From deterministic finite automata to infinite games

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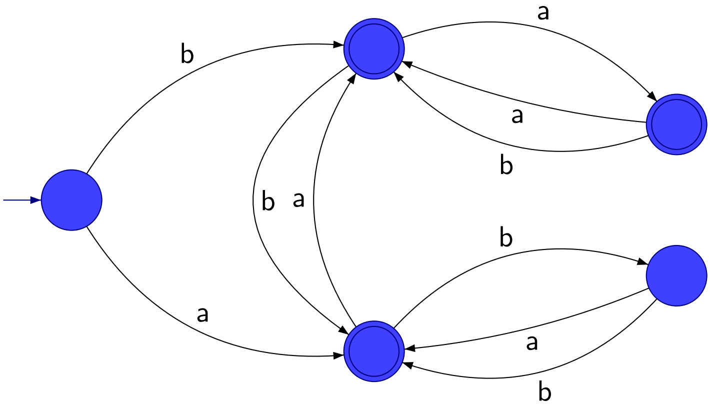
May 11, 2011

Deterministic finite automaton



used to recognize languages $L \subseteq \Sigma^*$, i.e., sets of words using letters from the set $\Sigma = \{a, b\}$

$$a \in L, ab \notin L, aba \in L, \ldots$$

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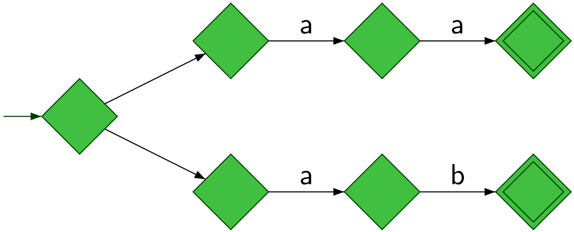
Non-determinism

In some states the automaton has a choice of where it will go

We assume that the automaton always makes the choice which leads to acceptance



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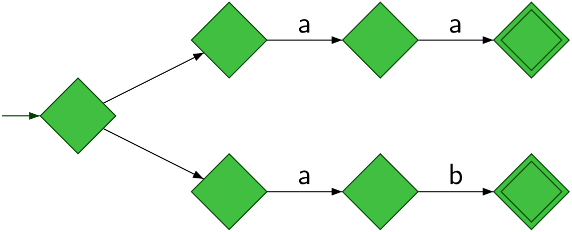

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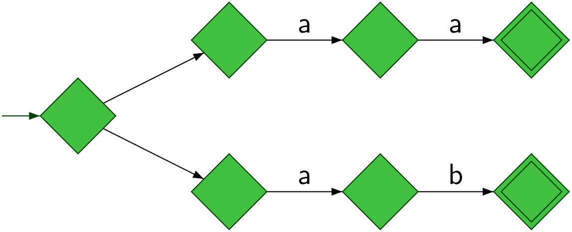

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subset construction: n non-det states $\rightarrow 2^n$ det states

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In terms of logic, non-determinism corresponds to disjunction and existential quantification

In terms of regular expressions, non-determinism allows to express union, concatenation, and Kleene star easily

More about formulae

What languages can we express using first order logic (FO)?

$$\exists i \quad \forall i \quad i = j \quad i < j \quad i \in A \quad \neg \quad \vee \quad \wedge$$

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To get all regular languages, we also need $\exists O \exists E$ (MSO logic)

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A try: **Accept all paths** – we can express $\wedge$, but not $\vee$

Solution: using games

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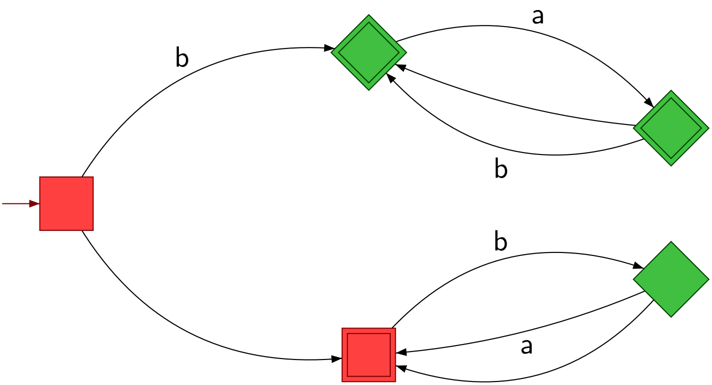
If both players play perfectly and Eva wins, then the sequence is convergent

Alternating automata



Alternation in complexity theory

What problems can be solved by a machine running in polynomial time?

deterministic	P

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any number of alternations	AP $=$ PSPACE

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- By using the subset construction we get a NFA with 2^n states, we have to determinize again to get a DFA with 2^{2^n} states
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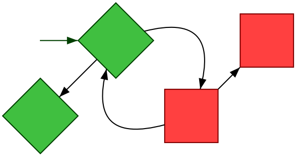
- Still an useful notion

Infinite cycles

What happens if we get into an infinite cycle?



Usually we assume infinite computations to be non-accepting
But in terms of games this means Eva loses – that's **not fair!**

Infinite computations in nature

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We actually **want** the computation to be infinite, but it is required to satisfy some property ϕ, for example:

for every request there is a response
there is no response if it was not requested
no deadlocks, no starvation...

We want Eva to win the game iff ϕ is satisfied

Infinite games in nature

How do other games solve the problem?

- Chess – the game is considered a draw after 50 moves (without an irreversible action such as moving a pawn or capturing)
- Go – ko rule

But we don't want draws!

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Answer: Eva has a winning strategy iff $\mathbb{R} - Z$ is a meager set

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But **reasonable** infinite games are determined (Martin '75 - Borel determinacy)

A non-determined game

XOR function:

$$X : \{0,1\}^* \to \{0,1\}$$

such that

$$X(0^*) = 0$$
$$X(u0v) \neq X(u1v)$$

A well known function

Non-determined game

Infinite XOR function:

$$X : \{0,1\}^\omega \to \{0,1\}$$

such that

$$X(0^\omega) = 0$$
$$X(u0v) \neq X(u1v)$$

Existence follows from the Axiom of Choice

Non-determined game

Infinite XOR game

- Eva chooses a finite sequence of bits w_1
- Adam chooses w_2
- Eva chooses w_3
- Adam chooses w_4
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- Eva wins if $X(w_1 w_2 w_3 w_4 \ldots) = 1$

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- Eva chooses w_1
- Adam chooses $w_2' w_3$
- Eva chooses w_4
- Adam chooses w_5
- ...
- Adam will win!

Back to computations: how to express ϕ?

In terms of logic (FO, MSO): no problem ($\forall i$ now quantifies not over a finite set of positions in a word, but an infinite set of integers $\mathbb{N}$); there are also special logics for that (e.g. LTL)

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In terms of ω-**languages**: we speak about subsets of Σ^ω instead of Σ^*

In terms of ω-**regular expressions**: for $L \in \Sigma^*$ we add an operation L^ω

$$\Sigma^\omega \ni baabbaabb(ab)^\omega$$

What about automata?

ω-regular expressions and MSO logic express the same class of languages (called ω-**regular languages**)

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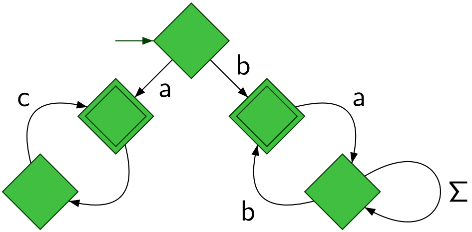
The simplest approach: make all infinite computations false/losing (or true/winning) – not powerful enough, we cannot express $a^\omega \in \{a, b\}^\omega$
We need to use some acceptance condition (or winning condition) to tell which infinite runs are accepted

Büchi automata ('60)

We again use F - the set of accepting states, but now the infinite computation is accepting if it visits the states in F infinitely often (Büchi condition)

$$ac^{\omega} \cup (ba\Sigma^*)^{\omega}$$



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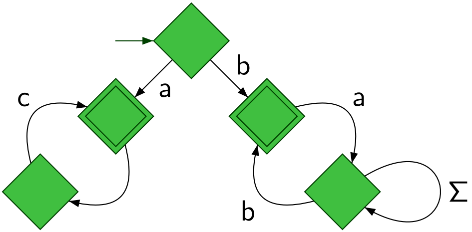

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Muller automata ('63)

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- Deterministic Muller automata recognize all ω-regular languages
- Negation is straightforward
- But the description is long (we have to define acceptance for each subset of Q)

Parity condition: motivation

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In practice, both good and bad things could happen...

the program seems to do its job
the program uses too much resources
the program hangs
the program works as it should
we lose some money
we break our moral rules
we earn some money
we become rich
newspapers write about us
we go to jail

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Büchi conditions allows us to define a good thing that has to happen infinitely often in order to make Eva a winner.
In practice, both good and bad things could happen...

the program seems to do its job	0
the program uses too much resources	1
the program hangs	1
the program works as it should	2
we lose some money	3
we break our moral rules	3
we earn some money	4
we become rich	4
newspapers write about us	4
we go to jail	5

Parity condition

We use $\mathrm{rank} : Q \to \mathbb{N}$
Run is accepted (Eva wins) if the greatest rank appearing infinitely often during the play is even, not accepted (Adam wins) if it is odd

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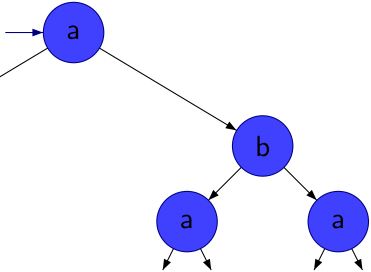
- Deterministic parity automata recognize all ω-regular languages (a nice translation from the Muller condition, LAR)
- Negation straightforward
- Effective description

More than words: ω-trees

In an ω-word, each position has **one successor**

In an ω-tree, a position can have **many successors**



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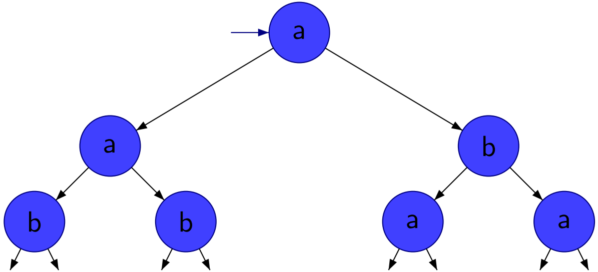
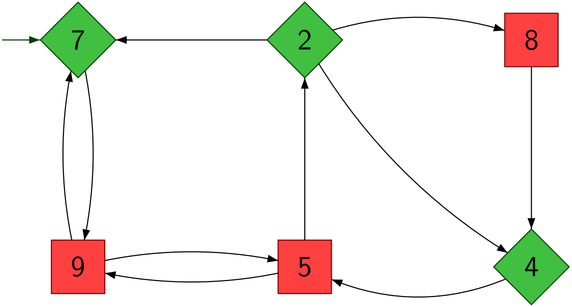


A tree can represent e.g. all possible runs of an operating system

Infinite game

Alternating automaton = a transition system, where transitions depends on decisions of two players and input
What happens if we remove the input?



Motivation I-II

- We can encode the input inside our automaton, getting an infinite game without input (for infinite "irregular" inputs this leads to an infinite transition system)

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- We can encode the input inside our automaton, getting an infinite game without input (for infinite "irregular" inputs this leads to an infinite transition system)
- We cannot run automata on infinite input in practice; but we want to solve problems like:
  - Given ϕ - a property that we want our program to satisfy during its execution
  - Given M - a model of our program
  - Question: does M satisfy ϕ? (**model checking**)

For ϕ in modal μ calculus, this reduces to a parity game

Motivation III

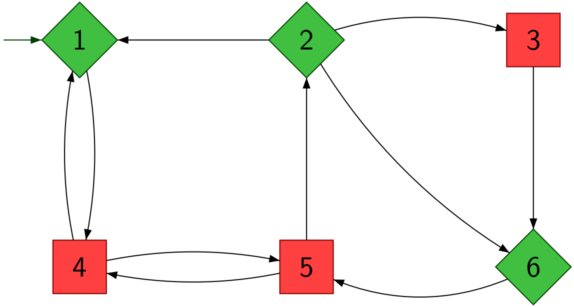
Our game models a system whose task is to provide outputs for given inputs

- States Q model possible states of our system
- Adam's moves model possible inputs
- Eva's moves model possible outputs
- The winning condition models whether Eva responded according to our needs (given by a formula ϕ)

If Eva wins such a game, then it is possible to implement a system which works according to ϕ

Parity games

Given: a parity game (an infinite game using the parity acceptance condition)



Question: who has a winning strategy?

Positional determinacy

A winning condition is determined if one of the players has a winning strategy

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Even on infinite arenas (useful theoretically, e.g. when complementing automata on ω-trees)

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- strategy improvement algorithms (Jurdziński, Vöge '00)
- $n^{O(\sqrt{n})}$ (Jurdziński, Paterson, Zwick '06)

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Parity games: algorithms

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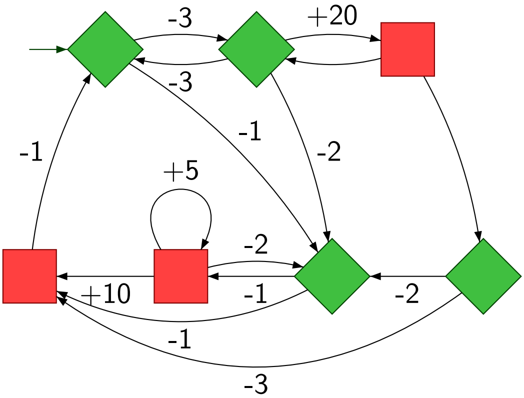
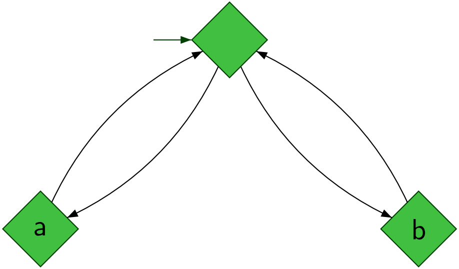
- No polynomial algorithm known yet

Mean payoff game

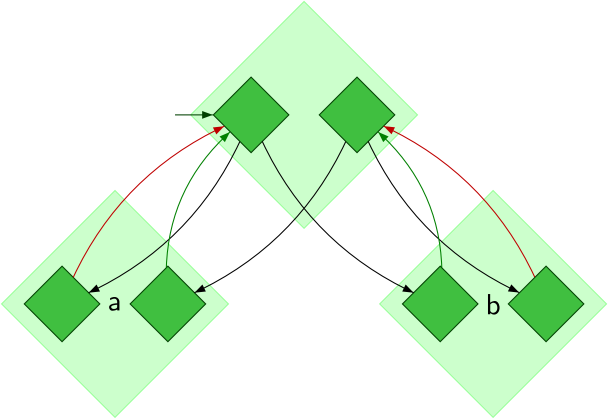


Muller game



Eva wants both a and b to appear infinitely often

Muller game



Eva wants both *a* and *b* to appear infinitely often

Muller game via LAR (DJW '97)

In Muller games we have winning strategies with finite memory:

Eva has a deterministic finite automaton which changes memory states depending on what happens in the game (i.e., the sequence of game states), and her move depends only on the current game state and the current memory state

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Strategies using a small amount of memory are good in practice (useful for automatic synthesis)

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- solvable in single polynomial time

Research problems

- Given a winning condition W, how to effectively decide who wins the game on given arena?
- Can the winner win using a simple strategy (positional, small memory)?
- Are there any characterizations which allow us to immediately tell that games using given winning condition are positionally determined?

Conclusion

Summary

- non-deterministic automata, FO and MSO logic
- alternating automata
- ω-regular languages
- infinite games
- parity games

thank you