Finding a Puiseux expansion of a curve in parametric form Segovia, YMIS 2010

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Question

Let us be given a curve in a parametric form

$$\begin{cases} x(t) &= t^4 + 2t^5 + 3t^6 + 4t^7 \\ y(t) &= t^6 + 3t^7 + 11t^8 + 30t^9 + 5t^{10}. \end{cases}$$

We ask what is the topological type of the singularity (Puiseux expansion)

 $y = x^{3/2} + c_1 x^{7/4} + c_2 x^{8/4} + c_3 x^{9/4} + c_4 x^{10/4} + c_5 x^{11/4} + \dots$

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We ask what is the topological type of the singularity (Puiseux expansion)

$$y = c_0 x^{3/2} + c_1 x^{7/4} + c_2 x^{8/4} + c_3 x^{9/4} + c_4 x^{10/4} + c_5 x^{11/4} + \dots$$

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Standard approach

We write

Therefore $y = x^{3/2}$ is equal to

 $5t^{0} + 20t^{0} + \frac{17}{9}t^{20} + \dots$

 $0_1 c_2 = 5$. Now we look at $y = x^{3/2} = 5x^2$. After "simple" computations we get

$\frac{417}{8}e^{10} - \frac{417}{8}e^{10} - \frac{821}{8}e^{11} - \dots$

We get that $c_2 = 0$, $c_1 \neq 0$ and $c_2 \neq 0$ (in the latter we have to compute also $\gamma = \pi^2/2 = 5\pi^2 + 4\frac{1}{2}\kappa^2/2$).

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Standard approach

We write

$$x^{3/2} = (t^4 + 2t^5 + 3t^6 + 4t^7)^{3/2}$$

Therefore $y - x^{3/2}$ is equal to

$$5t^8 + 20t^9 + \frac{17}{8}t^{10} + \dots$$

Hence $c_1 = 0, c_2 = 5$. Now we look at $y - x^{3/2} - 5x^2$. After "simple" computations we get

$$y - x^{3/2} - 5x^2 = -\frac{417}{8}t^{10} - \frac{821}{8}t^{11} - \dots$$

We get that $c_3 = 0$, $c_4 \neq 0$ and $c_5 \neq 0$ (in the latter we have to compute also $y - x^{3/2} - 5x^2 + \frac{417}{8}x^{5/2}$).

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We write

$$x^{3/2} = t^6 (1 + (2t + 3t^2 + 4t^3))^{3/2}$$

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$$x^{3/2} = t^6(1 + \frac{3}{2}(2t + 3t^2 + 4t^3) + \frac{3}{8}(2t + 3t^2 + 4t^3) + \dots)$$

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Standard approach

We write

$$x^{3/2} = t^6 + 3t^7 + 11t^8 + 30t^9 + \frac{457}{8}t^{10} + \frac{821}{8}t^{11} + \dots$$

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Look at the order at zero

Write

$$y = c_0 x^{q/p} + c_1 x^{(q+1)/p} + c_2 x^{(q+2)/p} + \dots$$

We divide both sides by $x^{q/p}$. We get

where

$$P_1 = \dot{y}x - \frac{q}{\rho}y\dot{x}.$$

If the order of P_1 at zero is $q + (p - 1) + r_1$, we know that $c_1 = \cdots = c_{r_1-1} = 0 \neq c_{r_1}$. In the above example $P_1 = 10t^{11} + 65t^{12} + 35t^{13} - 85t^{14} - 165t^{15} - 10t^{16}$, so $r_1 = 2$, $c_1 = 0$ and $c_2 = 5$.

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$$y = c_0 x^{q/p} + c_1 x^{(q+1)/p} + c_2 x^{(q+2)/p} + \dots$$

We divide both sides by $x^{q/p}$. We get

$$\frac{y}{x^{q/p}} = c_0 + c_1 x^{1/p} + c_2 x^{2/p} + \dots$$

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Look at the order at zero

Write

$$y = c_0 x^{q/p} + c_1 x^{(q+1)/p} + c_2 x^{(q+2)/p} + \dots$$

We divide both sides by $x^{q/p}$, differentiate with respect to t. We get

$$\frac{y}{x^{q/p}} = c_0 + c_1 x^{1/p} + c_2 x^{2/p} + \dots$$
$$\frac{\dot{y}x - \frac{q}{p}y\dot{x}}{x^{q/p+1}} = c_1 \frac{1}{p} \dot{x} x^{1/p-1} + c_2 \frac{2}{p} \dot{x} x^{2/p-1} + \dots$$

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$$\frac{\dot{y}x - \frac{q}{p}y\dot{x}}{x^{q/p+1}} = c_1 \frac{1}{p} \dot{x} x^{1/p-1} + c_2 \frac{2}{p} \dot{x} x^{2/p-1} + \dots$$
$$\dot{y}x - \frac{q}{p} y\dot{x} = \frac{c_1}{p} \dot{x} x^{(q+1)/p} + \frac{2c_2}{p} \dot{x} x^{(q+2)/p} + \dots$$

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$$P_1(t) = \frac{c_1}{p} \dot{x} x^{(q+1)/p} + \frac{2c_2}{p} \dot{x} x^{(q+2)/p} + \dots,$$

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Looking further at the order at zero

From the equation

$$P_{1}(t) = \frac{r_{1}c_{r_{1}}}{p} \dot{x} x^{(q+r_{1})/p} + \frac{(r_{1}+1)c_{r_{1}+1}}{p} \dot{x} x^{(q+r_{1}+1)/p} + \dots$$

we can go further dividing, differentiating and multiplying. We get
$$P_{2} = S_{2}(r_{2})c_{r_{2}}\dot{x}^{3} x^{(q+r_{2})/p} + S_{2}(r_{2}+1)c_{r_{2}+1}\dot{x}^{3} x^{(q+r_{2}+1)/p} + \dots,$$

where

We see that again $c_{r_1+1}=\cdots=c_{r_2-1}=0
eq c_{r_2}.$ In our case $P_2=-1680t^{19}-11520t^{20}-39060t^{21}-\ldots.$

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From the equation

$$P_1(t) = \frac{r_1 c_{r_1}}{p} \dot{x} x^{(q+r_1)/p} + \frac{(r_1+1)c_{r_1+1}}{p} \dot{x} x^{(q+r_1+1)/p} + \dots$$

we assume here that ord $P_1 = q + (p - 1) + r_1$ we can go further dividing, differentiating and multiplying. We get

 $P_2 = S_2(r_2)c_{r_2}\dot{x}^3 x^{(q+r_2)/p} +$

 $S_2(r_2+1)c_{r_2+1}\dot{x}^3x^{(q+r_2+1)/p}+\ldots,$

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where

- $P_2 = x\dot{x}P_1' (\frac{q+n}{p}\dot{x}^2 + x\ddot{x})P_1.$
- r_2 is such that $\operatorname{ord}_{t=0} P_2 = q + 3(p-1) + r_2$

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 $P_2 = -1680t^{19} - 11520t^{20} - 39060t^{21} - .$

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we can go further dividing, differentiating and multiplying. We get

$$\begin{split} P_2 &= S_2(r_2) c_{r_2} \dot{x}^3 x^{(q+r_2)/p} + \\ &\quad S_2(r_2+1) c_{r_2+1} \dot{x}^3 x^{(q+r_2+1)/p} + \dots, \end{split}$$

where

• $P_2 = x\dot{x}P_1' - (\frac{q+r_1}{p}\dot{x}^2 + x\ddot{x})P_1.$

• r_2 is such that $\operatorname{ord}_{t=0} P_2 = q + 3(p-1) + r_2$

• S_2 is some coefficient depending on q, p and r_1 .

We see that again $c_{r_1+1} = \cdots = c_{r_2-1} = 0 \neq c_{r_2}$. In our case

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$$P_1(t) = \frac{r_1 c_{r_1}}{p} \dot{x} x^{(q+r_1)/p} + \frac{(r_1+1)c_{r_1+1}}{p} \dot{x} x^{(q+r_1+1)/p} + \dots$$

we can go further dividing, differentiating and multiplying. We get

$$\begin{aligned} P_2 &= S_2(\mathbf{r_2}) c_{\mathbf{r_2}} \dot{x}^3 x^{(q+\mathbf{r_2})/p} + \\ &\quad S_2(\mathbf{r_2}+1) c_{\mathbf{r_2}+1} \dot{x}^3 x^{(q+\mathbf{r_2}+1)/p} + \dots, \end{aligned}$$

where

•
$$P_2 = x\dot{x}P_1' - (\frac{q+r_1}{p}\dot{x}^2 + x\ddot{x})P_1.$$

• r_2 is such that $\operatorname{ord}_{t=0} P_2 = q + 3(p-1) + r_2$

• S_2 is some coefficient depending on q, p and r_1 . We see that again $c_{r_1+1} = \cdots = c_{r_2-1} = 0 \neq c_{r_2}$. In our case

 $P_2 = -1680t^{19} - 11520t^{20} - 39060t^{21} - \dots$

Hence $c_3 = 0$, $c_4 \neq 0$.

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General formula

In general from the expression

$$P_k = S_k(r_k) \dot{x}^{2k-1} c_{r_k} x^{(q+r_k)/p} + \dots$$

upon applying the above procedure we get

$$P_{k+1} = S_{k+1}(r_{k+1})\dot{x}^{2k+1}c_{r_{k+1}}x^{(q+r_{k+1})/p} + \dots$$

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Advantages

• Quick and elegant way to compute.

- We can also get the Puiseux coefficients, not only whether they are zero or no.
- Hard-core example: $x(t) = t^{12} + t^{13} + \frac{37}{28}t^{14}$, $y(t) = t^{18} + \frac{3}{2}t^{19} + \frac{33}{14}t^{20} + \frac{13}{14}t^{21} + \frac{675}{1568}t^{22} - \frac{675}{3136}t^{23}$
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Deformations

- A deformation is a family $x_s(t) = a_p(s)t^p + a_{p+1}(s)t^{p+1} + \dots$, $y_s = b_q(s)t^q + b_{q+1}(s)t^{q+1} + \dots$, $s \in B(0, 1)$;
- If a_p(s) ≠ 0 for s ≠ 0, the Puiseux coefficients c_j(s) are well-defined.
- If a_p(0) = 0, the limit lim_{s→0} c_j(s) might not exist, or loose its topological meaning.
- The polynomials $P_k(t, s)$ behave well under passing to the limit.
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