Resolution of singularities
TiGR
07.05.2020

Kiedy mamy stożek, którego promienie nie rozpinają kraty

Lemat

Każdy wektor v prymitywny mogę dopełnić do bazy kraty

Mając wektor (m,n). Wystarczy, że znajdę taki wektor (k,l), że ml-kn=1

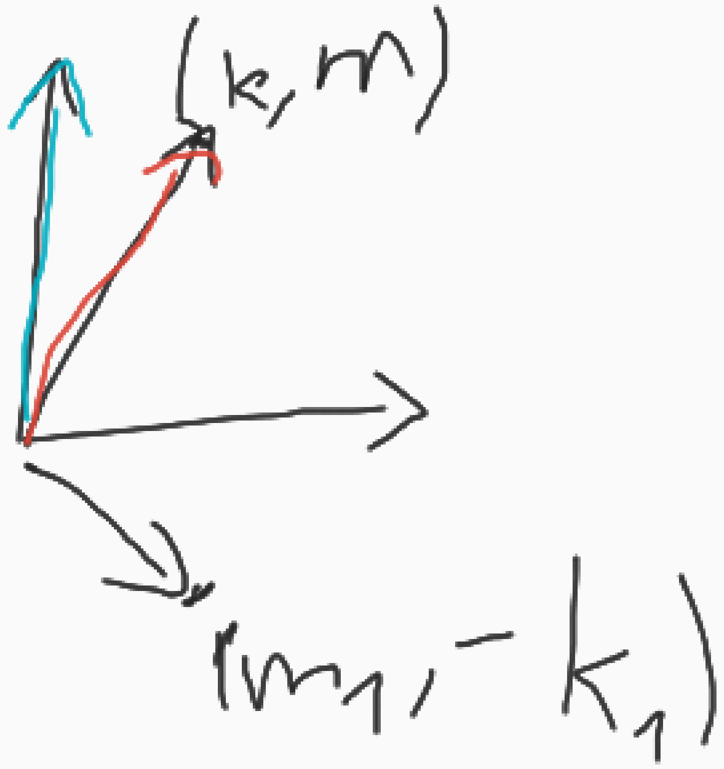
$$\begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix} \begin{pmatrix} m \\ k \end{pmatrix} = \begin{pmatrix} m \\ cm+k \end{pmatrix}$$

Przesuwamy się powyższym automorfizmem, tak, by k<m, i by leżały tak jak na rysunku

$(m_1, -k_1)$

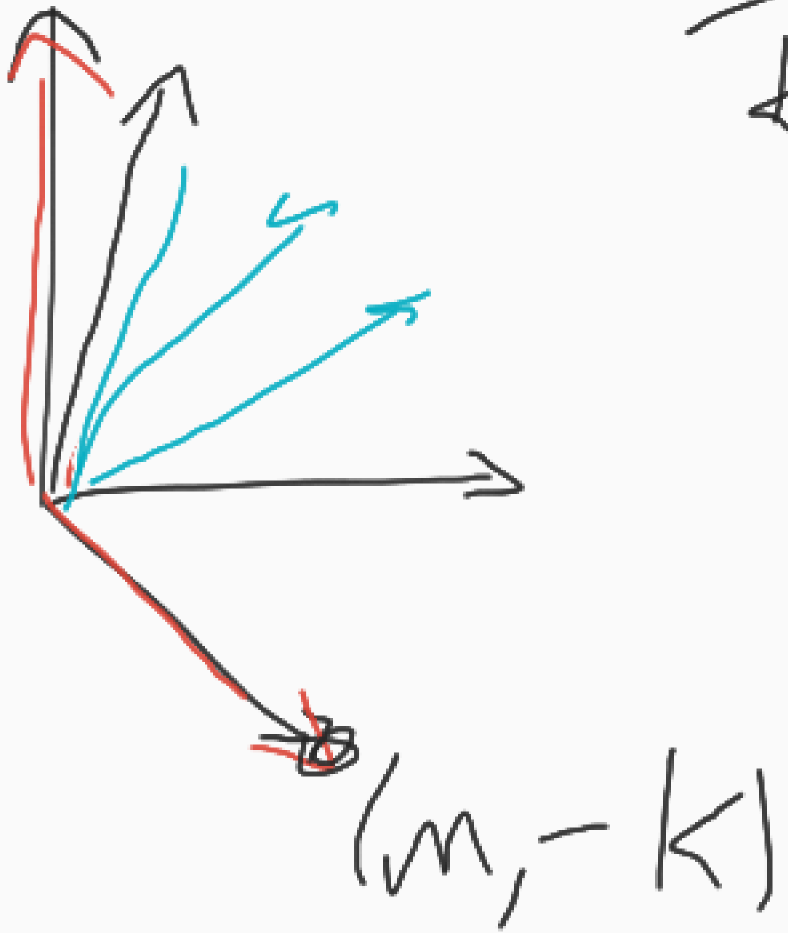
$m_1 = k$

$0 < k_1 < m_1$



$k_1 = a_1 k - m$

$a_1 \geq 2$



$$\frac{m}{k} = \alpha_1 - \frac{1}{\frac{k}{r}} =$$

$$= \alpha_1 - \cfrac{1}{\alpha_2 - \cfrac{}{\ddots - \cfrac{1}{\alpha_r}}}$$

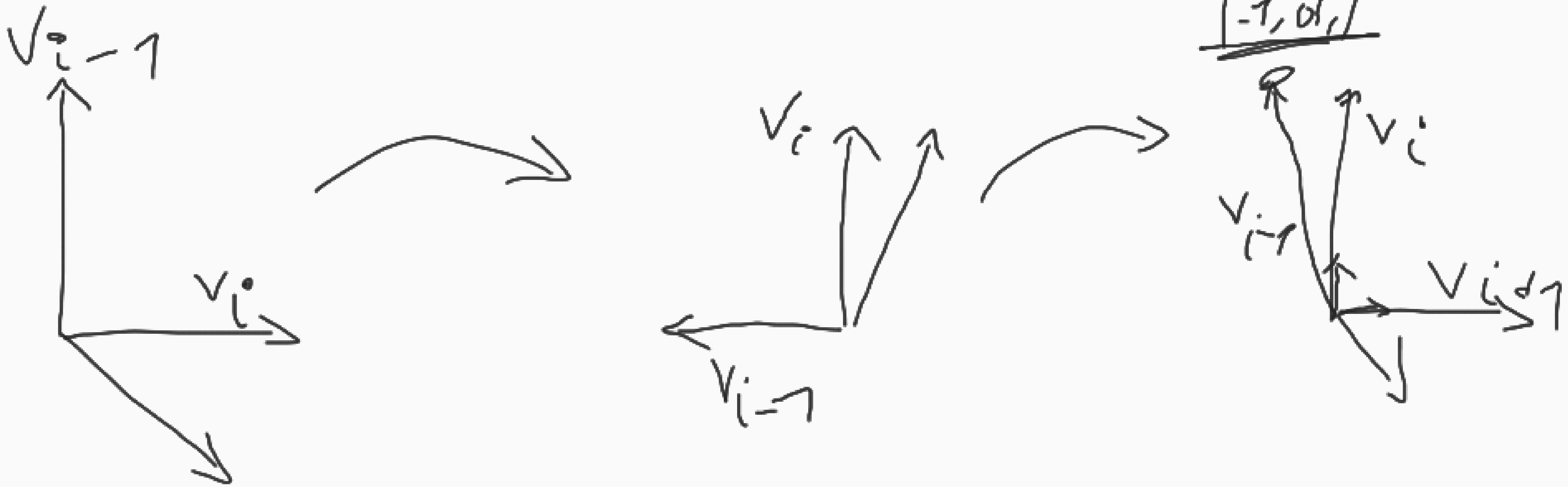
b)

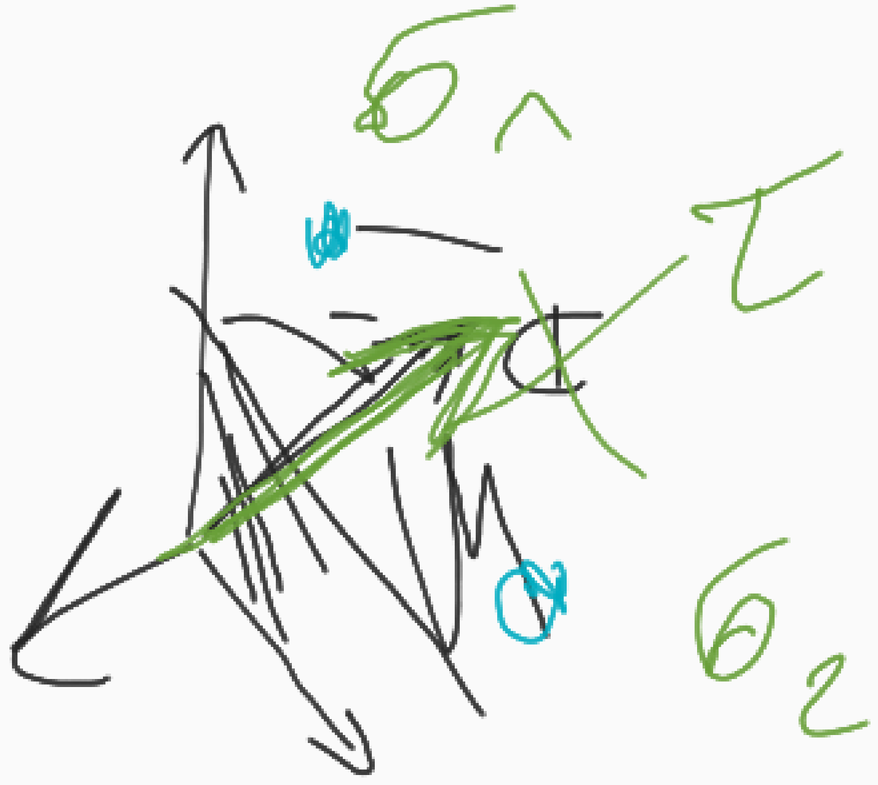
$$a_i v_i = v_{i-1} + v_{i+1}$$

V_{r+1}

$$\begin{pmatrix} 1 & 0 \\ a_i & 1 \end{pmatrix}$$

$(-1, a_i)$





$\sigma_1 \sim U_1 \ni P_1$

$U_1 \supset U_\tau$

$\sigma_2 \sim U_2 \ni P_2$

$U_2 \supset$

G_m^2

$\mathbb{A}^1_R = \operatorname{Spec} R[x] \hookrightarrow \subset Z^1$

$\overline{}$

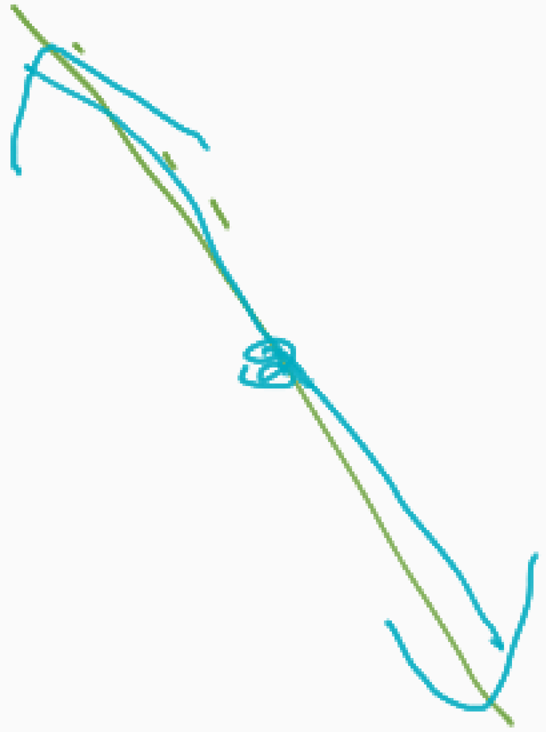
$T : (\theta, 1)$

$(0, t)$

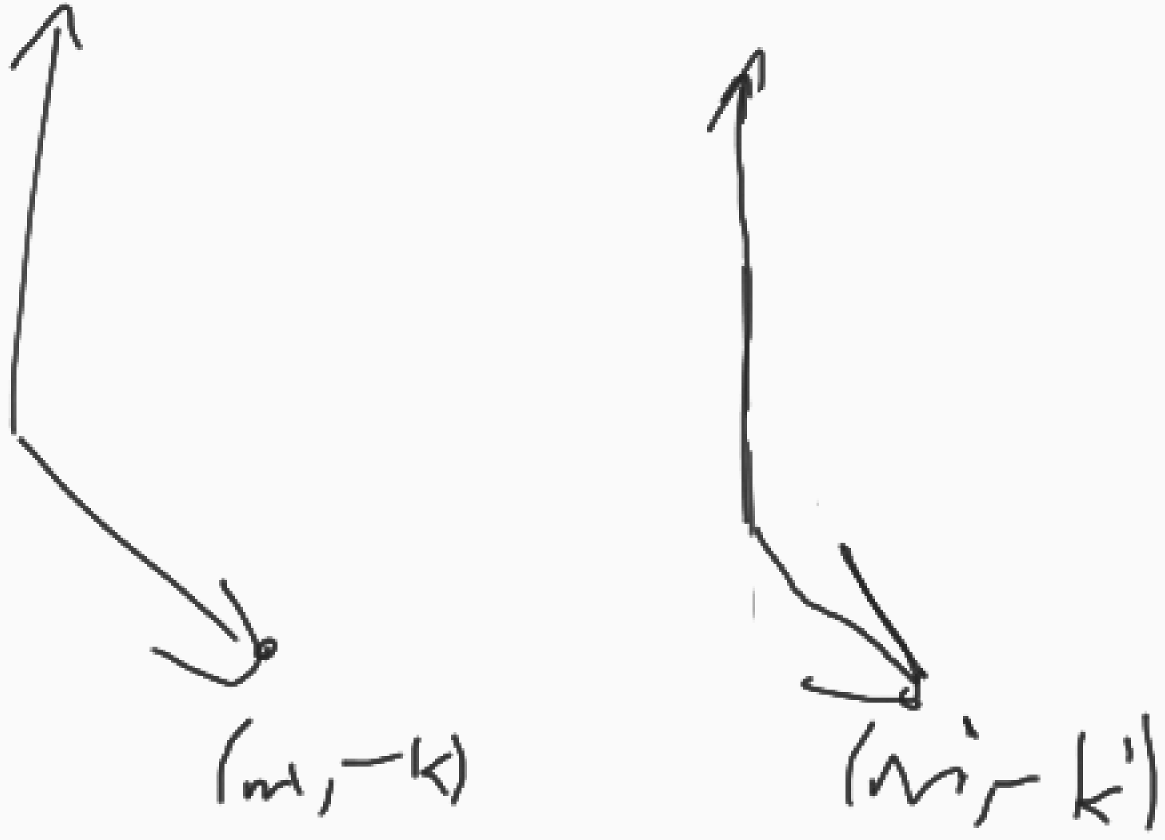
$\mathbb{A}^1_R \times (\mathbb{A}^1 \setminus \{0\})_R \hookrightarrow \subset Z^2$

$(\mathbb{C}_0 \times \mathbb{C}^*) = U_\tau \sim \tau$

NR



$NR/\langle v \rangle$



$$(m, -k) \longrightarrow (m', -k')$$

$$(0, 1) \longrightarrow (0, 1)$$

$$\begin{pmatrix} 1 & 0 \\ c & 1 \end{pmatrix}\begin{pmatrix} m \\ -k \end{pmatrix} = \begin{pmatrix} m' \\ -k' \end{pmatrix} \qquad m = m', \quad -k' = cm - k$$

$$\boxed{\begin{pmatrix} k & m \\ c & d \end{pmatrix}\begin{pmatrix} m \\ -k \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} k & m \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} m \\ d \end{pmatrix} \qquad k' = -\cancel{d}}$$

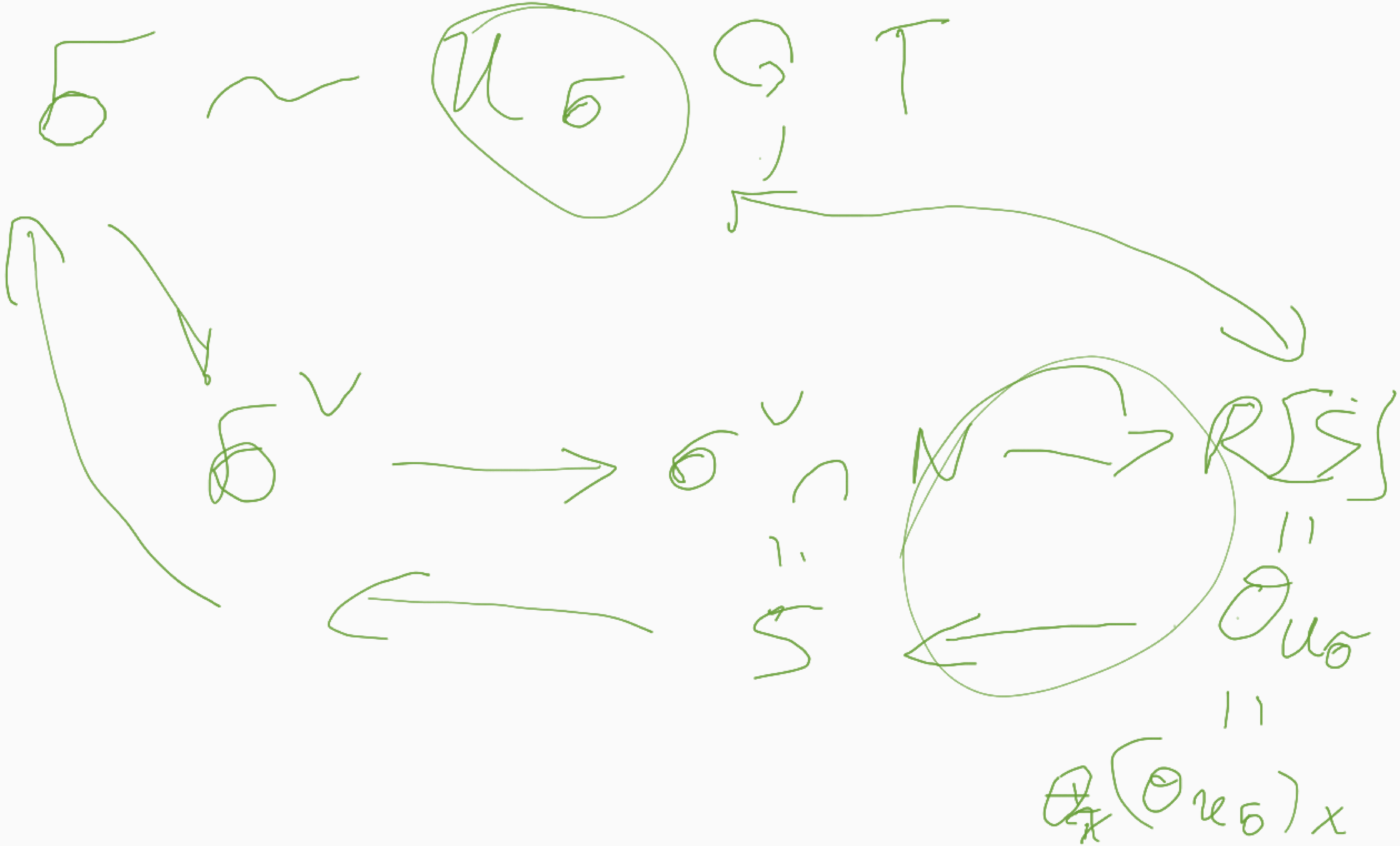
$$(m, -k) \longrightarrow (0, 1)$$

$$(0, 1) \longrightarrow (m', -k')$$

$$\begin{pmatrix} a & m' \\ b & -k' \end{pmatrix}\begin{pmatrix} m \\ -k \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} am - km' \\ bm + kk' \end{pmatrix}$$

$$kk' \equiv 1 \pmod{m}$$



σ ~ U_σ
T
σ^∨
σ^∨ ∩ N
R[S]
S
O_{U_σ}
O_X(O U_σ)_x

- $O \in U_\sigma$

$\sigma \longmapsto$ TYP OSOBLIWOŚCI W $O \in U_\sigma$

INIEKCJA

X — POW. ROZM. ALG / $\mathbb{C}$

(OSOBLIWA) S — POW. ALG.

$\exists\ \tilde{S} \Longrightarrow S$, 120

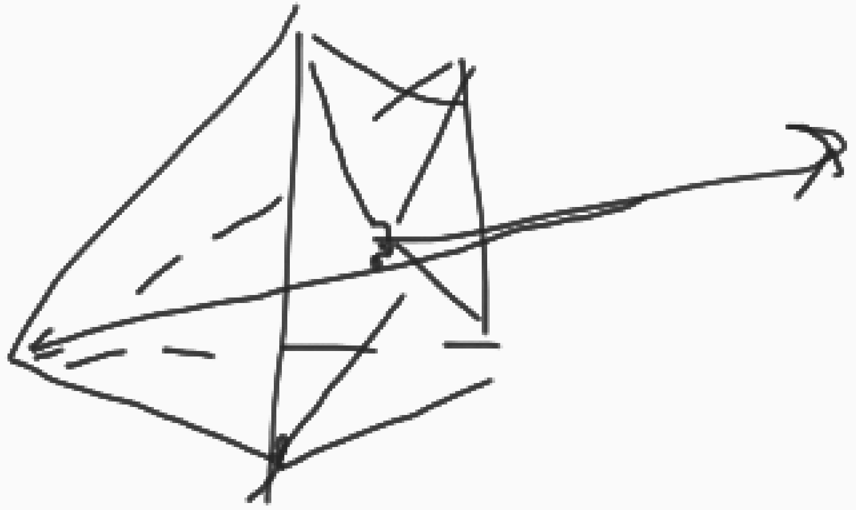
GŁADKIE, RZUTOWE NA PODZBIORZE

GŁADKIEJ $\rightarrow S_0 \subset S$

DLA Fów ∃ MINIM. ROZW. OSOBLIWOŚCI

$$\begin{tikzcd} & \tilde{E} \arrow[r] & * \\ \tilde{S} \arrow[rr] \arrow[dr, "\exists!"'] & & S \\ & \tilde{S}_{min} \arrow[ur] & \end{tikzcd}$$

$$\begin{tikzcd} \tilde{S} \supset E \; (-1)\text{-KRZYWA} \arrow[d] & \\ \tilde{S}_1 \arrow[r] & S \end{tikzcd}$$



k-1 ok.

$\downarrow$

k wyr stożek

k wymiarowy stożek rozpięty przez v_1, ... v_l

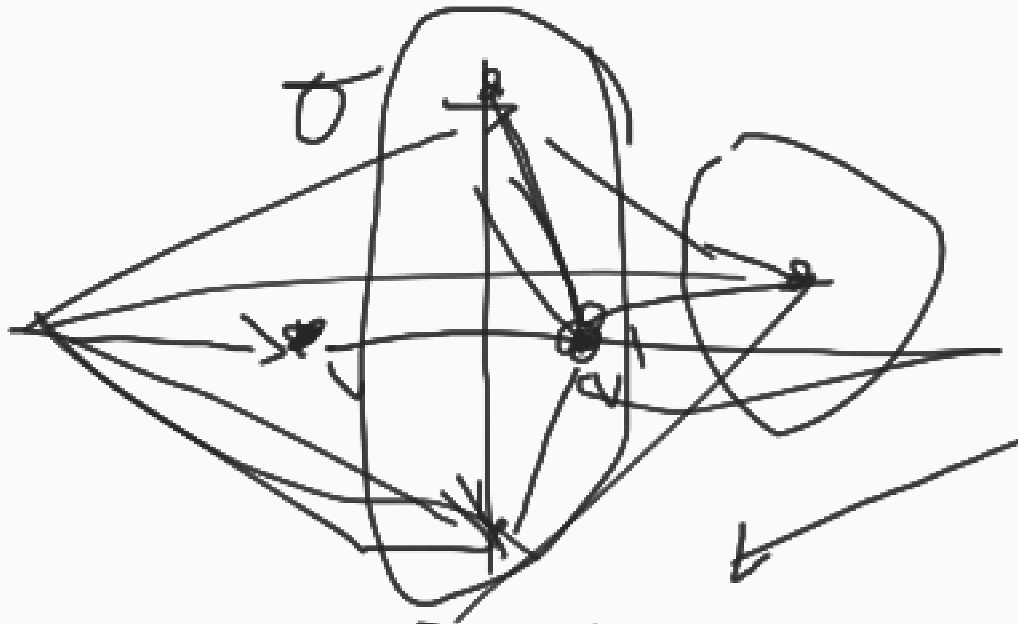
Wybieram wektor v z wnętrza stożka

dla k-1 elemntowego podzbioru D zbioruv_1, ... v_l, który jest zbiorem wektorów promieniowych pewnego k-1 stożka

tworzę k wymiarowy symplicjalny stożek z podzbioru D i wktora v

$\mathrm{mult}(\sigma) =$

$[N : \underbrace{\mathbb{Z}v_1 + \ldots + \mathbb{Z}v_k}]$



$v' = \sum \alpha_i v_i$

$0 < t_i$

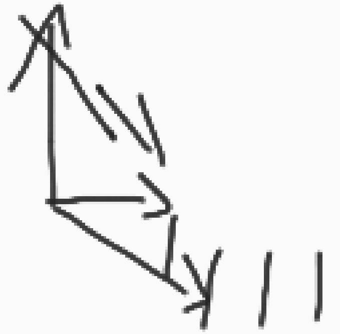
$v = \sum t_i v_i \quad 0 < t_i < 1$

$t_i \mid \mathrm{mult}(\sigma)$

$i = k$

$\frac{1}{t_i} \mid \mathrm{mult}(\sigma)$

$\sigma' = \mathrm{cone}(v_1, \ldots, v_{k-1}, v)$ $[N_0 : \mathbb{Z}v_1 + \mathbb{Z}v_{1} + \cdots + \hat{\mathbb{Z}}_i + \cdots + \mathbb{Z}_k]$



$N = N_\sigma = N_{\sigma'}$

$t_k = \frac{p}{q}$

$$p \cdot \text{mult}(\sigma)$$
$$= q\,\text{mult}(\sigma')$$

$$\mathbb{Z} \cdot \sigma(1) = \mathbb{Z}v_1 + \mathbb{Z}v_2 + \dots + \mathbb{Z}v_k$$

$$\mathbb{Z} \cdot \sigma'(1) = \dots + \mathbb{Z}v$$

$$\cancel{p \cdot \mathbb{Z}\sigma(1) = q\,\mathbb{Z}\sigma'(1)}$$

$$v = \sum \frac{p_i}{q} v_i$$

$$Z v_1 + Z v_2 + \ldots + Z v_{k-1} + Z$$

I V_k

II $p_k V_k$

V

III qV

IV

$$\text{mult}\, \sigma = [N : \mathrm{I}] = \frac{1}{p_k} \cdot (N : \mathrm{II}) =$$

$$= \frac{1}{p_k} [N : \mathrm{IV}] = \frac{q}{p_k} [N : \mathrm{III}] = \frac{q}{p_k} \text{mult}\, \sigma$$