# Data exchange and schema mappings 

S. Amano M. Arenas L. Libkin F. Murlak

Fagin-Kolaitis-Popa-Miller, ICDT'03
Fagin-Kolaitis-Popa-Tan, PODS’04

## Data exchange

- Source schema, target schema; need to transfer data between them.
- A typical scenario:
- Two organizations have their legacy databases, schemas cannot be changed.
- Data from organization 1 needs to be transfered to data from organization 2.
- Queries need to be answered against the transferred data.


## Data Exchange



## Outline

- data exchange problem
- universal solutions
- target constraints
- composing mappings

Data exchange problem

## An example

- We want to create a target database with the schema

$$
\begin{aligned}
& \text { Flight(city1,city2, aircraft,departure,arrival) } \\
& \text { Served(city,country,population, agency) }
\end{aligned}
$$

- We don't start from scratch: there is a source database containing relations

$$
\begin{aligned}
& \text { Route(source,destination,departure) } \\
& \text { BG(country, city) }
\end{aligned}
$$

- We want to transfer data from the source to the target.


## Relationships between source and target

How to specify the relationship?


## Relationships between source and target cont'd

- Formal specification: we have a relational calculus query over both the source and the target schema.
- The query is of a restricted form, and can be thought of as a sequence of rules:

Route $(c 1, c 2$, dept $) \rightarrow \operatorname{Flight}\left(c 1, c 2,,_{-}\right.$, dept, $)$
Route (city, _, $), B G($ city , country) $\rightarrow$ Served (city, country, , , $)$
Route(_, city, _ ), BG(city, country) $\rightarrow$ Served (city, country, _ , $)$

## Targets

- Target instances should satisfy the rules.
- What does it mean to satisfy a rule?
- Formally, a source $S$ and a target $T$ satisfy a rule

$$
\text { Route(c1, c2, dept) } \rightarrow \text { Flight(c1, c2, _, dept, _ ) }
$$

if they satisfy the constraint

$$
\forall c_{1}, c_{2}, d\left(\operatorname{Route}\left(c_{1}, c_{2}, d\right) \rightarrow \exists a_{1}, a_{2}\left(F \operatorname{light}\left(c_{1}, c_{2}, a_{1}, d, a_{2}\right)\right)\right)
$$

## Targets

- What happens if there no values for some attributes, e.g. aircraft, arrival?
- We put in null values or some real values.
- But then we may have multiple solutions!


## Targets

Source Database:

ROUTE:

| Source | Destination | Departure |
| :---: | :---: | :---: |
| Edinburgh | Amsterdam | 0600 |
| Edinburgh | London | 0615 |
| Edinburgh | Frankfurt | 0700 |

BG:

| Country | City |
| :---: | :---: |
| UK | London |
| UK | Edinburgh |
| NL | Amsterdam |
| GER | Frankfurt |

Look at the rule

$$
\text { Route(c1, c2, dept) } \rightarrow \operatorname{Flight(c1,~c2,~,~,~dept,~ـ~)~}
$$

The left hand side is satisfied by
Route(Edinburgh, Amsterdam, 0600)
But what can we put in the target?

## Targets

Rule: Route(c1, c2, dept) $\rightarrow$ Flight(c1, c2, _ , dept, _ ) Satisfied by: Route(Edinburgh, Amsterdam, 0600)
Possible targets:

- Flight(Edinburgh, Amsterdam, $\perp_{1}, 0600, \perp_{2}$ )
- Flight(Edinburgh, Amsterdam, B737, 0600, $\perp$ )
- Flight(Edinburgh, Amsterdam, $\perp$, 0600, 0845)
- Flight(Edinburgh, Amsterdam, $\perp, 0600, \perp$ )
- Flight(Edinburgh, Amsterdam, B737, 0600, 0845)

They all satisfy the constraints!

## Which target to choose?

- One of them happens to be right:
- Flight(Edinburgh, Amsterdam, B737, 0600, 0845)
- But in general we do not know this; it looks just as good as
- Flight(Edinburgh, Amsterdam, 'The Spirit of St Louis', 0600, 1300), or
- Flight(Edinburgh, Amsterdam, F16, 0600, 0620).
- Goal: look for the "most general" solution.
- How to define "most general": can be mapped into any other solution.
- It is not unique either, but the space of solution is greatly reduced.
- In our case Flight(Edinburgh, Amsterdam, $\perp_{1}, 0600, \perp_{2}$ ) is most general as it makes no additional assumptions about the nulls.


## Universal solutions

## Universal solutions

- A homomorphism is a mapping $h:$ Nulls $\rightarrow$ Nulls $\cup$ Constants.
- For example, $h\left(\perp_{1}\right)=B 737, h\left(\perp_{2}\right)=0845$.
- If we have two solutions $T_{1}$ and $T_{2}$, then $h$ is a homomorphism from $T_{1}$ into $T_{2}$ if for each tuple $t$ in $T_{1}$, the tuple $h(t)$ is in $T_{2}$.
- For example, if we have a tuple

$$
t=\text { Flight }\left(\text { Edinburgh, Amsterdam }, \perp_{1}, 0600, \perp_{2}\right)
$$

then
$h(t)=$ Flight(Edinburgh, Amsterdam, B737, 0600, 0845).

- A solution is universal if there is a homomorphism from it into every other solution.
- (We shall revisit this definition later, to deal with nulls properly.)


## Universal solutions: still too many of them

- Take any $n>0$ and consider the solution with $n$ tuples:

Flight(Edinburgh, Amsterdam, $\quad \perp_{1}, \quad 0600, \quad \perp_{2}$ )
Flight(Edinburgh, Amsterdam, $\perp_{3}, \quad 0600, \perp_{4}$ )
Flight(Edinburgh, Amsterdam, $\quad \perp_{2 n-1}, \quad 0600, \quad \perp_{2 n}$ )

- It is universal too: take a homomorphism

$$
h^{\prime}\left(\perp_{i}\right)= \begin{cases}\perp_{1} & \text { if } i \text { is odd } \\ \perp_{2} & \text { if } i \text { is even }\end{cases}
$$

- It sends this solution into

Flight(Edinburgh, Amsterdam, $\left.\perp_{1}, 0600, \quad \perp_{2}\right)$

## Universal solutions: cannot be distinguished by CQs

- There are queries that distinguish large and small universal solutions (e.g., does a relation have at least 2 tuples?)
- But these cannot be distinguished by conjunctive queries
- Because: if $\perp_{i_{1}}, \ldots, \perp_{i_{k}}$ witness a conjunctive query, so do $h\left(\perp_{i_{1}}\right), \ldots, h\left(\perp_{i_{k}}\right)$ - hence, one tuple suffices
- In general, if we have
- a homomorphism $h: T \rightarrow T^{\prime}$,
- a conjunctive query $Q$
- a tuple $t$ without nulls such that $t \in Q(T)$
- then $t \in Q\left(T^{\prime}\right)$


## Universal solutions and conjunctive queries

- If
- $T$ and $T^{\prime}$ are two universal solutions
- $Q$ is a conjunctive query, and
- $t$ is a tuple without nulls,
then

$$
t \in Q(T) \Leftrightarrow t \in Q\left(T^{\prime}\right)
$$

because we have homomorphisms $T \rightarrow T^{\prime}$ and $T^{\prime} \rightarrow T$.

- Furthermore, if
- $T$ is a universal solution, and $T^{\prime \prime}$ is an arbitrary solution, then

$$
t \in Q(T) \Rightarrow t \in Q\left(T^{\prime \prime}\right)
$$

## Universal solutions and conjunctive queries cont'd

- Now recall what we learned about answering conjunctive queries over databases with nulls:
- $T$ is a naive table
- the set of tuples without nulls in $Q(T)$ is precisely certain $(Q, T)$ - certain answers over $T$
- Hence if $T$ is an arbitrary universal solution

$$
\operatorname{certain}(Q, T)=\bigcap\left\{Q\left(T^{\prime}\right) \mid T^{\prime} \text { is a solution }\right\}
$$

- $\bigcap\left\{Q\left(T^{\prime}\right) \mid T^{\prime}\right.$ is a solution\} is the set of certain answers in data exchange under mapping $M$ : $\operatorname{certain}_{M}(Q, S)$. Thus

$$
\operatorname{certain}_{M}(Q, S)=\operatorname{certain}(Q, T)
$$

for every universal solution $T$ for $S$ under $M$.

## Universal solutions cont'd

- To answer conjunctive queries, one needs an arbitrary universal solution.
- We saw some; intuitively, it is better to have:

$$
\text { Flight(Edinburgh, Amsterdam, } \left.\perp_{1}, 0600, \quad \perp_{2}\right)
$$

than
Flight(Edinburgh, Amsterdam, $\perp_{1}, \quad 0600, \perp_{2}$ )
Flight(Edinburgh, Amsterdam, $\perp_{3}, 0600, \perp_{4}$ )
Flight(Edinburgh, Amsterdam, $\quad \perp_{2 n-1}, \quad 0600, \quad \perp_{2 n}$ )

- We now define a canonical universal solution.


## Canonical universal solution

- Convert each rule into a rule of the form:

$$
\varphi\left(x_{1}, \ldots, x_{k}, y_{1}, \ldots, y_{m}\right) \rightarrow \psi\left(x_{1}, \ldots, x_{k}, z_{1}, \ldots, z_{n}\right)
$$

For example,
Route(c1, c2, dept) $\rightarrow$ Flight(c1, c2, _ , dept, _ )
becomes
Route $\left(x_{1}, x_{2}, x_{3}\right) \rightarrow \operatorname{Flight}\left(x_{1}, x_{2}, z_{1}, x_{3}, z_{2}\right)$

- Evaluate $\varphi\left(x_{1}, \ldots, x_{n}, y_{1}, \ldots, y_{m}\right)$ in $S$.
- For each tuple $\left(a_{1}, \ldots, a_{n}, b_{1}, \ldots, b_{m}\right)$ that belongs to the result (i.e. $\varphi\left(a_{1}, \ldots, a_{n}, b_{1}, \ldots, b_{m}\right)$ holds in $S$ ), do the following:


## Canonical universal solution cont'd

- ... do the following:
- Create new (not previously used) null values $\perp_{1}, \ldots, \perp_{k}$
- Put tuples in target relations so that

$$
\psi\left(a_{1}, \ldots, a_{n}, \perp_{1}, \ldots, \perp_{k}\right)
$$

holds.

- What is $\psi$ ?
- It is normally assumed that $\psi$ is a conjunction of atomic formulae, i.e.

$$
R_{1}\left(\bar{x}_{1}, \bar{z}_{1}\right) \wedge \ldots \wedge R_{l}\left(\bar{x}_{l}, \bar{z}_{l}\right)
$$

- Tuples are put in the target to satisfy these formulae


## Canonical universal solution cont'd

- Example: no-direct-route airline:


## Oldroute $\left(x_{1}, x_{2}\right) \rightarrow \operatorname{Newroute}\left(x_{1}, z\right) \wedge \operatorname{Newroute}\left(z, x_{2}\right)$

- If $\left(a_{1}, a_{2}\right) \in \operatorname{Oldroute}\left(a_{1}, a_{2}\right)$, create a new null $\perp$ and put

```
Newroute(a1,\perp)
Newroute \(\left(\perp, a_{2}\right)\)
```

into the target.

- Complexity of finding this solution: polynomial in the size of the source $S$ :
$O\left(\sum_{\text {rules } \varphi \rightarrow \psi}\right.$ Evaluation of $\varphi$ on $\left.S\right)$


## Canonical universal solution and conjunctive queries

- Canonical solution: $\mathrm{CanSol}_{M}(S)$.
- We know that if $Q$ is a conjunctive query, then $\operatorname{certain}_{M}(Q, S)=\operatorname{certain}(Q, T)$ for every universal solution $T$ for $S$ under $M$.
- Hence

$$
\operatorname{certain}_{M}(Q, S)=\operatorname{certain}\left(Q, \operatorname{CanSol}_{M}(S)\right)
$$

- Algorithm for answering $Q$ :
- Construct $\mathrm{CanSol}_{M}(S)$
- Apply naive evaluation to $Q$ over $\operatorname{CanSol}_{M}(S)$


## Target constraints

## Data exchange and integrity constraints

- Integrity constraints are often specified over target schemas
- In SQL's data definition language one uses keys and foreign keys most often, but other constraints can be specified too.
- Adding integrity constraints in data exchange is often problematic, as some natural solutions - e.g., the canonical solution - may fail them.


## Target constraints cause problems

- The simplest example:
- Copy source to target
- Impose a constraint on target not satisfied in the source
- Schema mapping:
- $S(x, y) \rightarrow T(x, y)$ and
- Constraint: the first attribute is a key
- Instance S:

| 1 | 2 |
| :--- | :--- |
| 1 | 3 |

- Every target $T$ must include these tuples and hence violates the key.


## Target constraints: more problems

A common problem: an attempt to repair violations of constraints leads to a sequence of tuple insertions.

- Source DeptEmpl(dept_id,manager_name,empl_id)
- Target
- Dept(dept_id,manager_id,manager_name),
- Empl(empl_id,dept_id)
- Rule $\operatorname{DeptEmpl}(d, n, e) \rightarrow \operatorname{Dept}(d, z, n) \wedge \operatorname{Empl}(e, d)$
- Target constraints:
- Dept[manager_id] $\subseteq$ Empl[empl_id]
- Empl[dept_id] $\subseteq$ Dept[dept_id]


## Target constraints: more problems cont'd

- Start with (CS, John, 001) in DeptEmpl.
- Put Dept(CS, $\perp_{1}$, John) and Empl(001, CS) in the target
- Use the first constraint and add a tuple $\operatorname{Empl}\left(\perp_{1}, \perp_{2}\right)$ in the target
- Use the second constraint and put $\operatorname{Dept}\left(\perp_{2}, \perp_{3}, \perp_{3}{ }^{\prime}\right)$ into the target
- Use the first constraint and add a tuple $\operatorname{Empl}\left(\perp_{3}, \perp_{4}\right)$ in the target
- Use the second constraint and put $\operatorname{Dept}\left(\perp_{4}, \perp_{5}, \perp_{5}{ }^{\prime}\right)$ into the target
- this never stops....


## Target constraints: avoiding this problem

- Change the target constraints slightly:
- Target constraints:
- Dept[dept_id,manager_id] $\subseteq$ Empl[empl_id, dept_id]
- Empl[dept_id] $\subseteq$ Dept[dept_id]
- Again start with (CS, John, 001) in DeptEmpl.
- Put $\operatorname{Dept}\left(C S, \perp_{1}\right.$, John) and $\operatorname{Empl}(001, \mathrm{CS})$ in the target
- Use the first constraint and add a tuple Empl( $\left.\perp_{1}, \mathrm{CS}\right)$
- Now constraints are satisfied - we have a target instance!
- What's the difference? In our first example constraints are very cyclic causing an infinite loop. There is less cyclicity in the second example.

Bottom line: avoid cyclic constraints.

Composing mappings

## Schema mappings

- Rules used in data exchange specify mappings between schemas.
- To understand the evolution of data one needs to study operations on schema mappings.
- Most commonly we need to deal with composition.


## Semantics

## $\mathcal{M}_{1}$ composed with $\mathcal{M}_{2}$



## The closure problem



- Are mappings closed under composition?
- If not, what needs to be added?


## Composition: when it works

- Example:

$$
\begin{aligned}
\Sigma: & S\left(x_{1}, x_{2}, x_{3}\right)
\end{aligned} \rightarrow T\left(x_{1}, x_{2}\right) \wedge T\left(x_{2}, x_{3}\right) ~ 子 ~\left(x_{1}, x_{2}, z\right) .
$$

- First modify into:

$$
\begin{aligned}
& \Sigma: \quad S\left(x_{1}, x_{2}, x_{3}\right) \rightarrow T\left(x_{1}, x_{2}\right) \\
& S\left(x_{1}, x_{2}, x_{3}\right) \rightarrow T\left(x_{2}, x_{3}\right) \\
& \Delta: \quad T\left(x_{1}, x_{2}\right) \quad \rightarrow \quad W\left(x_{1}, x_{2}, z\right)
\end{aligned}
$$

- Then substitute in the definition of $W$ :

$$
\begin{aligned}
& S\left(x_{1}, x_{2}, y\right) \rightarrow W\left(x_{1}, x_{2}, z\right) \\
& S\left(y, x_{1}, x_{2}\right) \rightarrow W\left(x_{1}, x_{2}, z\right)
\end{aligned}
$$

to get $\boldsymbol{\Sigma} \circ \boldsymbol{\Delta}$.

## Composition: not without Skolem functions

$\begin{aligned} \text { sheep }(x) & \longrightarrow \operatorname{sheep}(x) \\ \text { true } & \longrightarrow \operatorname{dog}(y)\end{aligned}$
$\operatorname{sheep}(x), \operatorname{dog}(y) \longrightarrow \operatorname{guards}(y, x)$

| sheep |  |
| :--- | :--- | :--- | :--- |
| Shaun <br> Shirley |  |
| Shaun <br> Shirley | dog |

## Composition: not without equality

- Mapping $\Sigma$ :

$$
\operatorname{Empl}(e) \quad \rightarrow \operatorname{Mngr}(e, m)
$$

- Mapping $\Delta$ :

$$
\begin{aligned}
\operatorname{Mngr}(e, m) & \rightarrow \operatorname{Mngr}^{\prime}(e, m) \\
\operatorname{Mngr}(e, e) & \rightarrow \operatorname{SelfMng}(e)
\end{aligned}
$$

- Composition:

$$
\begin{aligned}
\operatorname{Empl}(e) & \rightarrow \operatorname{Mngr}^{\prime}(e, f(e)) \\
\operatorname{Empl}(e) \wedge e=f(e) & \rightarrow \operatorname{SelfMng}(e)
\end{aligned}
$$

## Composable class of mappings

Mappings with Skolem functions and equality compose!

- Replace all nulls by Skolem functions:
- Empl(e) $\rightarrow \operatorname{Mngr}(e, m)$ becomes
$\mathrm{Empl}(e) \rightarrow \operatorname{Mngr}(e, f(e))$
- $\Delta$ stays as before
- Use substitution:
- $\operatorname{Mngr}(e, m) \rightarrow \operatorname{Mngr}^{\prime}(e, m)$ becomes

$$
\operatorname{Empl}(e) \rightarrow \operatorname{Mngr}^{\prime}(e, f(e))
$$

- $\operatorname{Mngr}(e, e) \rightarrow$ SelfMng(e) becomes

$$
\operatorname{Empl}(e) \wedge e=f(e) \rightarrow \text { SelfMng }(e)
$$

## Complexity summery

- $(S, T) \in \llbracket \Sigma \rrbracket:$ PTIME (easy, relational query evaluation)
- $(S, T) \in \llbracket \Sigma \circ\lceil\rrbracket$ : NP-complete
(FKPT'04; improved examples of hardness in LS'08)
- certain answers:
- undecidable for RA
- PTIME for CQ
(folklore)

