LECTURE 5 Appendix
Tree-diagonal systems
Here lp is the number of processors used.

SCHUR VARIABLES

EVEN NUMBERS

$$Z_0 = e_0^T X_1$$

$$Z_2 = e_0^T X_2$$

$$Z_4 = e_0^T X_3$$

$$Z_6 = e_0^T X_4$$

$$\cdots$$

$$Z_{2(lp-1)-2} = e_0^T X_{lp-1}$$

Numeration algorithm:

$$Z_{2k-2} = e_0^T X_k \;\; k = 1, 2, \cdots, lp - 1$$

ODD NUMBERS

$$Z_1 = e_R^T X_0$$

$$Z_3 = e_R^T X_1$$

$$Z_5 = e_R^T X_2$$

$$Z_7 = e_R^T X_3$$

$$\cdots$$

$$Z_{2(l_p-1)+1} = e_R^T X_{lp-2}$$

Numeration algorithm:

$$Z_{2k+1} = e_R^T X_k \;\; k = 0, 1, \cdots, lp - 2$$

In the SCHUR SYSTEM THERE IS 2(lp-1) EQUATIONS.

SCHUR SYSTEM OF EQUATIONS for $lp = 4$

- how to number equations of the Schur System?

- look at the RHS!

$$b^0 e_R^T V^0 z_0 + z_1 = e_R^T \tilde{X}_0 \tag{0}$$

$$z_0 + a^1 e_0^T W^1 z_1 + b^1 e_0^T V^1 z_2 = e_0^T \tilde{X}_1 \tag{1}$$

$$a^1 e_R^T W^1 z_1 + b^1 e_R^T V^1 z_2 + z_3 = e_R^T \tilde{X}_1 \tag{2}$$

$$z_2 + a^2 e_0^T W^2 z_3 + b^2 e_0^T V_2 z_4 = e_0^T \tilde{X}_2 \tag{3}$$

$$a^2 e_R^T W^2 z_3 + b^2 e_R^T V^2 z_4 + z_5 = e_R^T \tilde{X}_2 \tag{4}$$

$$z_4 + a^3 e_R^T W^3 z_5 = e_0^T \tilde{X}_3 \tag{5}$$

SCHUR SYSTEM MATRIX WITH THE RHS COLUMN
($lp = 4$)

$$\begin{bmatrix}
b^0 e_R^T V^0 & 1 & \cdot & \cdot & \cdot & \cdot & e_R^T \tilde{X}_0 \\
1 & a^1 e_0^T W^1 & b^1 e_0^T V^1 & \cdot & \cdot & \cdot & e_0^R \tilde{X}_1 \\
\cdot & a^1 e_R^T W^1 & b^1 e_R^T V^1 & 1 & \cdot & \cdot & e_R^T \tilde{X}_1 \\
\cdot & \cdot & 1 & a^2 e_0^T W^2 & b^2 e_0^T V^2 & \cdot & e_0^T \tilde{X}_2 \\
\cdot & \cdot & \cdot & a^2 e_R^T W^2 & b^2 e_R^T V^2 & 1 & e_R^T \tilde{X}_2 \\
\cdot & \cdot & \cdot & \cdot & 1 & a^3 e_0^T W^3 & e_0^T \tilde{X}_3
\end{bmatrix}$$