MATHEMATICAL MODEL OF THE TURBULENT FLOW BY MAREK BURNAT

In the simplest case the equations of the model are as follows

$$\frac{\partial \rho}{\partial t}(t,\underline{x},\underline{\alpha}) + \underline{\alpha}\nabla_{\underline{x}}\rho(t,\underline{x},\underline{\alpha}) - \nu\Delta_{\underline{x}}\rho(t,\underline{x},\underline{\alpha}) + \tag{1}$$

$$-\kappa\int_{\mathcal{A}} M(t,\underline{x},\underline{\alpha},\underline{\beta},\rho(t,\underline{x},\underline{\alpha}),\rho(t,\underline{x},\underline{\beta}))d\underline{\beta} = 0.$$

Clearly the above equations have to be completed by certain convenient initial and boundary conditions. The function to be found is $\rho(t,\underline{x},\underline{\alpha})$, it is the *$\alpha$-mass density function*. So called 'Euler mass density function' can be computed by integration

$$\rho(t,\underline{x}) = \int_{\mathcal{A}} \rho(t,\underline{x},\underline{\alpha})d\underline{\alpha}.$$

The independent variables of the model are
t- time, $\underline{x} \in \Omega \subset R^3$- space variables, $\underline{\alpha} \in \mathcal{A} \subset R^3$ - "α-velocities". The nonlinear function M of several variables is called *the mixer*. It serves to mix together various elements of the set

$$\{\rho(t,\underline{x},\underline{\alpha})| \quad \underline{x} \in \Omega, \quad \underline{\alpha} \in \mathcal{A}\}$$

in such a way that the most important conservation laws are satisfied. Some details of this model and of its first one-dimensional numerical approximation can be found in [1].

Now, from the beginning of 2010 the new 2-dimensional approximation of the model have been implemented on the cluster halo2 (ICM U.W.) It is worth to note that in two-dimensional model the sets Ω and $\mathcal{A}$ are of the dimension 2. This means that the problem (if we don't take into consideration the time variable t) is of the dimension 4.

References

[1] Marek Burnat, Krzysztof Moszyski "ON SOME PROBLEMS IN MATHEMATICAL MODELING OF TURBULENT FLOW", Journal of Technical Physics, 48, 3-4, 171-192, 2007.