

RESEARCH PROJECT
TOPOLOGICAL METHODS OF GEOMETRIC
ANALYSIS

DESCRIPTION FOR THE GENERAL PUBLIC

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We should start by explaining what is geometric analysis. It is easy to begin: it is a field of mathematics, but then it gets more difficult, because geometric analysis does not have clear boundaries. We include there the applications of geometry (differential geometry and topology included) to problems originating in different areas of analysis (mainly in differential equations and variational calculus), but also the reverse: methods of differential equations, variational calculus or other areas of analysis applied to geometric problems.

In this broad sense the whole differential geometry, which studies curves, surfaces and their multi-dimensional counterparts, manifolds, is a part of geometric analysis. Classical differential geometry studies smooth manifolds, that is manifolds which can be described with functions that are smooth, in other words – differentiable (preferably infinitely many times).

Unfortunately, (or, perhaps, luckily) such functions are not sufficient to describe and study real-life phenomena. To study the shape of a soap film spanned on a bent loop of wire, to predict the location of cracks forming in a squeezed material, we need functions, which are not differentiable, and might have no derivative at any point.

In order to study such functions using the toolbox of analysis, the notion of weak derivative was introduced. We cannot fit here the precise definition; the important fact is that for smooth functions the weak and the classical (strong) derivative are the same, and that, from the point of view of partial differential equations and variational calculus, the weak derivative retains many properties of the usual derivative. Many, but not all.

We are particularly interested in topological properties of transformations, that is, roughly speaking, these properties which are not sensitive to small perturbations of the transformation. Does the transformation reverse the left and the right hand side (like a symmetry on a plane does), or not? If you wind an elastic band around your pencil (which can be described by a transformation from a circle onto itself), how many times does the band encircle the core? Does the transformation glue distant points? Does it move points which are very near away from each other? We know well how to check and study these properties using the derivative – that approach works fine with differentiable transformations. What if the transformation is only weakly differentiable? Can we use the weak derivative to answer such questions?

My project consists of several detailed problems, with the above difficulty the unifying theme: we study transformations between spheres, between spaces, between cubes of different dimensions, and we ask questions which could be easily answered, if the transformations in question were sufficiently many times (or at least one time) differentiable. We, however, are interested in transformations which are not smooth enough to use the known, classical methods.

The answers to these problems are important in the theory of elastic deformations and in other mathematical models in material science.

We expect that the material product of that research will be around 10 research papers, published in leading mathematical journals; the results of the research will be announced to the mathematical community on international conferences.