Exercises

1. Consider the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{Q} - \{0\}$, $\mathbb{R}$, $\mathbb{R} - \{0\}$,

$$A = \{3 - \frac{1}{2n} \mid n \in \mathbb{N} - \{0\}\},$$
$$B = \{\pi - \frac{2}{n} \mid n \in \mathbb{N} - \{0\}\} \cup \{4\}$$
$$C = \{0\} \cup \{\frac{1}{n} \mid n \in \mathbb{N} - \{0\}\} \cup \{2 - \frac{1}{n} \mid n \in \mathbb{N} - \{0\}\}$$

   ordered by the relation $\leqslant$. Which of them are well-founded?

2. Is the set $\langle \mathbb{N}^*, \leqslant_{lex} \rangle$ well-founded? And the set $\langle \mathbb{N}^2, \leqslant_{lex} \rangle$?

3. Consider partial ordering of the set $\{0, 1\}^{\mathbb{N}}$

$$f \leqslant g \text{ iff. } \forall x (f(x) \leqslant g(x)).$$

   (a) Is the ordering linear?
   (b) Is it well-founded?
   (c) Does there exist an infinite chain?
   (d) Does there exist an infinite antichain?
   (e) Is the set a complete lattice?