Selected topics in graph theory
17.12.2018 — homework, set 3

Problem 1. Characterize all d-regular connected multigraphs G for which the graph constructed from G^2 by removing all loops is bipartite.

Problem 2. A simple graph G is *super-regular* if there exist integers d, a, and b such that G is d-regular, every two adjacent vertices of G have exactly a common neighbors, and every two nonadjacent vertices of G have exactly b common neighbors. Prove that the adjacency matrix of a super-regular graph has at most three distinct eigenvalues. Furthermore, prove that if there are three distinct eigenvalues of this adjacency matrix and all of them have pairwise different multiplicities, then all three eigenvalues are integers.

Problem 3. Let G and H be two d-regular n-vertex graphs that have exactly the same spectrum, that is, their adjacency matrices have exactly the same eigenvalues with the same multiplicities. Show that the length of the shortest odd cycles in G and H are equal.