## Mathematical Olympiad in Poland
## Second Round, February 25–26, 2000

### First Day

**1.** Prove or disprove the following statement:
Each positive rational number can be written in the form

$$\frac{a^2+b^3}{c^5+d^7}$$

where $a$, $b$, $c$, $d$ are positive integers.

**2.** In the triangle $ABC$ the bisector of the angle $\sphericalangle BAC$ meets the circumcircle of the triangle $ABC$ in the point $D \neq A$. The points $K$ and $L$ are the perpendicular projections of the points $B$ and $C$ onto the line $AC$, respectively. Prove that

$$AD \geq BK + CL\,.$$

**3.** In the unit squares of the $n \times n$ chessboard are written $n^2$ different positive integers. In each column of the chessboard the unit square with the greatest number is coloured red. A set $S$ of $n$ unit squares is called admissible if any two unit squares from $S$ do not lie in the same column or in the same row of the chessboard. Prove that the admissible set with the greatest sum of numbers written in its unit squares contains at least one red unit square.

### Second Day

**4.** Point $I$ is the incentre of the triangle $ABC$ with $AB \neq AC$. The lines $BI$ and $CI$ meet the sides $AC$ and $AB$ in the points $D$ and $E$ respectively. Find all angles $\sphericalangle BAC$ for which the equality $DI = EI$ can be satisfied.

**5.** Let $N$ denote the set of all positive integers. Prove or disprove that:

There exists a function $f: N \to N$ such that the equality $f(f(n)) = 2n$ holds for all $n \in N$.

**6.** Let $w$ be a polynomial of degree two with integer coefficients. Suppose that for each integer $x$ the value $w(x)$ is the square of an integer. Prove that $w$ is the square of a polynomial.