Algebraic Geometry, Fall 2013

Homework, set 4, for January 20th, 2014

All varieties are projective (unless otherwise stated) and defined over an algebraically closed field k.

1. Show that a hypersurface of degree 2 with a singular point is a cone.

2. Prove that if a plane curve of degree 3 has 3 singular points then it breaks up as a union of 3 lines.

3. Let s_i be the i-th elementary symmetric polynomial in $x_0, ..., x_4$, i.e.,

$$s_i = \sum_{j_1 < ... < j_i} x_{j_1} ... x_{j_i}.$$

   Describe the singular points of the intersection of $s_2 = 0$ and $s_4 = 0$ in $\mathbb{P}^4$.

4. Do Exercise 5.3 from Chapter I of Hartshorne's "Algebraic Geometry".

5. Do Exercise 5.4 from Chapter I of Hartshorne's "Algebraic Geometry".