Khovanov homology and periodic knots

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Periodic knots

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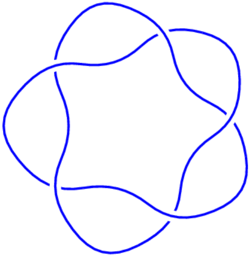

Alexander polynomial obstruction

Theorem (Murasugi criterion)

Suppose $K \subset S^3$ is a p-periodic knot with p a prime. Let Δ be the Alexander polynomial of K and Δ' be the Alexander polynomial of the quotient knot $K/\mathbb{Z}_p$. Let l be the absolute value of linking number of K with the symmetry axis. Then $\Delta_0 | \Delta$ and up to multiplication by a power of t we have

$$\Delta \equiv \Delta_0^p (1 + t + \ldots + t^{l-1})^{p-1} \bmod p. \tag{1}$$

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Often one can find factors of Δ over integers.

Naik's criterion

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- Restrictions for $H_1(\Sigma^m(K))$ of periodic knots.

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- $\mathbb{Z}_p$ acts on the spin-c structures.
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- These invariants appear with multiplicities.
- If K is quasi-alternating, then $\Sigma^2(K)$ is an L–space and d-invariants are computable.

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- Hillman, Livingston and Naik generalize the Murasugi's periodicity criterion for twisted Alexander polynomials.
- Computable, if we know a representation.
- It is known when knot group admits a representation into a dihedral group.
- Other representations are sometimes harder to find.

None of the above criteria can be used for $\Delta = 1$ knots. The TAP criterion is possible, but requires finding non-trivial representations.

Jones and HOMFLYPT

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Theorem (HOMFLYPT criterion)

Let $\mathcal{R}$ be a unital subring in $\mathbb{Z}[a^{\pm 1}, z^{\pm 1}]$ generated by $a, a^{-1}, \frac{a+a^{-1}}{z}$ and z. For a prime number p let $\mathcal{I}_p$ be the ideal in $\mathcal{R}$ generated by p and z^p. If a knot K is p-periodic and $P(a,z)$ is its HOMFLYPT polynomial, then

$$P(a,z) \equiv P(a^{-1}, z) \bmod \mathcal{I}_p.$$

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Przytycki shows an effective way of applying Theorem 4.

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For any Λ–module M define the equivariant Khovanov homology as

$$\mathsf{EKh}(L; M) = \mathsf{Ext}_\Lambda(M, \mathsf{CKh}(D; R)).$$

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- Most important example: $M = \Lambda$.

Equivariant Khovanov. Properties.

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- On the other hand we have a Schur decomposition of $\mathrm{Hom}_\Lambda(\Lambda; CKh(D))$.

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- Equivariant Lee homology for knots is easy.
- The equivariant Khovanov polynomial and the equivariant Lee polynomial differ by a specific polynomial.
- And $\mathrm{EKh}(L; \Lambda)$ splits as a sum over different representations of Λ.

Main criterion

Theorem (—,Politarczyk, 2017)

Let K be a p^n-periodic, where p is an odd prime. Suppose that $\mathbb{F} = \mathbb{Q}$ or $\mathbb{F}_r$, for a prime r such that $r \neq p$, and r has maximal order in the multiplicative group mod p^n. Set $c = 1$ if $\mathbb{F} = \mathbb{F}_2$ and $c = 2$ otherwise. Then the Khovanov polynomial $\mathrm{KhP}(K; \mathbb{F})$ decomposes as

$$\mathrm{KhP}(K; \mathbb{F}) = \mathcal{P}_0 + \sum_{j=1}^{n} (p^j - p^{j-1}) \mathcal{P}_j, \tag{2}$$

where

$$\mathcal{P}_0, \mathcal{P}_1, \dots, \mathcal{P}_n \in \mathbb{Z}[q^{\pm 1}, t^{\pm 1}],$$

are Laurent polynomials such that

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1. $\mathcal{P}_0 = q^{s(K,\mathbb{F})}(q + q^{-1}) + \sum_{j=1}^{\infty}(1 + tq^{2cj})\mathcal{S}_{0j}(t, q)$, and the polynomials $\mathcal{S}_{0j}$ have non-negative coefficients;

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3. If the width of $\mathrm{Kh}(K;\mathbb{F})$ is equal to w, then $S_{kj} = 0$ for $j > \frac{c}{2}w$.
4. $\mathcal{P}_k(-1,q) - \mathcal{P}_{k+1}(-1,q) \equiv \mathcal{P}_k(-1,q^{-1}) - \mathcal{P}_{k+1}(-1,q^{-1}) \pmod{q^{p^{n-k}} - q^{-p^{n-k}}}$;

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Without (4) the condition is trivial!

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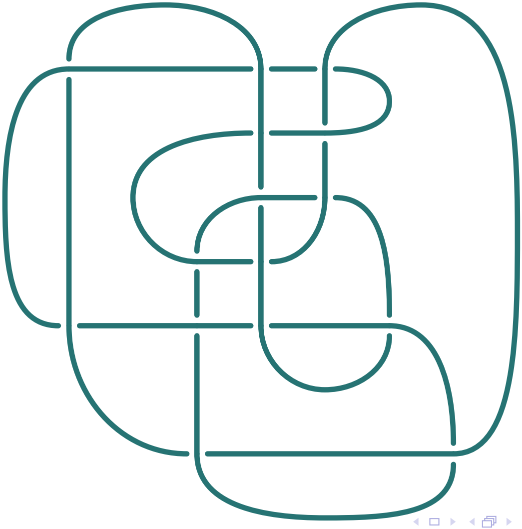

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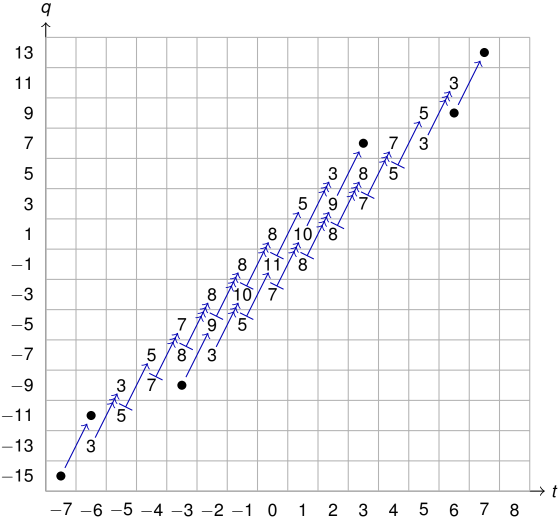
Khovanov homology



Khovanov Polynomial

$$\begin{aligned}\mathrm{KhP} = {} & q + q^{-1} + (1 + tq^4)(t^{-7}q^{-15} + 3t^{-6}q^{-13} + t^{-5}q^{-11} + \\ & + 3t^{-4}q^{-9} + t^{-3}q^{-9} + 3t^{-2}q^{-7} \\ & + t^{-1}q^{-5} + 3t^{-1}q^{-3} + q^{-3} + q^{-1} + 3tq + \\ & + t^2q^3 + 3t^3q^3 + t^4q^5 + 3t^5q^7 + t^6q^9 \\ & + 4(t^{-5}q^{-11} + t^{-4}q^{-9} + 2t^{-3}q^{-7} + 2t^{-2}q^{-5} + \\ & + t^{-1}q^{-5} + t^{-1}q^{-3} + 2tq^{-1} \\ & + q^{-3} + q^{-1} + 2t^2q + t^3q^3 + t^4q^5)).\end{aligned}$$

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$\widetilde{\Xi}(q) = (q + q^{-1} + (1 + tq^4)(\mathcal{S}'_{01} - \mathcal{S}'_{11})|_{t=-1}$ and set
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In general checking all possibilities requires 20736 possibilities.

Speed up

Let $\delta = at^u q^j$. The change $S'_{11} \mapsto S'_{11} - \delta$, $S'_{01} \mapsto S'_{01} + 4\delta$ induces the change

$$\Xi \mapsto \Xi + aT_{ij},$$

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Reducing modulo $q^5 - q^{-5}$ we get $T_{ij} = (-1)^i R_{j'}$ with $j' = j \bmod 10$ and

$$\begin{aligned} R_1 = R_5 &= 5(q - q^9), \\ R_3 &= 10(q^3 - q^7), \\ R_7 = R_9 &= 5(-q - q^3 + q^7 + q^9). \end{aligned}$$

Speed up 2

δ is such that $S'_{11} - \delta$ and δ have non-negative coefficients. Hence the question is, whether

$$\Xi = a_1 R_1 + a_3 R_3 + a_7 R_7$$

with

$$a_1 \in \{-1, 0, 1, 2, 3, 4, 5, 6\},$$
$$a_3 \in \{-3, -2, -1, 0\},$$
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This is impossible. The knot is not periodic.

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