INEQUALITIES FOR MARTINGALE TRANSFORMS AND RELATED CHARACTERIZATIONS OF HILBERT SPACES

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Abstract. Suppose that f is a martingale taking values in a Banach space $\mathbb{B}$ and g is its transform by a deterministic sequence of numbers in $\{-1,1\}$, such that $\sup_n ||g_n|| \geq 1$ almost surely. We show that a certain family of Φ-estimates for f holds true if and only $\mathbb{B}$ is a Hilbert space.

1. Introduction

Martingale theory provides insight into the structure of Banach spaces and is a powerful tool in the study of their properties, as evidenced in the works of many mathematicians, see e.g. Bourgain [1], Burkholder [6], Figiel [7], Godefroy [8], McConnell [12], Pisier [13] and references therein. The purpose of this paper is to investigate the geometry of Banach spaces by means of certain martingale inequalities which, in particular, lead to related characterizations of Hilbert spaces.

We start with introducing the necessary background and notation. Assume that $(\mathbb{B}, ||\cdot||)$ is a real or complex Banach space and let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, equipped with $(\mathcal{F}_n)_{n\geq 0}$, a nondecreasing sequence of sub-σ-algebras of $\mathcal{F}$. Let $f = (f_n)_{n\geq 0}$ be an adapted martingale taking values in $\mathbb{B}$, with the corresponding difference sequence $d = (d_n)_{n\geq 0}$ given by $d_0 = f_0$ and $d_n = f_n - f_{n-1}$ for $n \geq 1$. For any predictable sequence $v = (v_n)_{n\geq 0}$, we say that g is a transform of f by v, if $g_n = \sum_{k=0}^n v_k d_k$ for all $n \geq 0$. Here by predictability we mean that each v_n is measurable with respect to the algebra $\mathcal{F}_{(n-1)\vee 0}$. In the particular case when each term v_n is deterministic and takes values in $\{-1, 1\}$, we will say that g is a ± 1-transform of f. We shall use the notation $||f||_p = \sup_{n\geq 0} ||f_n||_p$ for the p-th moment of f $(1 \leq p \leq \infty)$ and $f^* = \sup_{n\geq 0} ||f_n||$ for the maximal function of f. If f converges almost everywhere, then its pointwise limit will be denoted by f_∞. We say that f is simple, if for any n the term f_n takes only a finite number of values and there is N such that $f_N = f_{N+1} = f_{N+2} = \ldots = f_\infty$ with probability 1.

A Banach space $\mathbb{B}$ is UMD (unconditional for martingale differences), if for some (equivalently, for all) $p \in (1,\infty)$ there is a finite constant $\beta = \beta_p$ such that

$$||g||_p \leq \beta \, ||f||_p$$

for all $\mathbb{B}$-valued martingales f, g such that g is a ± 1-transform of f. Here the filtration must vary as well as the probability space, unless it is assumed to be nonatomic. There is a beautiful geometric characterization of UMD spaces, due to Burkholder [3]. A function $\zeta : \mathbb{B} \times \mathbb{B} \to \mathbb{R}$ is said to be biconvex if both $\zeta(x, \cdot)$

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and $\zeta(\cdot, y)$ are convex for all x, $y \in \mathbb{B}$. Here is a slight modification of the principal result of [3] (see also [2]).

Theorem 1.1. *A Banach space $\mathbb{B}$ is UMD if and only if there is a biconvex function $\zeta : \mathbb{B} \times \mathbb{B} \to \mathbb{R}$ such that $\zeta(0,0) > 0$ and*

$$\zeta(x,y) \leq ||x+y|| \qquad \text{if } ||x|| = ||y|| = 1. \tag{1.1}$$

A biconvex function ζ which satisfies (1.1) must also satisfy $\zeta(0,0) \leq 1$. Indeed, if $||x|| = 1$, then $\zeta(x,x) \leq 2$, $\zeta(x,-x) \leq 0$ and $\zeta(0,0) \leq [\zeta(x,0) + \zeta(-x,0)]/2$, so

$$\zeta(0,0) \leq \big[\zeta(x,x) + \zeta(x,-x) + \zeta(-x,x) + \zeta(-x,-x)\big]/4 \leq 1.$$

This bound can be attained. If $\mathbb{B}$ is a Hilbert space with the inner product $\langle \cdot, \cdot \rangle$ and $\zeta(x,y) = 1 + \mathrm{Re}\langle x, y\rangle$, then ζ is a biconvex function satisfying (1.1) and $\zeta(0,0) = 1$. On the other hand, if $\mathbb{B}$ is UMD, but not a Hilbert space, then $\zeta(0,0) < 1$, as shown by Burkholder [4] and Lee [10]. This fact has a nice probabilistic interpretation (see Theorem 2.1 and Theorem 3.2 in [3]).

Theorem 1.2. *The following conditions are equivalent.*

(i) For any $\mathbb{B}$-valued martingale f, if there is a ± 1-transform g of f which satisfies $g^* \geq 1$ almost surely, then $||f||_1 \geq 1/2$.

(ii) $\mathbb{B}$ is isometric to a Hilbert space.

What can be said if we replace $||f||_1$ above by the p-th moment, or more generally, by the Orlicz norm of f? A partial answer (the implication (ii)$\Rightarrow$(i)) is contained in the following statement. Suppose that Φ is an increasing convex function on $[0,\infty)$ such that Φ is twice differentiable on $(0,\infty)$ and $\Phi(0) = \Phi'(0+) = 0$.

Theorem 1.3. *Suppose that f is a Hilbert-space-valued martingale for which there exists a ± 1-transform g satisfying $g^* \geq 1$ almost surely.*

(i) If Φ' is concave, then

$$\sup_n \mathbb{E}\Phi(||f_n||) \geq \frac{1}{2}\int_0^\infty \Phi(t)e^{-t}\,dt \tag{1.2}$$

and the constant on the right is the best possible, already in the real case.

(ii) If Φ' is convex, then

$$\sup_n \mathbb{E}\Phi(||f_n||) \geq \Phi(1) \tag{1.3}$$

and the constant on the right is the best possible, already in the real case.

For example, if we take $\Phi(t) = t^p$, then for f as in the theorem we have $||f||_p^p \geq \Gamma(p+1)/2$ for $1 \leq p \leq 2$, $||f||_p \geq 1$ for $p \geq 2$, and both estimates are sharp.

The first part of the theorem above is due to Burkholder; the proof in [5] concerns real martingales, but it can be easily transfered to the Hilbert setting. The second assertion is probably well known but, as we have not found any reference, we shall include an easy proof for the sake of completeness.

Motivated by (1.2) and (1.3), we establish the following result on UMD spaces (consult [3] and, for the reverse statement concerning Paley-Walsh martingales, see the proof of Theorem 3.1 in [4]).

Theorem 1.4. *Let Φ be as above and suppose that there is $c_\Phi > 0$ such that the following holds: we have $\sup_n \mathbb{E}\Phi(||f_n||) \geq c_\Phi$ for any $\mathbb{B}$-valued f which admits a ± 1-transform g with $g^* \geq 1$ almost surely. Then $\mathbb{B}$ is a UMD space.*

By the previous theorem, if Φ' is concave or convex, then the largest possible choice for c_Φ is given by the right-hand side of (1.2) or (1.3). If Φ is nontrivial (see below), this choice characterizes Hilbert spaces in the following sense.

Theorem 1.5. *Assume that $\mathbb{B}$ is* not *a Hilbert space and let Φ be as above.*

(i) If Φ' is concave and $\Phi \not\equiv 0$, then there is a $\mathbb{B}$-valued martingale f and its ± 1-transform g satisfying $g^* \geq 1$ almost surely, but

$$\sup_n \mathbb{E}\Phi(|f_n|) < \frac{1}{2}\int_0^\infty \Phi(t)e^{-t}\,dt. \tag{1.4}$$

(ii) If Φ' is convex and $\Phi'(1) > 0$, then there is a $\mathbb{B}$-valued martingale f and its ± 1-transform g satisfying $g^* \geq 1$ almost surely, but

$$\sup_n \mathbb{E}\Phi(|f_n|) < \Phi(1). \tag{1.5}$$

Obviously, neither of the conditions $\Phi \not\equiv 0$, $\Phi'(1) > 0$ can be removed, because the right-hand sides of (1.4), (1.5) must be strictly positive.

The results announced above will be proved in the next section. Surprisingly, parts (i) and (ii) of Theorem 1.5 require completely different arguments. The proof of the first part rests on Burkholder's biconvex characterization of Hilbert spaces, while the second is dealt with directly, using some basic facts from convex geometry.

2. Proofs

2.1. **Proof of** (1.3)**.** Of course, the sharpness of the estimate is clear. Pick f, g as in the statement and note that we may assume that both martingales are simple. In addition, we may restrict ourselves to f satisfying $\sup_n \mathbb{E}\Phi(||f_n||) < \infty$, since otherwise there is nothing to prove. If Φ is nonzero (as we may assume), this implies that f, and hence also g, are bounded in L^2: by the convexity of Φ', there are $a > 0$ and $b \in \mathbb{R}$ such that $\Phi(x) \geq ax^2 + b$ for all $x \geq 0$.

Let $\varepsilon \in (0,1)$ and consider a stopping time $\tau = \inf\{n \geq 0 : ||g_n|| \geq 1 - \varepsilon\}$. Then $\tau < \infty$ because $g^* \geq 1$ almost surely. Next, for any integer n we have $\mathbb{E}||f_{\tau\wedge n}||^2 = \mathbb{E}||g_{\tau\wedge n}||^2$, since $\mathbb{B}$ is a Hilbert space. Observe that the function $x \mapsto \Phi(\sqrt{x})$, $x \geq 0$, is convex: its second derivative at $x > 0$ equals $x^{-3/2}(\Phi''(\sqrt{x})\sqrt{x} - \Phi'(\sqrt{x}))/4$, which is nonnegative. Therefore $\Phi(\sqrt{s}) - \Phi(1) \geq \frac{1}{2}\Phi'(1)(s-1)$ for all $s \geq 0$ and

$$\begin{aligned}
\Phi(1) &= \Phi(1) + \frac{1}{2}\Phi'(1)\big(||f_{\tau\wedge n}||_2^2 - ||g_{\tau\wedge n}||_2^2\big)\\
&= \mathbb{E}\left[\Phi(1) + \frac{1}{2}\Phi'(1)(||f_{\tau\wedge n}||^2 - 1) + \frac{1}{2}\Phi'(1)(1 - ||g_{\tau\wedge n}||^2)\right]\\
&\leq \mathbb{E}\Phi(||f_{\tau\wedge n}||) + \Phi'(1)(1 - ||g_{\tau\wedge n}||_2^2) \leq \sup_k \mathbb{E}\Phi(||f_k||) + \Phi'(1)(1 - ||g_{\tau\wedge n}||_2^2),
\end{aligned}$$

where the latter bound follows from Jensen inequality. Let $n \to \infty$ to obtain $\Phi(1) \leq \sup_n \mathbb{E}\Phi(||f_n||) + 2\Phi'(1)\varepsilon$ and take $\varepsilon \to 0$ to get the claim.

2.2. **A zigzag martingale and a biconvex function.** Suppose that $Z = (Z_0, Z_1, Z_2, \ldots)$ is a martingale with values in $\mathbb{B} \times \mathbb{B}$ and let us write $Z_n = (X_n, Y_n)$, where both X_n and Y_n have their values in $\mathbb{B}$. We say that Z is *zigzag* if for any positive integer n, either $X_{n+1} - X_n \equiv 0$ or $Y_{n+1} - Y_n \equiv 0$. For example, if f is a $\mathbb{B}$-valued martingale with difference sequence d and g is a transform of f by a sequence v of numbers in $\{-1,1\}$, then

$$X_n = f_n + g_n = \sum_{k=0}^{n}(1+v_k)d_k, \qquad Y_n = f_n - g_n = \sum_{k=0}^{n}(1-v_k)d_k,$$

$n = 0, 1, 2, \ldots$, define a zigzag martingale Z. For any $(x,y) \in \mathbb{B}\times\mathbb{B}$, let $\mathbf{Z}(x,y)$ denote the class of all simple zigzag martingales $Z = (X, Y)$ satisfying $Z_0 \equiv (x,y)$ and $||X_\infty - Y_\infty|| \geq 2$ almost surely. Introduce the function $\eta : \mathbb{B}\times\mathbb{B} \to \mathbb{R}$ by

$$\eta(x,y) = \inf\big\{\mathbb{E}\Phi(||X_\infty + Y_\infty||/2) : Z \in \mathbf{Z}(x,y)\big\}.$$

Using the "splicing argument"of Burkholder (see page 77 in [2]), one shows that η is a biconvex function. For a fixed odd integer $N = 2n+1$, a number $\delta \in (0,1/2)$ and a pair $(x,y) \in \mathbb{B}\times\mathbb{B}$ with $||x|| = ||y|| = 1$, we shall construct an important zigzag martingale $Z = (Z_0, Z_1, Z_2, \ldots, Z_{N-1}, Z_N, Z_N, \ldots) \in \mathbf{Z}(x,y)$. Consider two sequences $(r_k)_{k\geq 1}$, $(s_k)_{k\geq 1}$ of independent centered random variables such that

$$r_{2k+1} \equiv 0, \qquad \mathbb{P}(s_{2k+1} = -x-y) = 1 - \mathbb{P}(s_{2k+1} = \delta(x+y)) = \frac{\delta}{1+\delta},$$

$$s_{2k+2} \equiv 0, \qquad \mathbb{P}(r_{2k+2} = (x+y)(\delta-1)) = 1 - \mathbb{P}(r_{2k+2} = \delta(x+y)) = \delta$$

for $k = 0, 1, 2, \ldots$, and let ε be an independent Rademacher variable. Define $\tau = \inf\{k \geq 1 : r_k = (x+y)(\delta-1) \text{ or } s_k = -x-y\}$,

$$Z_k = \big(x + r_1 + r_2 + \ldots + r_{\tau\wedge k}, y + s_1 + s_2 + \ldots + s_{\tau\wedge k}\big)$$

for $k = 0, 1, 2, \ldots, N-1$, and, for $k \geq N$,

$$Z_k = \big(x + r_1 + \ldots + r_{\tau\wedge(N-1)}, y + s_1 + \ldots + s_{\tau\wedge(N-1)} + \varepsilon(x+y)1_{\{\tau\geq N\}}\big).$$

To gain some intuition about Z, observe that Z starts from (x,y); at the first step, it moves either to $(x,-x)$ (and stays there forever), or to $(x, y+\delta(x+y))$. If the second possibility occurs, then Z moves either to $(\delta(x+y)-y, y+\delta(x+y))$ (and stops), or to $(x+\delta(x+y), y+\delta(x+y))$. If the latter happens, then the pattern of the movement is repeated; in particular, after $2k$ steps ($k \leq n$), the process reaches the set $F = \{(u,v) \in \mathbb{B}\times\mathbb{B} : ||u-v|| = 2\}$ or takes the value $(x+k\delta(x+y), y+k\delta(x+y))$. Finally, at the N-th move, if $||X_{N-1} - Y_{N-1}|| = ||X_{2n} - Y_{2n}|| \neq 2$, then Z_N goes to $(x+n\delta(x+y), -x+n\delta(x+y))$ or to $(x+n\delta(x+y), x+2y+n\delta(x+y))$, and both points belong to F. Consequently, we see that $Z \in \mathbf{Z}(x,y)$ and $\tau\wedge N$ is the first moment when Z enters the set F.

Directly from the probabilities defining the distribution of the sequences $(r_k)_{k\geq 1}$ and $(s_k)_{k\geq 1}$, we infer that $\mathbb{P}(||X_\infty + Y_\infty|| = 0) = \mathbb{P}(\tau = 1) = \delta/(1+\delta)$,

$$\mathbb{P}(||X_\infty + Y_\infty|| = 2k\delta||x+y||) = \mathbb{P}(\tau \in \{2k, 2k+1\}) = \frac{2\delta}{(1+\delta)^2}\cdot\left(\frac{1-\delta}{1+\delta}\right)^{k-1}$$

for $k = 1, 2, \ldots, n-1$, and

$$\mathbb{P}(||X_\infty + Y_\infty|| = 2n\delta||x+y||) = \mathbb{P}(\tau \geq 2n) = \frac{1}{1+\delta}\left(\frac{1-\delta}{1+\delta}\right)^{n-1}.$$

In consequence, we have

$$\eta(x,y) \leq \sum_{k=1}^{n-1}\Phi(n\delta||x+y||)\cdot\frac{2\delta}{(1+\delta)^2}\left(\frac{1-\delta}{1+\delta}\right)^{k-1} + \Phi(n\delta||x+y||)\cdot\frac{1}{1+\delta}\left(\frac{1-\delta}{1+\delta}\right)^{n-1}.$$

Now we treat the right-hand side as a Riemann sum and let $\delta \to 0$, $n \to \infty$ to get

$$\eta(x,y) \leq \int_0^\infty \Phi(t||x+y||/2)e^{-t}\mathrm{d}t \leq \frac{||x+y||}{2}\int_0^\infty \Phi(t)e^{-t}\mathrm{d}t. \tag{2.1}$$

The latter bound follows from the observation that the function

$$\psi(t) = \Phi\left(\frac{t||x+y||}{2}\right) - \frac{||x+y||}{2}\Phi(t), \qquad t \geq 0,$$

is nonpositive. Indeed, $\psi(0) = 0$ and

$$\psi'(t) = \frac{||x+y||}{2}\left[\Phi'(t||x+y||/2) - \Phi'(t)\right] \leq 0,$$

because $||x+y|| \leq ||x|| + ||y|| = 2$.

Proof of Theorem 1.4. Observe that we have

$$\eta(0,0) \geq c_\Phi. \tag{2.2}$$

To prove this, pick $Z = (X,Y) \in \mathbf{Z}(0,0)$ and note that $g = (X-Y)/2$ is a ±1-transform of $f = (X+Y)/2$. Since $||X_\infty - Y_\infty|| \geq 2$ almost surely, we have $\mathbb{P}(g^* \geq 1) = 1$ and hence, by the assumptions of Theorem 1.4,

$$\mathbb{E}\Phi(||X_\infty + Y_\infty||/2) \geq c_\Phi > 0.$$

This yields (2.2), since $Z \in \mathbf{Z}(0,0)$ was arbitrary. Now let $\zeta : \mathbb{B}\times\mathbb{B} \to \mathbb{R}$ be given by $\zeta(x,y) = 2\eta(x,y)/\int_0^\infty \Phi(t)e^{-t}\mathrm{d}t$. Then ζ is a biconvex function satisfying $\zeta(0,0) > 0$ and, by (2.1), $\zeta(x,y) \leq ||x+y||$ for $||x|| = ||y|| = 1$. Thus, $\mathbb{B}$ is UMD. □

Proof of Theorem 1.5 (i). Suppose that the claim does not hold. Then, using the same argumentation as previously, we get the following version of (2.2):

$$\eta(0,0) \geq \frac{1}{2}\int_0^\infty \Phi(t)e^{-t}\mathrm{d}t.$$

Thus, the function ζ just defined above not only satisfies (1.1), but also $\zeta(0,0) \geq 1$. In consequence, by Burkholder's characterization theorem, $\mathbb{B}$ is a Hilbert space. □

2.3. **Proof of Theorem 1.5 (ii).** We shall need the following two well-known lemmas from the theory of convex bodies (see e.g. Istrăţescu [9]).

Lemma 2.1. *Let $\mathbb{B}$ be a two-dimensional real Banach space. Then the norm of $\mathbb{B}$ comes from an inner product if and only if the unit sphere of $\mathbb{B}$ is an ellipse.*

Lemma 2.2. *If C is symmetric (with respect to the origin) closed convex curve in the plane, then there is a unique ellipse of maximal area inscribed in C. The maximal ellipse touches C in at least four points which are symmetric pairwise.*

We turn to the proof. We may assume that $\mathbb{B}$ is a real Banach space of dimension two. Let $S_\mathbb{B}$ denote the unit sphere of $\mathbb{B}$ and let S_0 be the ellipse of maximal area inscribed in $S_\mathbb{B}$, with at least four distinct contact points, symmetric pairwise. We may take S_0 to be the unit circle, applying affine transformation if necessary. Let $|\cdot|$ be the norm induced by S_0 and denote by A, C be two contact points, with no contact points in the interior of the shorter arc AC. Using rotation and reflection, we may assume that $A = (1,0)$ and $C = (\cos 2\theta, \sin 2\theta)$ for some $\theta \in (0,\pi/2)$. Let $D = s(\cos\theta, \sin\theta)$, where $s > 1$ is such that $||D|| = 1$. Consider the probability space $([0,1], \mathcal{B}(0,1), \mathbb{P})$, where $\mathbb{P}$ is the Lebesgue's measure. For a given $0 < r < 1/2$, define the mean zero $\mathbb{B}$-valued random variable $\xi = x + A1_{[0,r)} + C1_{[r,2r)} - D1_{[2r,1]}$, with $x = -w(\cos\theta, \sin\theta)$ and $w = 2r\cos\theta + s(2r-1) \in (-s, \cos\theta)$. Let f, g be two $\mathbb{B}$-valued martingales given by $f_0 = -g_0 = x$, $f_1 = f_2 = \ldots = x + \xi$, $g_1 = g_2 = \ldots = -x + \xi$. Then g is a transform of f by $v = (-1, 1, 1, 1, \ldots)$ and $g_1 \in \{A, C, -D\}$, so $g^* \geq 1$ almost surely. On the other hand,

$$\begin{aligned}
\sup_n \mathbb{E}\Phi(||f_n||) &= \mathbb{E}\Phi(||x+\xi||)\\
&= r\Phi(||2x+A||) + r\Phi(||2x+C||) + (1-2r)\Phi(||2x-D||)\\
&\leq r\Phi(|2x+A|) + r\Phi(|2x+C|) + (1-2r)\Phi(||2x-D||)\\
&= 2r\Phi(\sqrt{1-4w\cos\theta+4w^2}) + (1-2r)\Phi\left(\frac{s+2w}{s}\right)\\
&= \frac{w+s}{s+\cos\theta}\Phi(\sqrt{1-4w\cos\theta+4w^2}) + \left(1 - \frac{w+s}{s+\cos\theta}\right)\Phi\left(\frac{s+2w}{s}\right).
\end{aligned}$$

Denoting the right-hand side by $G(w)$, we compute that $G(0) = \Phi(1)$ and

$$G'(0) = \frac{2\Phi'(1)(-s^2+1)\cos\theta}{s(s+\cos\theta)} < 0.$$

This implies $\sup_n \mathbb{E}\Phi(||f_n||) < \Phi(1)$ for small $w > 0$. The claim is proved.

References

[1] J. Bourgain, *Some remarks on Banach spaces in which martingale difference sequences are unconditional*, Ark. Mat. **21** (1983), 163–168.

[2] D. L. Burkholder, *Martingales and Fourier analysis in Banach spaces*, Probability and Analysis (Varenna, 1985) Lecture Notes in Math. **1206**, Springer, Berlin (1986), pp. 61–108.

[3] D. L. Burkholder, *A geometrical characterization of Banach spaces in which martingale difference sequences are unconditional*, Ann. Probab. 9 (1981), 997-1011.

[4] D. L. Burkholder, *Martingale transforms and geometry of Banach spaces*, Proceedings of the Third International Conference on Probability in Banach Spaces, Tufts University, 1980, Lecture Notes in Math., 860 (1981), 35-50.

[5] D. L. Burkholder, *Boundary value problems and sharp inequalities for martingale transforms*, Ann. Probab. **12** (1984), 647–702.

[6] D. L. Burkholder, *Martingales and singular integrals in Banach spaces*, Handbook of the Geometry of Banach Spaces, Vol. 1, 2001, pp. 233-269.

[7] T. Figiel, *Singular integral operators: a martingale approach*, Geometry of Banach spaces (Strobl, Austria, 1989), London Math. Soc. Lecture Notes Series 158, Cambridge University Press, Cambridge (1990), 95–110.

[8] G. Godefroy, *Renorming of Banach spaces*, Handbook of the Geometry of Banach Spaces, Vol. 1, W. B. Johnson and J. Lindenstrauss, eds. Elsevier, Amsterdam (2001), 781–835.

[9] V. I. Istrăţescu, *Inner product structures*, Reidel, Boston, MA, 1987.

[10] J. M. Lee, *On Burkholder's biconvex-function characterization of Hilbert spaces*, Proc. Amer. Math. Soc. **118** (1993), no. 2, 555–559.

[11] B. Maurey, *Système de Haar*, Séminaire Maurey-Schwartz, 1974-1975, École Polytechnique, Paris (1975).

[12] T. R. McConnell, *On Fourier multiplier transformations of Banach-valued functions*, Trans. Amer. Math. Soc. **285** (1984), 739–757.

[13] G. Pisier, *Martingales with values in uniformly convex spaces*, Israel J. Math. **20** (1975), 326–350.

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