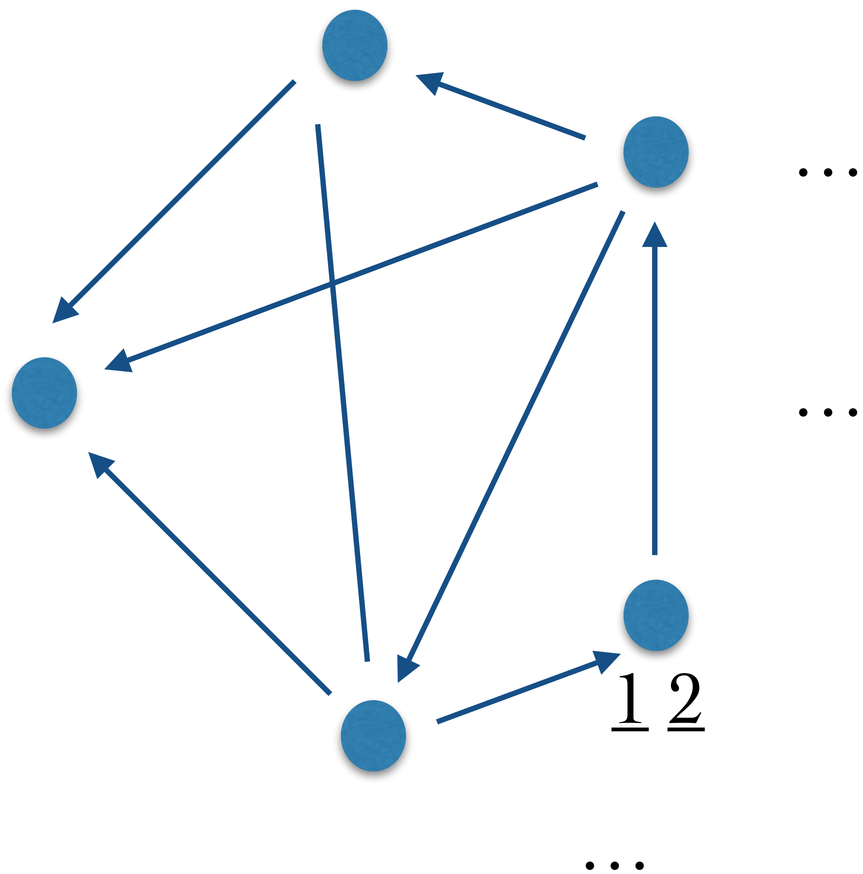
$$\left.\begin{array}{l}
\{\, (ab, bc) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
\{\, (ab, cb) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
\{\, (ba, bc) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
\{\, (ba, cb) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\}
\end{array}\right\} E$$

$\{\, (ab, cd) : \ a,b,c,d \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a,\ \ldots \,\}$

$\{\, (ab, ba) : \ a,b \text{ in } \mathcal{A},\ a\neq b \,\}$

$\{\, (ba, ab) : \ a,b \text{ in } \mathcal{A},\ a\neq b \,\}$

$$V = \{\, ab : \ a, b \text{ in } \mathcal{A},\ a \neq b \,\}$$



orbits in V×V:

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\{\, (ba,\ bc) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
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\end{array}\right\} E$$

$$\{\, (ab,\ cd) : \ a,b,c,d \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a,\ \ldots \,\}$$

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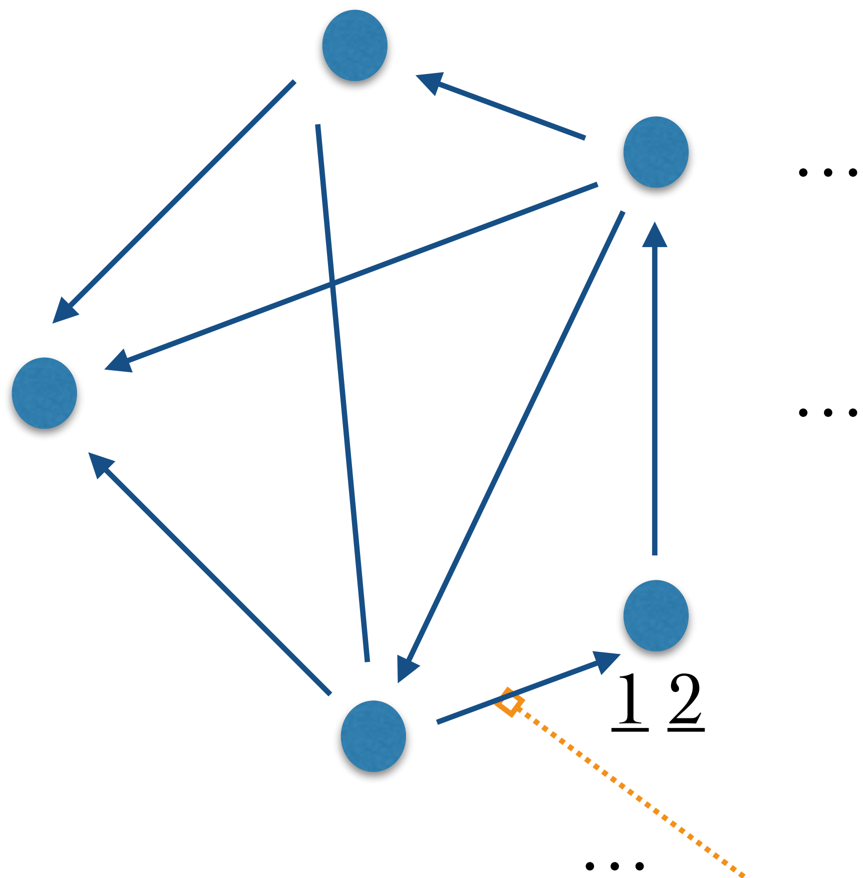
$$V = \{\, ab : \ a, b \text{ in } \mathcal{A},\ a \neq b \,\}$$



orbits in V×V:

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\{\ (ab,\ cb) :\ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a\ \} \\
\{\ (ba,\ bc) :\ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a\ \} \\
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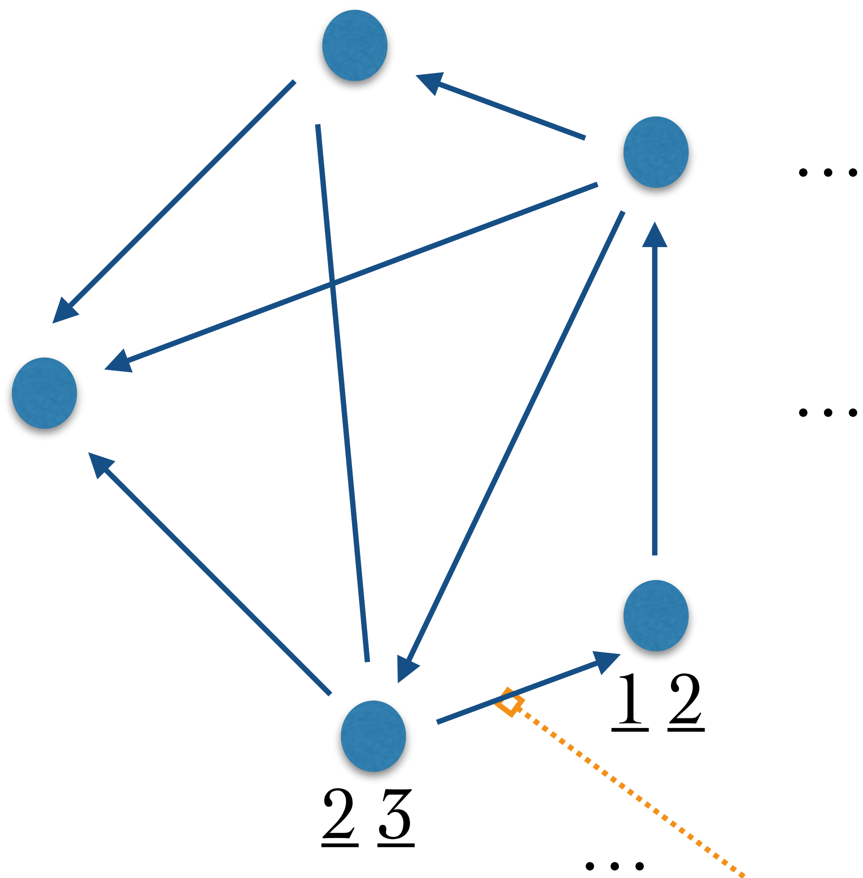
\end{array}\right\} E$$

$$\{\ (ab,\ cd) :\ a,b,c,d \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a,\ \ldots\ \}$$

$$\{\ (ab,\ ba) :\ a,b \text{ in } \mathcal{A},\ a\neq b\ \}$$

$$\{\ (ba,\ ab) :\ a,b \text{ in } \mathcal{A},\ a\neq b\ \}$$

$$V = \{\, ab : \ a, b \text{ in } \mathcal{A},\ a \neq b \,\}$$



orbits in V×V:

$$\left.\begin{array}{l}
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\{\, (ab, cb) : \ a,b,c \text{ in } \mathcal{A},\ a \neq b,\ b \neq c,\ c \neq a \,\} \\
\{\, (ba, bc) : \ a,b,c \text{ in } \mathcal{A},\ a \neq b,\ b \neq c,\ c \neq a \,\} \\
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\end{array}\right\} E$$

$$\{\, (ab, cd) : \ a,b,c,d \text{ in } \mathcal{A},\ a \neq b,\ b \neq c,\ c \neq a,\ \ldots \,\}$$

$$\{\, (ab, ba) : \ a,b \text{ in } \mathcal{A},\ a \neq b \,\}$$

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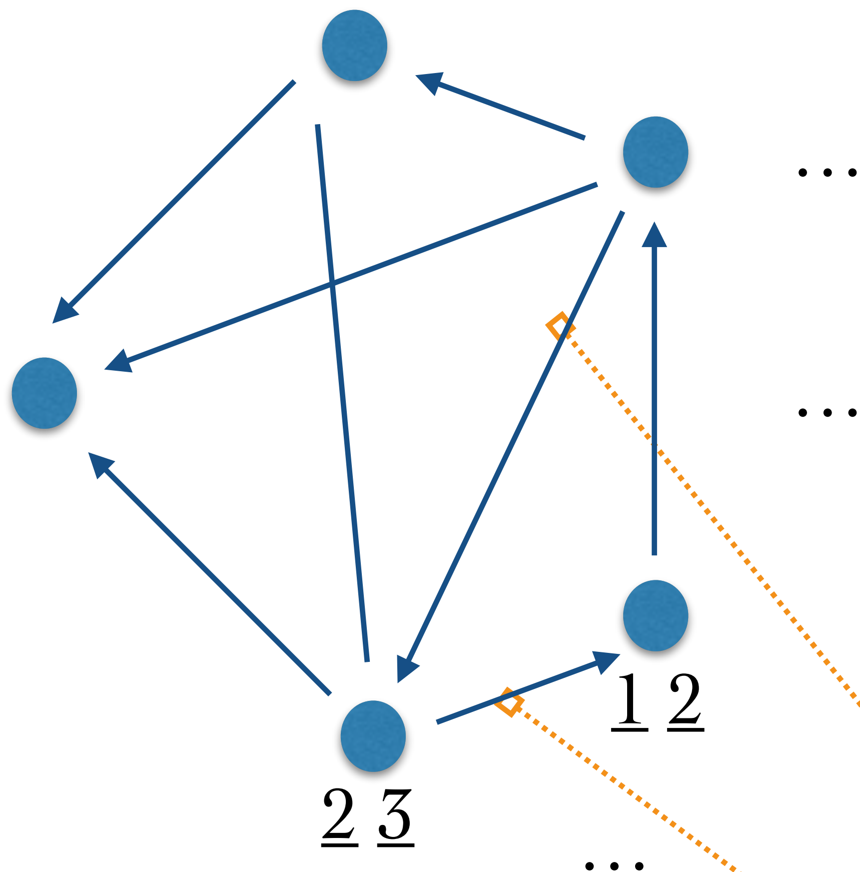
$$V = \{\, ab : \ a, b \text{ in } \mathcal{A},\ a \neq b \,\}$$



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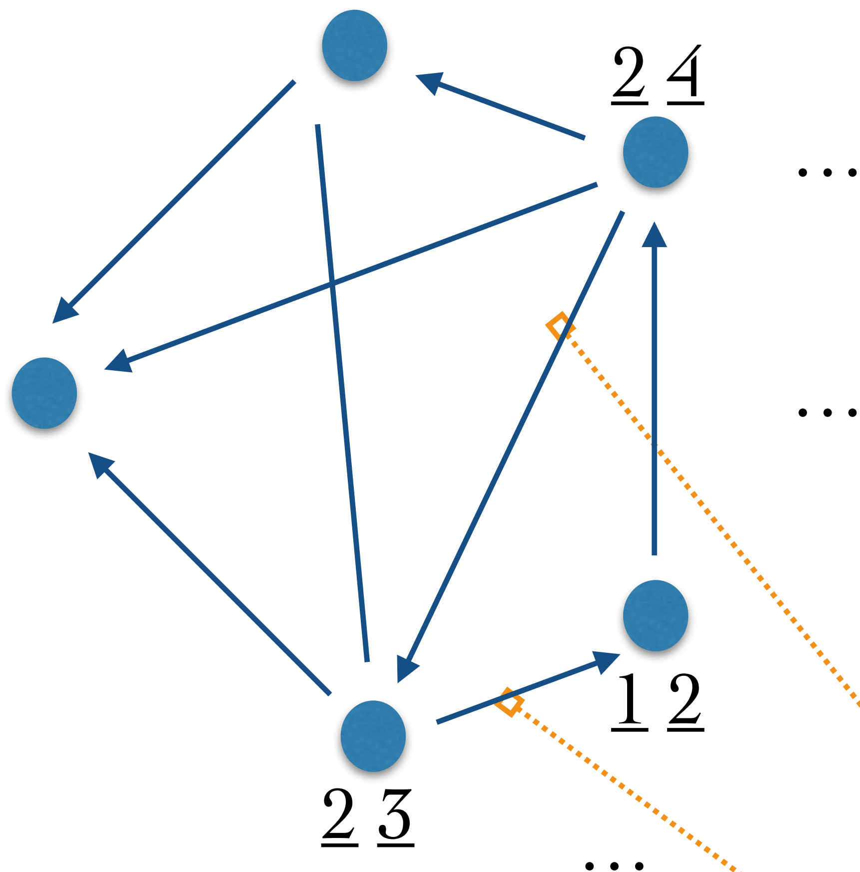
\{\, (ab,\ cb) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
\{\, (ba,\ bc) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\
\{\, (ba,\ cb) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\}
\end{array}\right\} E$$

$$\{\, (ab,\ cd) : \ a,b,c,d \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a,\ \ldots \,\}$$

$$\{\, (ab,\ ba) : \ a,b \text{ in } \mathcal{A},\ a\neq b \,\}$$

$$\{\, (ba,\ ab) : \ a,b \text{ in } \mathcal{A},\ a\neq b \,\}$$

V = { ab : a, b in $\mathcal{A}$, a ≠ b }



orbits in V×V:

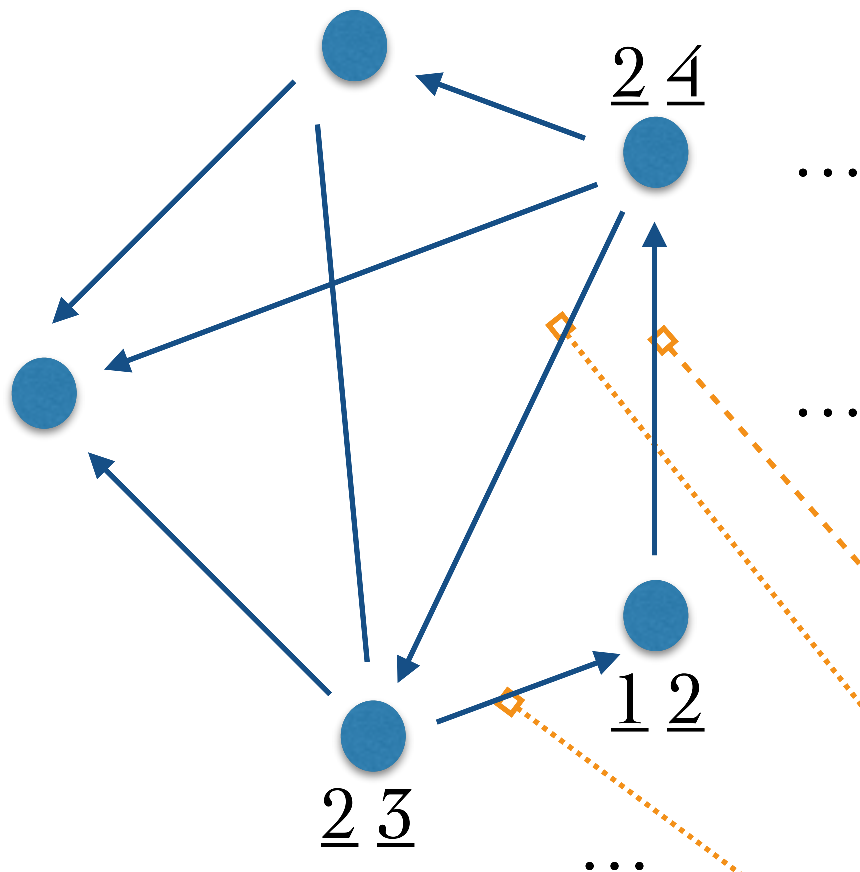
$$\left.\begin{array}{l}
\{\ (ab,\ bc) :\ a,b,c \text{ in } \mathcal{A},\ a{\neq}b,\ b{\neq}c,\ c{\neq}a\ \} \\
\{\ (ab,\ cb) :\ a,b,c \text{ in } \mathcal{A},\ a{\neq}b,\ b{\neq}c,\ c{\neq}a\ \} \\
\{\ (ba,\ bc) :\ a,b,c \text{ in } \mathcal{A},\ a{\neq}b,\ b{\neq}c,\ c{\neq}a\ \} \\
\{\ (ba,\ cb) :\ a,b,c \text{ in } \mathcal{A},\ a{\neq}b,\ b{\neq}c,\ c{\neq}a\ \}
\end{array}\right\}\ E$$

{ (ab, cd) : a,b,c,d in $\mathcal{A}$, a≠b, b≠c, c≠a, … }

{ (ab, ba) : a,b in $\mathcal{A}$, a≠b }

{ (ba, ab) : a,b in $\mathcal{A}$, a≠b }

$V = \{\, ab : \ a, b \text{ in } \mathcal{A},\ a \neq b \,\}$



orbits in V×V:

$$\left.\begin{array}{l} \{\, (ab, bc) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\ \{\, (ab, cb) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\ \{\, (ba, bc) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \\ \{\, (ba, cb) : \ a,b,c \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a \,\} \end{array}\right\} E$$

$\{\, (ab, cd) : \ a,b,c,d \text{ in } \mathcal{A},\ a\neq b,\ b\neq c,\ c\neq a,\ \dots \,\}$

$\{\, (ab, ba) : \ a,b \text{ in } \mathcal{A},\ a\neq b \,\}$

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restrictions and variants

Theorem: For finite signatures:

- 😀 $\emptyset$-definable homomorphism problem decid.
- ☹️ definable homomorphism problem undecid.
- 😀 homomorphism problem decid.
- ☹️ homomorphic extension problem undecid.

restrictions and variants

- definable homomorphisms vs. arbitrary ones

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still undecidable if target structure is fixed and ϕ-definable?

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- restriction on homomorphisms: injective, embedding

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still undecidable if target structure is fixed, and embedding is required?

restrictions and variants

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- **structured atoms**

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- restriction on homomorphisms: injective, embedding
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we prove decidability
only under assumptions
on atoms

Plan

- homomorphism problem
- orbit-finite homomorphism problem
- decidability/undecidability results
- **open problem**

isomorphism problem

$$\mathbb{A} \xrightarrow{\textbf{iso ?}} \mathbb{B}$$

isomorphism problem

$$\mathbb{A} \xrightarrow{\textbf{iso ?}} \mathbb{B}$$

definable or arbitrary?

isomorphism problem

$$\mathbb{A} \xrightarrow{\textbf{iso ?}} \mathbb{B}$$

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Thank you!