

17:GR G - finite group
 \mathbb{C}^n - set of G -complexes
 \mathbb{C}^n - set of G -complexes with two parts
 \mathbb{C}^n - set of pairs (K, L) with $L \subset K$ subp.

Def: An equivariant cobordism theory (generalization) \mathcal{Y}^G is a collection of functors

- with axioms:
 1° if f_0 is a equiv. homotopy to f_1 then $\mathcal{Y}^G(f_0) = \mathcal{Y}^G(f_1)$
 2° $(K, K \cup L) \in \mathcal{Y}^G(K, L)$ induces a isomorphism
 $\mathcal{Y}^G(K, K \cup L) \xrightarrow{\cong} \mathcal{Y}^G(K, L)$
 3° $(K, L) \in \mathbb{C}^2 \xrightarrow{\text{incl.}} \mathcal{Y}^G(K, L) \xrightarrow{\text{incl.}} \mathcal{Y}^G(K, K \cup L) \xrightarrow{\text{incl.}} \mathcal{Y}^G(K, L) \xrightarrow{\text{incl.}} \mathcal{Y}^G(K, L)$

Remark: G is abelian $\mathcal{Y}^G(K, L)$ has a natural structure of a G -module.

$\mathbb{C}^n \rightarrow \mathbb{C}^m$ $\mathbb{C}^n \rightarrow \mathbb{C}^m$ base point of K
 $(x, y) \mapsto (x, z)$ $(x, y) \mapsto (x, z)$

\mathbb{C}^n is a base point in \mathbb{C}^n (if L is G -free, base point of $\mathbb{C}^n = K^*$ is disjoint from L)

$K \in \mathbb{C}^n$, ZK - reduced suspension of K . Exp. cob. theory on \mathbb{C}^n is:

- $\tilde{\mathcal{Y}}^G: \mathbb{C}^n \rightarrow \mathcal{A}b$, $\sigma^G(K): \tilde{\mathcal{Y}}^G(K) \rightarrow \tilde{\mathcal{Y}}^G(ZK)$
 1° $f_0 \sim f_1$ $\tilde{\mathcal{Y}}^G(f_0) = \tilde{\mathcal{Y}}^G(f_1)$
 2° $\sigma^G(K)$ is iso for $V, W \in \mathbb{C}^n$
 3° $\tilde{\mathcal{Y}}^G(K \cup L) \rightarrow \tilde{\mathcal{Y}}^G(K) \rightarrow \tilde{\mathcal{Y}}^G(L)$

Cor: $H^*(pt)$.

In equiv. case we identify maps and coset spaces of G

$\frac{\mathbb{C}^n}{G} \cong \text{Hom}(G, \mathbb{C}^n)$
 \uparrow "action"
 $G/H \rightarrow G/K$
 $\mathcal{Y}^G(G/H) \rightarrow \mathcal{Y}^G(G/K)$

A theory on \mathbb{C}^n is called equivariant when it satisfies an additional axiom:

4° $\mathcal{Y}^G(G/H) = 0$ $\forall H \leq G$
 5° $\mathcal{Y}^G(G/H) = 0$ $\forall H \leq G$

Category of equivariant abelian $\mathcal{A}b$:

objects - left $\mathcal{A}b$ -spaces $\mathcal{A}b^G$
 Morphisms - equiv. maps.

Character of morphisms in $\mathcal{A}b$:

$f: \mathcal{A}b^G \rightarrow \mathcal{A}b^G$ - any map, $f(x) = gx$ for some $g \in G$

f equivariant $\Leftrightarrow f(gx) = gf(x) \forall g \in G$

$f(gx) = gf(x) \Leftrightarrow f(gx) = gf(x) \Leftrightarrow f(gx) = gf(x) \Leftrightarrow f(gx) = gf(x)$

Let $a \in G$ s.t. $a^{-1}Ha \in K$ $\hat{a}: \mathcal{A}b^G \rightarrow \mathcal{A}b^G$ $\hat{a}(gx) = gax$

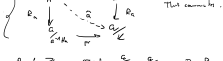
Then \hat{a} is equiv. and any equiv. map is of this form.

$\hat{a} \in \text{Con}(K) \subset \text{Con}(G) \cong G/K$

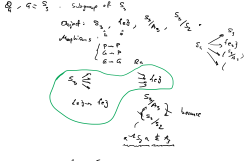
Suppose that $a^{-1}Ha \in K$. This induces $\mathcal{A}b^{a^{-1}Ha} \rightarrow \mathcal{A}b^K$ (inj)

and $H \in \text{Con}(K) \Rightarrow \mathcal{A}b^H \rightarrow \mathcal{A}b^K$

$R_a: \mathcal{A}b^H \rightarrow \mathcal{A}b^{a^{-1}Ha}$ ($gH \mapsto gHa = ga(a^{-1}Ha)$) and $R_a: \mathcal{A}b^{a^{-1}Ha} \rightarrow \mathcal{A}b^K$
 $gaKa^{-1} \mapsto gaKa^{-1} = gaK$

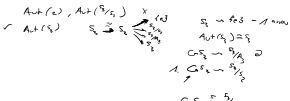


$\mathcal{A}b$ for $\mathcal{A}b$ objects $\mathcal{A}b^G, \mathcal{A}b^{G/H}$ $P \rightarrow P$
 $G \rightarrow P$
 $G \rightarrow G, P$



$\text{Aut}(S_0) \subset \text{Aut}(S_1/S_0)$

Why? $\text{Group} \cong \frac{N(S_0)}{S_0} \cong \frac{N(S_1)}{S_1} \rightarrow \frac{N(S_2)}{S_2}$
 $1 = \frac{N(S_0)}{S_0} \cong \frac{N(S_1)}{S_1} \cong \frac{N(S_2)}{S_2}$
 $N(S_0) \cong S_0$ s.t. $\mathcal{A}b_{S_0} = \mathcal{A}b_{S_0}$



$\left\{ \begin{aligned} \text{Aut}(\frac{S_0}{S_0}, \frac{S_1}{S_0}) &= \text{Aut}(S_0/S_0) \\ \text{Aut}(S_0/S_0) &= \text{Aut}(S_0/S_0) \end{aligned} \right\}$

$\text{Aut}(S_0, S_1/S_0) = S_1/S_0$
 $\text{Aut}(S_0, S_2/S_0) = S_2/S_0$
 $\text{Aut}(S_0, S_1/S_1) = S_1/S_1$
 $\text{Aut}(S_0, S_2/S_2) = S_2/S_2$

$\mathcal{A}b_{S_0}$

Generic coefficient systems:

Def: A G -coeff. sys. for G is a contravariant functor $\mathcal{A}b^G \rightarrow \mathcal{A}b$

A morphism between M, N is a unit transformation $\tau: M \rightarrow N$

G -coeff. sys. form a category \mathcal{C}_G (abelian).

Example: 1° \mathcal{Y}^G -equiv. cob. theory and $g \in \mathcal{Y}^G$

$h^G: \mathcal{A}b^G \rightarrow \mathcal{A}b^G$ $h^G(\frac{S_0}{S_0}) = \mathcal{Y}^G(\frac{S_0}{S_0})$

$f: \mathcal{A}b^G \rightarrow \mathcal{A}b^G$ equiv. $\Rightarrow h^G(f) = \mathcal{Y}^G(f)$.

2° A - G -module $H: \mathcal{A}b^G \rightarrow \mathcal{A}b$

$H(\frac{S_0}{S_0}) = A^H$

For $g \in G$ s.t. $H(g) = g^{-1}Hg$ we see that $\rho: A \rightarrow A$

then $A^H \rightarrow A^H$ ($g \in A^H \Rightarrow H(g) = g^{-1}Hg = gHg = gA$)

Define $\rho: A^H \rightarrow A^H$

if $\hat{g} = \hat{g}$ ($\hat{g}^{-1}\hat{g} \in K$) then $\rho(\hat{g}) = \hat{g}K$.

$\hat{g}: \mathcal{A}b^G \rightarrow \mathcal{A}b^G$ $H(\hat{g}) = \rho \hat{g}K$.

3° V - G space with a base point \ast

Def: $\tilde{\mathcal{Y}}^G(V): \mathcal{A}b^G \rightarrow \mathcal{A}b$

$\tilde{\mathcal{Y}}^G(V)(\frac{S_0}{S_0}) = \tilde{\mathcal{Y}}^G(V^H, \ast)$ $\tilde{\mathcal{Y}}^G(V^H, \ast)$ - abelian

$\tilde{\mathcal{Y}}^G(V)(\hat{g}) = \rho \hat{g}: \tilde{\mathcal{Y}}^G(V^H, \ast) \rightarrow \tilde{\mathcal{Y}}^G(V^H, \ast)$

Remark: $H_0(\frac{S_0}{S_0}, \frac{S_1}{S_0}) \cong \frac{N(H)}{H}$ $\forall H \in \mathcal{C}_G$ $H(\frac{S_0}{S_0})$ has a structure of $\frac{N(H)}{H}$ -module

$H \in \mathcal{C}_G$ $\mathcal{A}b^G$ "invariant" $\frac{S_0}{S_0}, \frac{S_1}{S_0}$ $P \xrightarrow{A} P$, $V: G \rightarrow P$, $\hat{a}: G \rightarrow G$

$H(P), H(G)$, homeomorphism $H(\hat{a}) = 1$

$E = H(G) \cdot H(P) \rightarrow H(G)$

$\hat{a} = H(G) \cdot H(G) \rightarrow H(G)$

$H(\hat{a}) = H(G)H(G)$ $H(\hat{a})H(G) = H(G)$

$H(G)$ may be considered as a G -module $(G, H) \rightarrow \mathcal{A}b^G$

and $H(P)$ is a kind of G -module.

E is an equiv. homeomorphism $E: H(P) \rightarrow H(G)$

$\hat{a}: \mathcal{A}b^G \rightarrow \mathcal{A}b^G$ is just H_0, H_1 G -module

with homeomorphism $H_0 \rightarrow H_1$

$\mathcal{A}b^G(H, H)$

$H_0 \xrightarrow{E} H_1$
 $\downarrow \cong \downarrow \cong$
 $H_0 \xrightarrow{E} H_1$

Example: $\tilde{\mathcal{Y}}^G(V)$, $G = \mathcal{A}b^G$, $V = G$ -space with base pt. \ast

$\tilde{\mathcal{Y}}^G(V)$ consists of $\tilde{\mathcal{Y}}^G(V^H, \ast)$, $\tilde{\mathcal{Y}}^G(V)$

and $E: \tilde{\mathcal{Y}}^G(V^H, \ast) \rightarrow \tilde{\mathcal{Y}}^G(V^H, \ast) \subset \tilde{\mathcal{Y}}^G(V)$ induced by $V^H \subset V$.